Optimal Control of Rotary Crane Using the Straight Transfer Transformation Method to Eliminate Residual Vibration[†]

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This paper presents a model based on the load position of a rotary crane, validated by experimental results. Specifically, it provides a method for controlling the sway of a rotary crane using a straight transfer transformation (STT) model. The STT model was built and its parameters were geometrically derived. Taking into account variations in rope length, the optimal control of a rotary crane for reduction of residual sway was solved using the Davidon-Fletcher-Powell (DFP) optimization method. The proposed control method using the STT model was demonstrated to be effective in eliminating the influence of centrifugal force through simulation and experiments.

Key Words: rotary crane, modeling, nonlinear optimal control, vibration control, DFP method

1. Introduction

Rotary cranes are indispensable machines for transferring loads in factories, construction sites, harbors and so on. The fundamental motions of a rotary crane are rotation, boom hoisting and load lifting. Fig.1 shows a laboratory model of a rotary crane. The load sway grows from the centrifugal force that rotary motion induces. As shipping cost is related to a ship's anchorage time, it is desirable to minimize anchorage time by eliminating sway at the end of a load transfer.

Fig. 1 A laboratory apparatus of a rotary crane built in this paper

Many studies on controlling the sway of rotary crane loads have been published. Sakawa and Nakazumi applied the open-loop plus feedback control scheme to let the sway of the load decay at the end point of transfer¹⁾.

Kimiaghalam et al. developed a state-space model of the crane from an implicit description without simplifying assumptions, and they designed a fuzzy controller²⁾. Souissi and Koivo discussed the problem in terms of controlling the crane load's oscillations³⁾. Stefan et al. presented a feedback method to control the load rotation⁴⁾. Chung et al. used the method of algebraic equations of motion to solve the optimal problem of load swing⁵⁾. But these papers did not consider the condition of simultaneous rotary motion and boom hoisting motion. Using a rotary crane, Yamazaki et al. presented the idea of a straight transfer without centrifugal force that makes the crane tip and the load move on a straight line in the X-Y coordinates by the simultaneous motions of rotation and boom hoisting⁶⁾. Although straight transfer is limited to working spaces within 180 degrees, it is obvious that straight transfer by a rotary crane uses less space to transfer a load from starting point to end point than rotational transfer. In the literature⁶⁾, the problem of bang-bang minimum time control was only considered for a constant rope length by the optimum technique of open-loop inputs. Further, the advantage of the straight transfer method has not been discussed from the practical viewpoint of calculation time.

Hence, this paper applies a general optimal control method to handle not only minimum time control but also the general control problem. The focus is on the motion of a load position in the X-Y coordinates for a rotary crane with a straight transfer, accompanied by a rope length change during the load's transfer.

First, a nonlinear model of the load position and a tip position of the rotary crane was built in three-dimensional space. The model allowed us to determine the optimum

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Fig. 2 A schematic diagram of the rotary crane

control input for moving a load position along a straight line, i.e., a straight transfer, if the performance index is given such that Eq. (12) is satisfied, for example. When this nonlinear model built in three-dimensional space was used, an enormous calculation was needed to determine the optimum control input to eliminate residual vibration, due to the model's centrifugal force.

For the straight transfer, a nonlinear model of the swing of a load and the tip position of a boom was embedded in two-dimensional space. When a load is transferred by two-dimensional transformation, the boom tip's trajectory draws a circle in two-dimensional space. Therefore an imaginary boom angle, an imaginary boom length and an imaginary swing angle of the load are created, as explained in the next section. In the present paper, this embedded model in two-dimensional space is called a straight transfer transformation (STT) model. In actual sites, it is sometimes desired that the load not change its distance from the ground. This means that the load should be moved downwards in order to keep the load position at a constant height from the ground while the boom tip moves upwards. For example, the load is sometimes required to be moved as ordered by a human. This requires varying the rope length to compensate for the boom tip elevation. We are the first to incorporating this point into our research. The optimum control problem can then be resolved using the Davidon-Fletcher-Powell (DFP) method for the STT model considering variations in rope length. It is shown that actual control inputs for a rotary crane can be obtained by transforming the coordination system from two-dimensional imaginary coordination in the STT model into the three-dimensional coordination of the actual rotary crane.

Finally, the validity of the model and the effectiveness of this method were evaluated by simulation and experimental results.

Table 1 Symbolic notation in Fig.2

Symbol	Unit	Explanation (Values)
θ	rad	rotary angle
$\tilde{\alpha}$	rad	swing angle on XY plane
ϕ	rad	boom angle
$\tilde{\beta}$	rad	swing angle from Z direction
l	m	rope length
r	m	radius of the drum (0.049)
L_B	m	length of the boom (0.845)
H	m	height of the turn table (0.627)
m	kg	mass of the load (1.5)
g	m/s ²	gravity (9.8)
$(\tilde{x}, \tilde{y}, \tilde{z})$	m	position of the crane tip
F	N	tension of the rope
(x, y, z)	m	position of the load
l_{b1}	m	length of line OA (0.74)
ρ	rad	angle between OA and boom (0.118)
l_{m1}	m	length of line OB (0.573)
γ_1	rad	angle between OB and axis Z (0.5)
l_{m2}	m	length of line OC (0.435)
γ_2	rad	angle between OC and axis Z (0.557)

2. Load Position (LP) Model of Rotary Crane

2.1 Derivation of Load Position Model

The laboratory experimental rotary crane built in this paper is shown in Fig.1 and Fig.2.

For simplicity, the following assumptions were made:

- The body of the crane can be regarded as a rigid body, and the load can be regarded as a point mass.
- Frictional torques which may exist in torque-transfer mechanisms can be neglected.
- Weight and elongation of ropes due to tensile force are neglected.
- The driving motor used in this paper is assumed to have enough power for the weight of the load.

The properties of the driving motors are regarded as the following systems:

$$\ddot{\theta} = -\frac{1}{T_\theta} \dot{\theta} + \frac{K_\theta}{T_\theta} u_\theta \quad (1)$$

$$E_\phi \ddot{\phi} = A_\phi \dot{\phi} + B_\phi u_\phi + F_\phi \dot{\phi}^2 \quad (2)$$

where

$$E_\phi = -\frac{l_{m1}}{\sqrt{L_{x1}}} \cos(\phi + \rho - \gamma_1) - \frac{l_{m2}}{\sqrt{L_{x2}}} \cos(\phi + \rho - \gamma_2)$$

$$A_\phi = -\frac{E_\phi}{T_\phi}, \quad B_\phi = \frac{rK_\phi}{l_{b1}T_\phi}$$

$$\begin{aligned}
F_\phi &= \frac{l_{m1}^2 l_{b1}}{L_{x1} \sqrt{L_{x1}}} \cos^2(\phi + \rho - \gamma_1) \\
&+ \frac{l_{m1}}{\sqrt{L_{x1}}} \sin(\phi + \rho - \gamma_1) \\
&+ \frac{l_{m2}^2 l_{b1}}{L_{x2} \sqrt{L_{x2}}} \cos^2(\phi + \rho - \gamma_2) \\
&+ \frac{l_{m2}}{\sqrt{L_{x2}}} \sin(\phi + \rho - \gamma_2)
\end{aligned}$$

$$L_{x1} = l_{m1}^2 + l_{b1}^2 - 2l_{m1}l_{b1} \sin(\phi + \rho - \gamma_1)$$

$$L_{x2} = l_{m2}^2 + l_{b1}^2 - 2l_{m2}l_{b1} \sin(\phi + \rho - \gamma_2)$$

$$\ddot{l} = -\frac{1}{T_l} \dot{l} + \frac{K_l}{T_l} u_l \quad (3)$$

Here, T_θ , T_ϕ and T_l are respectively the time constant of the rotary motor, the hoisting motor for the boom and the lifting motor for the load. K_θ , K_ϕ and K_l are respectively the gain of each motor, u_θ is the rotary input voltage, u_ϕ is the hoisting input voltage, and u_l is the load lifting input voltage.

The model based on load position is comprised of Eqs.(1) ~ (3), (A. 16), (A. 17) and (A. 20) ~ (A. 23). The derivation of the load position model is described in Appendix A.

Fig. 3 Measurement of swing angle

When the load is transferred, the swing angles α' and β' can be measured by the two pairs of forks and encoders, as shown in Fig.3. It must be noted that in the simulation program, the swing angles of the load are α and β . According to Fig.3, the relationship between (α, β) and the load position (x, y, z) , as well as the position of the crane tip $(\tilde{x}, \tilde{y}, \tilde{z})$, can be obtained as follows:

$$\alpha = \sin^{-1} \left(\frac{x - \tilde{x}}{\sqrt{l^2 - (y - \tilde{y})^2}} \right) \quad (4)$$

Table 2 Restricted condition for the actual apparatus

Condition	V_{max}	v_{max}	a_{max}
Rotary	± 10 V	± 1 rad/s	± 2 rad/s ²
Hoisting	± 10 V	± 0.3 rad/s	± 2 rad/s ²
Rope Lifting	± 10 V	± 0.12 m/s	± 1 m/s ²

$$\beta = \sin^{-1} \left(\frac{y - \tilde{y}}{\sqrt{l^2 - (x - \tilde{x})^2}} \right) \quad (5)$$

The measured swing angles from the sensors are α' and β' . From Fig.3, the relationship between (α, β) and (α', β') can be obtained.

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} \quad (6)$$

The restricted conditions for the laboratory apparatus are shown in Table 2. Here V_{max} means the maximum voltage, v_{max} means the maximum velocity, and a_{max} means the maximum acceleration.

In order to check the validity of the above model, a series of experiments were carried out. Due to the limitations of space, only one result is given here.

Fig.4 shows the results of experiment and simulation with rotation from $\theta_i = 0^\circ$ to $\theta_f = 90^\circ$, boom hoisting from $\phi_i = 30^\circ$ to $\phi_f = 30^\circ$, and rope length varying from $l_i = 0.9$ m to $l_f = 0.9$ m. Three motions are performed synchronously. The input voltage of u_θ , u_ϕ and u_l are the trapezoidal voltage. Considering the restricted conditions, the transfer time was decided to be $t_c = 7.0$ s. The solid line indicates the experiment results and the dashed line indicates the results of simulation. Fig.5 shows the locus of the load on the horizontal surface. From Fig.4 and Fig.5, it is clear that the experimental result is almost the same as the simulation result. The validity of the model based on load position is thus demonstrated.

2.2 Optimal Control Strategy

In this paper, a popular approach to optimal control is applied, and summarized as follows:

Consider the nonlinear dynamical system described by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad (7)$$

where $\dot{\mathbf{x}} = (x_1, \dots, x_n)^T$ is a state vector, $\mathbf{f} = (f_1, \dots, f_n)$ is a nonlinear n-vector function, and $\mathbf{u}(t) = (u_1, \dots, u_m)$ is a control input m-vector.

The performance index (or cost function) is given by

$$J = \mathbf{g}(t_f, \mathbf{x}(t_f)) + \int_{t_0}^{t_f} \mathbf{f}_0(t, \mathbf{x}(t), \mathbf{u}(t)) dt \quad (8)$$

where t_0 is an initial time, and t_f is a final time. If a Hamiltonian is given by

$$\mathbf{H} = -\mathbf{f}_0(t, \mathbf{x}(t), \mathbf{u}(t)) + \mathbf{p}^T(t) \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (9)$$

Fig. 4 Experimental and simulation results on validity of load position model

Fig. 5 Locus of the load and the crane tip on xy plane in the case of Fig.4

then an adjoint equation is derived as

$$\left. \begin{aligned} \dot{\mathbf{p}}(t) &= -\left(\frac{\partial \mathbf{H}}{\partial \mathbf{x}}\right)^T = -\mathbf{f}_x^T(t, \mathbf{x}(t), \mathbf{u}(t))\mathbf{p}(t) \\ &\quad + \mathbf{f}_{0x}(t, \mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{p}(t_f) &= \frac{\partial g(t_f, \mathbf{x}(t_f))}{\partial \mathbf{x}(t_f)} \equiv \mathbf{p}_f \end{aligned} \right\} \quad (10)$$

where $\mathbf{f}_x \equiv (\partial \mathbf{f})^T / (\partial \mathbf{x})$, and $\mathbf{f}_{0x} \equiv (\partial \mathbf{f}_0)^T / (\partial \mathbf{x})$

The gradient of the performance index J is defined by

$$\mathbf{J}_{\mathbf{u}} = -\frac{\partial \mathbf{H}}{\partial \mathbf{u}} = -\mathbf{f}_{\mathbf{u}}(t, \mathbf{x}(t), \mathbf{u}(t))\mathbf{p}(t) + \mathbf{f}_{0\mathbf{u}}(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (11)$$

Therefore, an optimal control problem is reduced to a two-point boundary value problem constructed of Eqs.(7) and (10). The optimal solution can be numerically obtained on a fixed time interval using the DFP method⁷⁾,⁸⁾, the Fletcher-Reeves (FR) method, and so on. Because the DFP method is said to be one of the most effective methods at present, it was adopted as the optimization technique in this paper. A comparison between the DFP

and FR methods is presented in Section 4. The procedure for obtaining an optimal solution is given in the following:

[Procedure of obtaining an optimal solution for nonlinear system by means of the DFP method:]

(Step1) Given the initial control $\mathbf{u}_0(t)$ on time interval $t \in [t_0, t_f]$ and let $\mathbf{B}_0 = \mathbf{I}$.

(Step2) Obtain the solution of Eq.(7) using $\mathbf{u}_0(t)$, and then $\mathbf{x}_0(t) = \mathbf{x}(t, \mathbf{u}_0)$. Using $\mathbf{u}_0(t)$ and $\mathbf{x}_0(t)$, solve $\mathbf{p}_0(t)$ by Eq.(10). Then, calculate $\mathbf{J}_{\mathbf{u}_0}$ as the gradient of the performance index. Let $\mathbf{g}_0 = \mathbf{J}_{\mathbf{u}_0}$, $\mathbf{d}_0 = -\mathbf{g}_0$ and $i=1$.

(Step3) Choose α to minimize $J(\mathbf{u}_{i-1} + \alpha \mathbf{d}_{i-1})$. Let α be α_{i-1} and $\mathbf{u}_i = \mathbf{u}_{i-1} + \alpha_{i-1} \mathbf{d}_{i-1}$.

(Step4) Memory $\mathbf{s}_{i-1} = \mathbf{u}_i - \mathbf{u}_{i-1}$, $\mathbf{g}_{i-1}$ and $\mathbf{s}_{i-1}$.

(Step5) Use $\mathbf{u}_i(t)$ to calculate $\mathbf{x}_i(t)$ and $\mathbf{p}_i(t)$ until $J_{\mathbf{u}_i} < \mathbf{E}$, where $\mathbf{E} = [\epsilon, \dots, \epsilon]^T$ is a specified vector positive number (Here, $\epsilon = 10^{-6}$). Otherwise let $\mathbf{g}_i = \mathbf{J}_{\mathbf{u}_i}$, and memory $\mathbf{y}_{i-1} = \mathbf{g}_i - \mathbf{g}_{i-1}$.

(Step6) If $i=1$, then go to (Step7), otherwise

$$\mathbf{B}_{i-1} \mathbf{y}_{i-1} = \mathbf{y}_{i-1} +$$

$$\sum_{k=0}^{i-2} \left[\frac{(\mathbf{s}_k, \mathbf{y}_{i-1})}{(\mathbf{y}_k, \mathbf{y}_k)} \mathbf{s}_k - \frac{(\mathbf{B}_k \mathbf{y}_k, \mathbf{y}_{i-1})}{(\mathbf{y}_k, \mathbf{B}_k \mathbf{y}_k)} \mathbf{B}_k \mathbf{y}_k \right], i > 1$$

and memory $\mathbf{B}_{i-1} \mathbf{y}_{i-1}$.

(Step7)

Calculate

$$\mathbf{d}_i = -\mathbf{g}_i - \sum_{k=0}^{i-1} \left[\frac{(\mathbf{s}_k, \mathbf{g}_i)}{(\mathbf{y}_k, \mathbf{y}_k)} \mathbf{s}_k - \frac{(\mathbf{B}_k \mathbf{y}_k, \mathbf{g}_i)}{(\mathbf{y}_k, \mathbf{B}_k \mathbf{y}_k)} \mathbf{B}_k \mathbf{y}_k \right]$$

(Step8) let $i=i+1$, then return to (Step3). $\square$

(Remark) $(y_k, y_k) = \int_{t_0}^{t_f} y_k^T y_k dt$

[Procedure of obtaining an optimal solution for nonlinear system by means of the FR method:]

(Step1) Given the initial control $\mathbf{u}_0(t)$ on time interval $t \in [t_0, t_f]$.

(Step2) Obtain the solution of Eq.(7) by using $\mathbf{u}_0(t)$, then $\mathbf{x}_0(t) = \mathbf{x}(t, \mathbf{u}_0)$. Using $\mathbf{u}_0(t)$, $\mathbf{x}_0(t)$, solve $\mathbf{p}_0(t)$ by Eq.(10). Then, calculate $\mathbf{J}_{\mathbf{u}_0}$ as the gradient of the performance index. Let $\mathbf{s}_0 = -\mathbf{J}_{\mathbf{u}_0}$, and $i = 0$.

(Step3) If $i=0$, go to (Step4). If $i \geq 1$, calculate $\beta_i = (J_{\mathbf{u}_i}, J_{\mathbf{u}_i}) / (J_{\mathbf{u}_{i-1}}, J_{\mathbf{u}_{i-1}})$, and let $\mathbf{s}_i = -\mathbf{J}_{\mathbf{u}_i} + \beta_i \mathbf{s}_{i-1}$.

(Step4) Choose α to minimize $J(\mathbf{u}_i + \alpha \mathbf{s}_i)$. Let α be α_i and $\mathbf{u}_{i+1} = \mathbf{u}_i + \alpha_i \mathbf{s}_i$.

(Step5) Similar to (Step2), let $\mathbf{x}_{i+1}(t) = \mathbf{x}(t : \mathbf{u}_{i+1})$ and solve $\mathbf{p}_{i+1}(t) = \mathbf{p}(t : \mathbf{u}_{i+1})$. Then, calculate $J_{\mathbf{u}_{i+1}}$.

(Step6) Stop if $J_{\mathbf{u}_{i+1}} < \mathbf{E}$, where $\mathbf{E} = [\epsilon, \dots, \epsilon]^T$ is a specified vector positive number (Here, $\epsilon = 10^{-6}$). Otherwise let $i=i+1$ and return to (Step3). $\square$

It is desired that the sway of the load be small during the transfer, and that there be no residual sway when the load arrives at its destination. In this paper, the performance index is given by:

$$J = \mathbf{X}(t_f)^T \mathbf{W} \mathbf{X}(t_f) + \int_{t_0}^{t_f} \mathbf{x}(t)^T \mathbf{W}_1 \mathbf{x}(t) dt \quad (12)$$

where $\mathbf{X}(t_f) = [\theta - \theta_{t_f}, \dot{\theta}, \phi - \phi_{t_f}, \dot{\phi}, \alpha, \dot{\alpha}, \beta, \dot{\beta}]$, $\mathbf{W} = \text{diag} [10^5, 10^5, 10^5, 10^5, 10^5, 10^5, 10^5, 10^5]$, $\mathbf{x}(t) = [\alpha, \dot{\alpha}, \beta, \dot{\beta}]$, and $\mathbf{W}_1 = \text{diag} [10^3, 10^3, 10^3, 10^3]$. Thus, for the model based on load position, an optimum control problem can be solved by the DFP method for Eqs.(A. 16), (A. 17), (A. 20) and (A. 21)~(A. 23).

Fig. 6 Optimal control simulation results for the load position model

Fig. 7 Locus of the load on xy plane for the optimal control simulation

In Fig.6 and Fig.7, the solid line shows the simulation result for the optimal control of the load position model

Fig. 8 Calculating result about Performance Index J

with rotation from $\theta_i = 0^\circ$ to $\theta_f = 90^\circ$, boom hoisting from $\phi_i = 30^\circ$ to $\phi_f = 30^\circ$, and rope length varying from $l_i = 0.9\text{m}$ to $l_f = 0.9\text{m}$. It can be easily understood that the locus of the load on the XY plane is approximately a straight line. This means that the straight transfer is thought to be the best method to decrease the sway of the load due to the elimination of centrifugal force. Nevertheless, because the load position model is used in Fig.6 and Fig.7, the number of the variables in the state equation is 18, and it takes a long time (48.8 seconds) to obtain the optimal control input such that $J_{u_\theta} < 10^{-6}$, $J_{u_\phi} < 10^{-6}$ and $J_{u_l} < 10^{-6}$, Fig.8 shows the convergence case about the Performance Index J. In order to obtain the optimal control input as quickly as possible, it is necessary to propose an equally effective method that reduces calculation time. In the next section, such a method will be proposed.

On the other hand, if optimal control is calculated by only rotary motion without the hoisting motion of boom from $\theta_i = 0^\circ$ to $\theta_f = 90^\circ$, and the hoisting angle $\phi_i = 30^\circ$ is kept constant, of course the rope length stays constant, as well. The simulation result is shown as a dotted line in Fig.6 and Fig.7, It can be seen that in the case of rotation-only, the sway of the load during the transfer is continuously generated and there is some residual sway. It is also clear that the straight transfer is better than a general rotary transfer. In fact, in order to compare the difference between the straight transfer and the rotary transfer, we simulated results for various θ_i , θ_f and ϕ_i , ϕ_f , as shown in Table 3 and 4. Here, α_{res} and β_{res} mean the residual swing angle, α_{max} and β_{max} mean the maximum swing angle during the transfer. The calculation to obtain an optimum control input was conducted until $J_{u_i} < 10^{-6}$ was satisfied. From these results, it is clear that the straight transfer is better than the general transfer with respect to sway reduction. Further, Table 5 shows the effect of the weighting factor for $\mathbf{W}$ and $\mathbf{W}_1$ in a cost function, where simulation condition was conducted under $\theta_i = 0^\circ$, $\theta_f = 90^\circ$, $\phi_i = 30^\circ$, $\phi_f = 30^\circ$. Then, $\mathbf{W} = \text{diag} [10^5, 10^5, 10^5, 10^5, 10^5, 10^5, 10^5, 10^5]$ and

Table 3 Straight transfer (with boom hoisting)

θ_i	θ_f	ϕ_i	ϕ_f	J	α_{res}	α_{max}	β_{res}	β_{max}
0°	120°	10°	10°	2.432	0.005°	0.99°	0.011°	0.57°
0°	90°	30°	30°	1.163	0.012°	0.52°	0.012°	0.52°
0°	45°	50°	50°	0.202	0.003°	0.12°	0.008°	0.28°
0°	80°	70°	70°	0.163	0.005°	0.17°	0.007°	0.21°

Table 4 Rotary transfer (without boom hoisting)

θ_i	θ_f	ϕ_i	ϕ_f	J	α_{res}	α_{max}	β_{res}	β_{max}
0°	120°	10°	10°	7.099	0.019°	1.25°	0.035°	1.26°
0°	90°	30°	30°	2.499	0.04°	0.54°	0.05°	0.87°
0°	45°	50°	50°	0.262	0.007°	0.13°	0.015°	0.28°
0°	80°	70°	70°	0.318	0.02°	0.19°	0.018°	0.35°

Table 5 The effect of weighting factor for W and W_1 in the case of boom hoisting

Case	W	W_1	J	α_{res}	β_{res}
1	$[10^5 \dots 10^5]$	$[10^4 \dots 10^4]$	10.74	0.101°	0.101°
2	$[10^5 \dots 10^5]$	$[10^3 \dots 10^3]$	1.163	0.012°	0.012°
3	$[10^5 \dots 10^5]$	$[10^2 \dots 10^2]$	0.139	0°	0°

$W_1 = \text{diag} [10^2, 10^2, 10^2, 10^2]$ showed the best results for control among the selected condition. So, these values were chosen at last in this paper.

Incidentally, by the procedure described above, the minimum time control by bang-bang control input can be obtained, when the end time t_f is sequentially reduced. However, the result of the minimum time control is omitted from this paper because it was not our main purpose.

3. Derivation of the STT model

The STT method means that the projection of the crane tip and the load move along a straight line in the X-Y coordinates by the simultaneous motions of rotation and boom hoisting (see Fig.9). In the analysis of this problem, an embedding method is used in order to reduce the dimensions of the model. Actual boom length L_B is replaced by the imaginary boom length R . Actual rotary input u_θ and hoisting input u_ϕ are also replaced by the imaginary hoisting input u_ψ , while actual swing angles α and β are replaced by the imaginary swing angle ξ as seen in Fig.10, so that the differential equations describing the model are reduced to only three variables. In the literature⁶⁾, the rope length was kept constant, while this paper enabled us to arbitrarily change the rope length, for example, as shown in Fig.9

3.1 Derivation of the STT model

The concept of STT is shown in Fig.9. Supposing the initial point of the boom tip is $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$, and the end point is $(\tilde{x}_f, \tilde{y}_f, \tilde{z}_f)$, then the projection of the orbit of

the boom tip is a line I-F on the X-Y plane. In order to simply analyze the problem of STT, a new coordinate, called the straight transfer coordinate, is built according to the following principle:

Fig. 9 The concept of straight transfer transformation

- The initial place of the boom tip $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$ is the original point of the new coordinate o' .

Fig. 10 Straight transfer transformation model of rotary crane

- $o'x'$ is parallel to the line IF.
- Axis z' is parallel to the axis z .

Fig.10 shows the concepts of imaginary boom length R , the imaginary hoisting angle ψ and the imaginary swing angle ξ .

In the straight transfer coordinates, the position of the crane tip can be expressed by the imaginary hoisting angle ψ and the length of the imaginary boom R ,

$$\tilde{x}' = R \{ \sin(\psi - \psi_i) + \sin \psi_i \} \quad (13)$$

$$\tilde{z}' = R \{ \cos(\psi - \psi_i) - \cos \psi_i \} \quad (14)$$

The position of the load is expressed by the swing angle in the direction of straight transfer ξ and the length of the

Table 6 Symbolic notation in Fig.10

symbol	unit	Explanation
l	m	Rope length
ψ_i	rad	Initial imaginary boom angle
ξ	rad	Swing angle
ψ_f	rad	Final imaginary boom angle
R	m	Imaginary boom length
$(\tilde{x}', \tilde{z}')$	m	Position of the crane tip
ψ	rad	Imaginary boom angle
(x', z')	m	Position of the load
M	kg	mass of cart
m	kg	mass of load

rope l :

$$x' = \tilde{x}' - l \sin \xi \quad (15)$$

$$z' = \tilde{z}' - l \cos \xi \quad (16)$$

The motion equation of Lagrange is given by

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = 0 \quad (17)$$

Using Eqs.(15) and (16), Eq.(17) is transformed into

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\xi}} \right) - \frac{\partial T}{\partial \xi} + \frac{\partial U}{\partial \xi} = 0 \quad (18)$$

where

$$T = T_x + T_z$$

$$T_x = \frac{1}{2} M \dot{x}'^2 + \frac{1}{2} m \left(\frac{d}{dt} (\tilde{x}' - l \sin \xi) \right)^2$$

$$T_z = \frac{1}{2} M \dot{z}'^2 + \frac{1}{2} m \left(\frac{d}{dt} (\tilde{z}' - l \cos \xi) \right)^2$$

$$U = Mg\tilde{z}' + mg(\tilde{z}' - l \cos \xi)$$

From Eq.(18), $\ddot{\xi}$ can be solved. Suppose that the control input u_ψ is the acceleration $\ddot{\psi}$ of the imaginary hoisting angle ψ . And in Eq.(16), when z' is the constant, $l \cos \xi = \tilde{z}'$, and then $\ddot{l}$ can be derived as shown in Eq.(21). Then the STT model follows that

$$\ddot{\psi} = u_\psi \quad (19)$$

$$\ddot{\xi} = -\frac{g}{l} \sin \xi + \frac{\ddot{x}'}{l} \cos \xi - \frac{\ddot{z}'}{l} \sin \xi - 2\frac{\dot{l}\dot{\xi}}{l} \quad (20)$$

$$\ddot{l} = \frac{\ddot{z}' + \xi \dot{l} \sin \xi + \dot{\xi}^2 l \cos \xi + 2\dot{l}\dot{\xi} \sin \xi}{\cos \xi} \quad (21)$$

$$\ddot{x}' = R \{ \ddot{\psi} \cos(\psi - \psi_i) - \dot{\psi}^2 \sin(\psi - \psi_i) \} \quad (22)$$

$$\ddot{z}' = -R \{ \ddot{\psi} \sin(\psi - \psi_i) + \dot{\psi}^2 \cos(\psi - \psi_i) \} \quad (23)$$

3.2 Derivation of the Parameter of STT Model

The parameters of the straight transfer, such as the length of the imaginary boom R , the initial imaginary hoisting angle ψ_i and the final imaginary hoisting angle ψ_f , are easily derived as follows:

$$R = \sqrt{L_B^2 - \frac{(\tilde{x}_i \tilde{y}_f - \tilde{x}_f \tilde{y}_i)^2}{(\tilde{x}_f - \tilde{x}_i)^2 + (\tilde{y}_f - \tilde{y}_i)^2}} \quad (24)$$

The final imaginary hoisting angle ψ_f is expressed by

$$\psi_f = \cos^{-1} \left\{ 1 - \frac{S_{if}}{2L_B^2} \right\} \quad (25)$$

where

$$S_{if} = (\tilde{x}_f - \tilde{x}_i)^2 + (\tilde{y}_f - \tilde{y}_i)^2 + (\tilde{z}_f - \tilde{z}_i)^2$$

The initial imaginary hoisting angle ψ_i becomes

$$\psi_i = \begin{cases} \cos^{-1} \frac{\tilde{z}_i - H}{R} & \psi_f \neq \psi_D \\ -\cos^{-1} \frac{\tilde{z}_i - H}{R} & \psi_f = \psi_D \end{cases} \quad (26)$$

$$\psi_D = \cos^{-1} \frac{\tilde{z}_f - H}{R} - \cos^{-1} \frac{\tilde{z}_i - H}{R}$$

3.3 Derivation of the Actual Control Inputs

The variables in the absolute coordinate can be transformed from the variables of the STT coordinates.

Fig. 11 Rotation angle γ

As shown in **Fig.11**, γ is the angle between direction $\vec{B}$ and direction $\vec{A}$, and it follows:

$$\gamma = \cos^{-1} \frac{\tilde{x}_f - \tilde{x}_i}{\sqrt{(\tilde{x}_f - \tilde{x}_i)^2 + (\tilde{y}_f - \tilde{y}_i)^2}} \quad (27)$$

Then, the following relation holds

$$\begin{bmatrix} \tilde{x} & \dot{\tilde{x}} & \ddot{\tilde{x}} \\ \tilde{y} & \dot{\tilde{y}} & \ddot{\tilde{y}} \\ \tilde{z} & \dot{\tilde{z}} & \ddot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} \tilde{x}_i & 0 & 0 \\ \tilde{y}_i & 0 & 0 \\ \tilde{z}_i & 0 & 0 \end{bmatrix} + \begin{bmatrix} \cos \gamma & 0 \\ \sin \gamma & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}' & \dot{\tilde{x}}' & \ddot{\tilde{x}}' \\ \tilde{z}' & \dot{\tilde{z}}' & \ddot{\tilde{z}}' \end{bmatrix} \quad (28)$$

From Eq.(28), the position $(\tilde{x}, \tilde{y}, \tilde{z})$ of the crane tip in the absolute coordinates, velocity $(\dot{\tilde{x}}, \dot{\tilde{y}}, \dot{\tilde{z}})$ and acceleration $(\ddot{\tilde{x}}, \ddot{\tilde{y}}, \ddot{\tilde{z}})$ can be calculated. Dividing Eq.(A.2) in the Appendix A by Eq.(A.1), it follows that:

$$\tan \theta = \frac{\tilde{y}}{\tilde{x}}$$

so that:

$$\theta = \tan^{-1}\left(\frac{\tilde{y}}{\tilde{x}}\right) \quad (29)$$

Differentiating Eq.(29), it follows that:

$$\dot{\theta} = \frac{\dot{\tilde{y}}\tilde{x} - \dot{\tilde{x}}\tilde{y}}{\tilde{x}^2 + \tilde{y}^2} \quad (30)$$

Further, differentiating Eq.(30), it follows that:

$$\ddot{\theta} = \frac{\ddot{\tilde{y}}\tilde{x} - \ddot{\tilde{x}}\tilde{y}}{\tilde{x}^2 + \tilde{y}^2} - \frac{2(\dot{\tilde{y}}\tilde{x} - \dot{\tilde{x}}\tilde{y})(\dot{\tilde{x}}\tilde{x} + \dot{\tilde{y}}\tilde{y})}{(\tilde{x}^2 + \tilde{y}^2)^2} \quad (31)$$

For the hoisting angle, from Eq.(A.3), it follows that:

$$\phi = \sin^{-1} \frac{\tilde{z} - H}{L_B} \quad (32)$$

Differentiating Eq.(32), it follows that:

$$\dot{\phi} = \frac{\dot{\tilde{z}}}{[L_B^2 - (\tilde{z} - H)^2]^{1/2}} \quad (33)$$

Further, differentiating Eq.(33), it follows that:

$$\ddot{\phi} = \frac{\ddot{\tilde{z}}}{[L_B^2 - (\tilde{z} - H)^2]^{1/2}} + \frac{\dot{\tilde{z}}^2(\tilde{z} - H)}{[L_B^2 - (\tilde{z} - H)^2]^{3/2}} \quad (34)$$

From Eqs.(1)~(3), the actual rotary input voltage u_θ , the hoisting input voltage u_ϕ and that of the rope lifting input voltage u_l becomes:

$$u_\theta = \frac{T_\theta}{K_\theta} \ddot{\theta} + \frac{1}{K_\theta} \dot{\theta} \quad (35)$$

$$u_\phi = \frac{E_\phi \ddot{\phi} - A_\phi \dot{\phi} - F_\phi \phi^2}{B_\phi} \quad (36)$$

$$u_l = \frac{T_l}{K_l} \ddot{l} + \frac{1}{K_l} \dot{l} \quad (37)$$

In order to obtain the optimal control input for the STT model comprised of Eqs.(19)~(21), the performance index is chosen as:

$$J = \mathbf{x}(t_f)^T W \mathbf{x}(t_f) + \int_{t_0}^{t_f} (\mathbf{x}(t)^T W_1 \mathbf{x}(t)) dt \quad (38)$$

,where

$$\mathbf{x}(t_f) = [\psi - \psi_{t_f}, \psi, \xi, \dot{\xi}], W = \text{diag} [10^5, 10^5, 10^5, 10^5], \\ \mathbf{x}(t) = [\xi, \dot{\xi}], \text{ and } W_1 = \text{diag} [10^3, 10^3].$$

Thus, for the STT model, an optimum control problem can be solved by the DFP method shown in Eqs.(19)~(21). Hereafter, actual control inputs could be calculated via Eqs.(28)~(37).

4. Experimental Results

The initial position of the crane tip is $(x_i, y_i, z_i) = (0.73m, 0m, 1.05m)$, the initial rotary angle $\theta_i = 0^\circ$, the initial hoisting angle $\phi_i = 30^\circ$. The final position is $(x_f, y_f, z_f) = (0m, 0.73m, 1.05m)$, the final rotary angle $\theta_f = 90^\circ$, and the final hoisting angle $\phi_f = 30^\circ$. The initial rope length $l_i = 0.9m$, and the control time is $t_c = 7.0s$. According to Eqs.(24)~(27), the imaginary boom length: $R = 0.67m$, the initial imaginary hoisting angle: $\psi_i = 50.8^\circ$ (0.89 rad), the final imaginary hoisting angle: $\psi_f = 101.5^\circ$ (1.77 rad), and $\gamma = 135^\circ$ (2.36 rad).

Fig.12 shows the simulation results obtained using the STT model. From the relationship derived in section 3.3 between the control input in the STT model and the actual control input, the actual control inputs can be obtained. Both the control experiment results and simulation results using the load position model are shown in Fig. 13. Compared with Fig.6, it is clear that the actual control inputs in Fig.13 obtained via the STT model are the same as the ones in Fig.6 obtained directly from the LP model, but only 1.38 seconds were required to calculate Fig.13 via Fig.12, while 48.8 seconds were required for Fig.6. Certainly the calculation of control inputs using the STT model is faster than that using the LP model. Fig.13 and Fig.14 show the results of vibration control under the above conditions. Through Fig.13 and Fig.14, it is clear that the experimental results are as same as the results of simulation. During the transfer, the swing angles were smaller than the case without control in Fig.4. At the end point of the transfer, the sway of the load was scarcely apparent in Fig.13 carried out by the proposed vibration control method.

Fig. 12 Control simulation results in STT model

The advantages of control design for straight transfer

using the STT model, as compared to control design using the three-dimensional load position model, are summarized in Table 7.

Fig. 13 Control experimental and simulation results using load position model by means of the optimal control obtained in STT model

Fig. 14 Locus of the load on xy plane in the case of Fig. 13

From Table 7, it can be understood that the STT model has only six variables and the repeated number of step 1 ~ 8 in the DFP algorithm is 8 times; this allows the cost function to converge to a minimum value of 0.138 and the calculating time to be reduced to 1.38 seconds, where the CPU of the computer is an Intel Pentium III 1GHz. Due to this advantage of the STT method, it is possible to practically apply the proposed algorithm for various tasks on the actual site. The load moves along a straight line without centrifugal force, so that it is easy to control. Finally, the residual swing angle when the load arrives to the goal point is almost zero by the proposed method.

In order to compare the calculating speed of the DFP method and FR method for the same problem, the FR method was also used to obtain the optimal input; these results are also shown in Table 7. Obviously, the DFP method is more useful as an optimization method in the

Table 7 The advantage of STT model

Content	STT Model	LP Model
Transfer Method	Straight	Straight
Variable Number	6	18
Calculating method	DFP FR	DFP
Calculating Number	8 11	40
Calculating Time	1.38 s 3.55s	48.8 s
Cost Function J	0.138 0.223	1.163
Residual Swing Angle	0° 0°	0.012°

present problems. Here the convergence condition is chosen as $J_{u_\theta} < 10^{-6}$, $J_{u_\phi} < 10^{-6}$ and $J_{u_l} < 10^{-6}$, as well.

5. Conclusion

Nonlinear optimal control using the straight transfer transformation (STT) model was presented to control the sway of a rotary crane. The experimental results agreed well with the simulation results. Both the validity of the model and the effectiveness of the control by the STT model were also demonstrated. Using a rotary crane model represented in three-dimensional space, it was shown that the optimization calculation of the proposed approach using the STT model embedded in two-dimensional space was faster than that of the conventional method because the STT model eliminates centrifugal force and thereby reduces the number of variables in the optimization.

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Appendix A.

(Derivation of 3-Dimensional Load Position Model)

From Fig.2, the position of the crane tip $(\tilde{x}, \tilde{y}, \tilde{z})$ and the load (x, y, z) in the absolute coordinate are given as the following equations

$$\tilde{x} = L_B \cos \phi \cos \theta \quad (\text{A.1})$$

$$\tilde{y} = L_B \cos \phi \sin \theta \quad (\text{A.2})$$

$$\tilde{z} = H + L_B \sin \phi \quad (\text{A.3})$$

$$x = \tilde{x} + l \sin \tilde{\beta} \cos(\theta + \tilde{\alpha}) \quad (\text{A.4})$$

$$y = \tilde{y} + l \sin \tilde{\beta} \sin(\theta + \tilde{\alpha}) \quad (\text{A.5})$$

$$z = \tilde{z} - l \cos \tilde{\beta} \quad (\text{A.6})$$

The relationship between load and the tension of the rope is

$$m\ddot{x} = -F \sin \tilde{\beta} \cos(\theta + \tilde{\alpha}) \quad (\text{A.7})$$

$$m\ddot{y} = -F \sin \tilde{\beta} \sin(\theta + \tilde{\alpha}) \quad (\text{A.8})$$

$$m\ddot{z} = F \cos \tilde{\beta} - mg \quad (\text{A.9})$$

From Eq. (A.9), it follows that

$$F = \frac{m(\ddot{z} + g)}{\cos \tilde{\beta}} \quad (\text{A.10})$$

From Eq.(A.4)~Eq.(A.6), it follows that

$$\sin \tilde{\beta} = \frac{\sqrt{(x - \tilde{x})^2 + (y - \tilde{y})^2}}{l} \quad (\text{A.11})$$

and

$$\cos \tilde{\beta} = -\frac{z - \tilde{z}}{l} \quad (\text{A.12})$$

Using Eq.(A.12), it can be obtained from Eq.(A.10)

$$F = -\frac{ml(\ddot{z} + g)}{z - \tilde{z}} \quad (\text{A.13})$$

From Eq.(A.4) and Eq.(A.5), it can be obtained that

$$\sin \tilde{\beta} \cos(\theta + \tilde{\alpha}) = \frac{x - \tilde{x}}{l} \quad (\text{A.14})$$

$$\sin \tilde{\beta} \sin(\theta + \tilde{\alpha}) = \frac{y - \tilde{y}}{l} \quad (\text{A.15})$$

From Eqs.(A.7),(A.8),(A.13),(A.14) and (A.15), it follows that

$$\ddot{x} = \frac{x - \tilde{x}}{z - \tilde{z}}(\ddot{z} + g) \quad (\text{A.16})$$

$$\ddot{y} = \frac{y - \tilde{y}}{z - \tilde{z}}(\ddot{z} + g) \quad (\text{A.17})$$

From Eq.(A.11), it follows that

$$\cos \tilde{\beta} = \frac{\sqrt{l^2 - (x - \tilde{x})^2 - (y - \tilde{y})^2}}{l} \quad (\text{A.18})$$

From Eq.(A.6) and Eq.(A.18), it follows that

$$z = \tilde{z} - l \cos \tilde{\beta} \quad (\text{A.19})$$

Differentiating Eq.(A.19) two times, it follows that

$$\ddot{z} = \frac{e_z^2}{l^2} \ddot{z} - \frac{e_z^2}{l^2} \left\{ \frac{e_x^2 + e_y^2}{e_z^2} g \right\} + \frac{e_z^2}{l^2} \Delta \quad (\text{A.20})$$

where

$$\Delta = \frac{\ddot{l}l + \dot{l}^2 + \ddot{x}e_x + \ddot{y}e_y - \dot{e}_x^2 - \dot{e}_y^2 - \dot{e}_z^2}{e_z}$$

and $e_x = x - \tilde{x}$, $e_y = y - \tilde{y}$, $e_z = z - \tilde{z}$

Differentiating Eqs.(A.1)~(A.3) two times respectively, it can be obtained that

$$\ddot{\tilde{x}} = -L_B \{ \ddot{\phi} \sin \phi \cos \theta + (\dot{\phi}^2 + \dot{\theta}^2) \cos \phi \cos \theta - 2\dot{\phi}\dot{\theta} \sin \phi \sin \theta + \ddot{\theta} \cos \phi \sin \theta \} \quad (\text{A.21})$$

$$\ddot{\tilde{y}} = -L_B \{ \ddot{\phi} \sin \phi \sin \theta + (\dot{\phi}^2 + \dot{\theta}^2) \cos \phi \sin \theta + 2\dot{\phi}\dot{\theta} \sin \phi \cos \theta - \ddot{\theta} \cos \phi \cos \theta \} \quad (\text{A.22})$$

$$\ddot{\tilde{z}} = L_B (\ddot{\phi} \cos \phi - \dot{\phi}^2 \sin \phi) \quad (\text{A.23})$$

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