Fictitious Reference Iterative Tuning in the Two-Degree of Freedom Control Scheme and Its Application to a Facile Closed Loop System Identification

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In 19) and 20), the authors proposed the tuning method of a controller by using only one-shot experimental data, which is referred to as Fictitious Reference Iterative Tuning (which is abbreviated to FRIT). FRIT requires only one-shot experimental data for an off-line non-linear optimization in order to obtain the optimal parameter minimizing the error between the desired output and the real one, so this is a useful and effective method in the sense that the cost and time for tuning of a controller can be drastically reduced. In this paper we expand FRIT into the two-degree of freedom control scheme. We show that our FRIT works effectively in the two-degree of freedom control systems and the minimization of the error in the fictitious area by using an off-line non-linear optimization corresponds to that in the real area. Moreover, by focusing the role of a feedforward controller in the two-degree of freedom control scheme, we also provide a new and facile closed loop system identification as one of the applications of the obtained result. Finally, we give an experimental result in order to show the validity of the proposed method in this paper.

Key Words: Fictitious Reference Iterative Tuning (FRIT), Iterative Feedback Tuning (IFT), two-degree of freedom control systems, closed loop system identification, facility

1. Introduction

Recently, the control system synthesis based on the direct use of the measured input/output data attracts attentions from the practical view points. Since the real measured input/output data of a plant includes fruitful informations on the dynamics of the plant more directly than mathematical models obtained in system identifications, it is to be expected that such direct approaches provide effective controllers reflecting the dynamics of a plant. In the case where the structure of a controller is fixed (e.g., PID controllers), this direct approach is regarded as a parameter tuning based on the direct use of the data without a mathematical model of a plant. As one of these tuning schemes, iterative feedback tuning (which is abbreviated to IFT in the following) was proposed by Hjalmarsson et.al in 8) and was studied in 2),7),9),14),15) and so on. In the case where a mathematical model of a plant in the closed loop with a (not necessarily desired) controller is unknown or performing an experiment for an identification is difficult, IFT is a useful and effective tuning method. IFT iteratively updates the variable parameter of an implemented controller so as to minimize a performance index, e.g., the sum of the squared error signal between the desired reference signal and the real output, by using the input/output data obtained in the iterative closed loop experiments. This minimization can be achieved by performing a non-linear optimization technique like Gauss-Newton method in which Hessian and Jacobian consist of the experimental data. Thus, many experiments must be performed in order to update the parameter of a controller so as to achieve the minimization of the performance index. This means that the use of IFT requires considerable cost and time, which is a serious problem from the practical points of view. In order to overcome such a drawback of IFT, we provided a new method of parameter tuning by using only one-shot experiment for the sake of reducing cost and time required for arriving at the optimum parameter of the controller in the iterative tuning in the references 19) and 20). The key concept of this approach was to use the fictitious reference signal generated by one-shot (initial) experimental data for the off-line tuning of the parameter of the controller, so this is referred to as fictitious reference iterative tuning (it is abbreviated to FRIT in the following). FRIT is more useful than IFT in the sense that we can reduce the number of the experiments for tuning of a controller drastically. In 19) and 20), FRIT was proposed and studied in the one-degree of freedom control scheme. In real
plants, there are many cases in which two-degree freedom controllers are used in order to improve the performance on tracking for the desired output as well as the feedback properties. If a tuning method of controllers based on the direct use of the one-shot experimental data like FRIT is also developed in the two-degree of freedom of control scheme, it is to be expected that such a method becomes more useful procedure for tuning of a controller. Thus, it is meaningful to expand FRIT into the two degree of freedom control scheme from the practical points of view.

Recently, the importance of closed loop identification has increased. The reason for this is that there are many cases where a plant has already been operated in the closed loop using an appropriate controller, or a plant is unstable. From these reasons, there are many literatures and studies on closed loop system identification and we can find novel techniques on this issue, e.g., (1), (6), (10), (12), (13), (18), (21), and so on. Since the proposed useful methods in these literatures are deeply supported by the basis of system theory, statistics, stochastics, signal processing, and so on, the experiments should be pre-designed elaborately. This means that these well-studied methods also requires much time for planning an experiment for an identification of a plant. On one hand, for engineers working in real plants, it is desired to execute a facile method for an identification of a plant operated by a controller in the case where they do not have so much time to perform an experiment for a system identification and it is not necessary for them to obtain a highly precise mathematical model.

Under these backgrounds, in this paper, we expand FRIT into the two-degree of freedom control scheme. Moreover, we also provide a new and facile closed loop system identification as one of the applications of our FRIT in the two degree of freedom control scheme. Roughly speaking, the later application can be summarized as follows. It is well-known that a feedforward-controller in the two degree of freedom control scheme, it is to be expected that such a method becomes more useful procedure for tuning of a controller.

This paper is organized as follows. In Section 2, we give some preliminaries. In Section 3, as our main results, we expand FRIT into the two-degree of freedom control scheme. We then provide a new and facile method of closed loop system identification method as one of the applications of FRIT together with some remarks. In Section 4, we show an experimental result in order to illustrate the validity of our result. In Section 5, we give concluding remarks.

2. Preliminaries

2.1 Notation

Let \( R \) and \( Z \) denote the set of real numbers and the integers, respectively. Let \( R^n \) denote the set of real vectors of size \( n \). Let \( (R)^Z \) denote the set of the maps from the time axis \( Z \) onto \( R \), i.e., \( w \in (R)^Z \) implies that \( w \) is a discrete time series. For \( w \in (R)^Z \), the value of \( w \) at the time \( t \) is denoted with \( w_t \). For \( w \in (R)^Z \) and \( a, b \in Z \) such that \( a \leq b \), \( w_{[a,b]} \) denote the finite time series of \( w \) defined in the time interval \([a,b] \). In this paper, we regard \( w_{[a,b]} \) as an element of \( R^{b-a+1} \), i.e., \( w_{[a,b]} = [w_a, w_{a+1}, \ldots, w_b]^T \).

Consider a single-input single-output, linear, time-invariant, and finite dimensional system in discrete time described by the (causal) transfer function \( G(q) \). Let \( u_{[1,N]} \) and \( y_{[1,N]} \) denote the input and output data, respectively, obtained in the interval \([1,N] \). Normally, the output \( y_t \) of an operator \( G(q) \) for the input \( u_{[1,t]} \) is written by the form of \( y_t = \sum_{k=0}^{t} \frac{g_k}{q-1} u_{t-k} \) by using the fact that \( G(q) \) can be written by Markov parameter expression \( G(q) = \sum_{k=0}^{\infty} g_k q^{-k} \). However, we use an abbreviated description as \( y_t = (G(q)u_t) \) or \( y_{[1,N]} = (G(q)u_{[1,N]} \) for the sake of enhancing the readability. As for the latter, \( y_{[1,N]} \in R^N \) is the range of the following Toeplitz type
In Eq. (1), the invertibility of \( G(q) \) is equivalent to the non-singularity of the Toeplitz matrix because of \( \det |g_0| \neq 0 \). It is easy to see that \( \langle G(q)u \rangle_{1,N} = G(q)u_{1,N} \). Moreover, for the other transfer function \( H(q) \), it follows from the well-known commutative property of the product of Toeplitz matrices that \( H(q)G(q)u_{1,N} = G(q)H(q)u_{1,N} \) holds. This fact is used in the calculations required in the proof of our main theorem.

2.2 Assumption

In this paper, we treat a linear, time-invariant, finite dimensional, and single input and single output system in discrete time, denoted with \( G(q) \). Assume that \( G(q) \) is strictly proper and a minimum phase system. Moreover, we assume that \( G(q) \) is unknown except the relative degree of \( G(q) \), and the plant has already been operated by implementing a stabilizing (and not necessarily desired) controller based on intuitions and experiences of engineers of the plant\(^{(1)}\). Here, particularly, we treat two degree of freedom control system\(^{(5)}\) as illustrated in Fig. 1. In this figure, \( C_r(\rho_r, q) \) and \( C_r(\rho_r, q) \) are a feedback controller and a feedforward one, respectively. \( T_d(q) \) is a desired transfer function from the reference to the output. It is well-known fact that the feedback controller is used to ensure the stability and robustness of the feedback loop, and the feedforward controller is used to yield the desired output \( T_d(q)r \) asymptotically. In this paper, they are parameterized by using \( \rho_e \in \mathbb{R}^{n_e} \) and \( \rho_r \in \mathbb{R}^{n_r} \) (where \( n_e \) and \( n_r \) are the numbers of the parameters of the feedback controller and the feedforward one, respectively). In the following, we often use the notation

\[
\rho = (\rho_e^T, \rho_r^T)^T \in \mathbb{R}^{n_e+n_r}.
\]

Moreover, \( \rho_e(k) \) (\( \rho_r(k) \)) denotes the \( k \)-th element of \( \rho_e \) (\( \rho_r \), respectively). Let \( y(\rho) \) and \( u(\rho) \) denote the output and the input, respectively, obtained in the closed loop with the controller with a parameter \( \rho \).

In addition, in order to explain the basic concept of FRIT and its role in the framework of closed loop system identification in the two-degree of freedom control scheme, which is proposed in this paper, we disregard the noise in the following explanation\(^{(2)}\).

3. FRIT in the two-degree of freedom control scheme and its application to closed loop system identification

3.1 FRIT in the two-degree of freedom control scheme

In this subsection, we expand our FRIT method proposed in \(^{(19)}\) and \(^{(20)}\) into the two-degree of freedom control scheme.

By contrast to IFT that must perform a lot of experiments for updating the controller parameter so as to achieve the optimum value in the sense that the performance index is minimized, our FRIT requires only one-shot experiment, and then the offline Gauss-Newton method by using the fictitious reference signal yields the optimum parameter in the fictitious space. Moreover, it was shown in Theorem 1 in \(^{(19)}\) that this optimum parameter corresponds to the optimum one in the real closed loop system.

In order to enable the real output to track the desired response, the aim of the controller parameter tuning is to find the optimum parameter \( \rho^* \) in the sense that

\[
\rho^* = \arg\min_\rho \sum_{i=1}^{N} \| e(\rho)_i \|^2
\]

where \( e(\rho) \) is the error between the desired response and the real output of the closed loop system associated with the closed loop is stable) \( \rho^* \), perform the first
on the closed loop system with \( \rho_0 \) and obtain the data
\((u(\rho_0), y(\rho_0))\). By using \((u(\rho_0), y(\rho_0))\), compute the fictitious reference signal \( \tilde{r}(\rho_i) \) at the \( i \)-th step as
\[
\tilde{r}(\rho_i) = \frac{u(\rho_i) + C_r(\rho_i, q) y(\rho_i)}{C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q)}
\]
and compute the error written by
\[
e(\rho_i) = (y(\rho_0) - T_d(q) \tilde{r}(\rho_i)).
\]
Observe that \( \tilde{r}(\rho_i) \) is a reference signal that yields \( u(\rho_0) \) and \( y(\rho_0) \). Note that \( e(\rho_i) \) can be computed off-line at each \( i \)-th step. Consider the following performance index in the fictitious domain:
\[
J_e(\rho) = \sum_{t=1}^{N} \| e(\rho_i) \|^2.
\]
Note that Eq. (6) consists of already-known information \((y(\rho_0), u(\rho_0))\). For the sake of minimization of \( J_e(\rho) \), that is, in order to find
\[
\rho^* = \arg\min_{\rho} J_e(\rho),
\]
we perform the following Gauss-Newton algorithm. Consider the gradient
\[
\frac{\partial J_e(\rho)}{\partial \rho} = \sum_{t=1}^{N} e(\rho_i) \left( \frac{\partial \tilde{e}(\rho)}{\partial \rho} \right)_{\rho = \rho_i},
\]
and the following update equation
\[
\rho^{i+1} = \rho^i - \gamma \frac{\partial J_e(\rho)}{\partial \rho} \bigg|_{\rho = \rho_i},
\]
where \( R_i \) is a Hessian approximated by
\[
R_i = \left( \frac{\partial J_e(\rho)}{\partial \rho} \right)_{\rho = \rho_i}^T \left( \frac{\partial J_e(\rho)}{\partial \rho} \right)_{\rho = \rho_i},
\]
and \( \gamma \) is a parameter that tunes the speed of the convergence. In Eq. (9) and Eq. (8), \( \frac{\partial \tilde{e}(\rho)}{\partial \rho} \bigg|_{\rho = \rho_i} \) can be computed as
\[
\frac{\partial \tilde{e}(\rho)}{\partial \rho} \bigg|_{\rho = \rho_i} = \frac{T_d(q)}{T_d(q) C_e(\rho_i, q) + C_r(\rho_i, q)} \times \left( \frac{\partial C_e(\rho_i, q)}{\partial \rho} y(\rho_i)^{\rho_i} \right)_{\rho = \rho_i}.
\]
In this point, it should be noted that the off-line computation can yield \( \tilde{e}(\rho_i) \) and \( \frac{\partial \tilde{e}(\rho_i)}{\partial \rho} \), so we do not have to perform an experiment at each step in the Gauss-Newton method, differently from IFT.

The remained problem is to see whether the minimization of Eq.(6) is equivalent to that of Eq.(2). The following theorem gives a solution to this problem from the practical points of view.

**Theorem 1.** Let \( \mathcal{P} \subseteq \mathbb{R}^{n+1} \) be the parameter set such that \( C_r(\rho_i, q) \) is invertible and \( G(q) C_e(\rho_i, q) \neq -1 \) for any \( \rho = [\rho_i, \rho_0]^T \in \mathcal{P} \). Assume that \( T_d(q) \) is strictly proper. Let \((u(\rho_0), y(\rho_0))\) be the input/output data in the feedback system described in Fig.1 by using the initial parameter \( \rho_0 \in \mathcal{P} \). Then \( \sum_{t=1}^{N} \| e(\rho_i) \|^2 = 0 \) if and only if \( \sum_{t=1}^{N} \| \tilde{e}(\rho_i) \|^2 = 0 \).

**Proof.** Let \( S(\rho, q) \) denote the sensitivity function described by
\[
S(\rho, q) := \frac{1}{1 + G(q) C_e(\rho, q)}. \tag{12}
\]
At first, we compute the error in the desired cost function:
\[
e(\rho) = S(\rho, q) (G(q) C_r(\rho, q) - T_d(q) r). \tag{13}
\]
Similarly, we also compute the error in the fictitious cost function:
\[
\tilde{e}(\rho) = S(\rho, q) \left( \frac{C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q)}{C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q)} \right) \times (G(q) C_r(\rho_i, q) - T_d(q) r). \tag{14}
\]
(As for the above computations, see appendix). Eliminating \( r \) in Eq.(13) and Eq.(14) enables us to obtain the relation between \( e(\rho) \) and \( \tilde{e}(\rho) \) described by
\[
\tilde{e}(\rho) = S(\rho_i, q) \left( \frac{C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q)}{C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q)} \right) \times (G(q) C_r(\rho_i, q) - T_d(q) r). \tag{15}
\]
Note that \( \sum_{t=1}^{N} \| e(\rho_i) \|^2 = 0 \) is equivalent to \( e(\rho_i) = 0 \) for \( t = 1, \ldots, N \). From Eq.(15), it is clear that \( e(\rho) = 0 \) implies \( \tilde{e}(\rho) = 0 \). Thus \( \sum_{t=1}^{N} \| e(\rho_i) \|^2 = 0 \) implies \( \sum_{t=1}^{N} \| \tilde{e}(\rho_i) \|^2 = 0 \). Conversely, from the invertibility of \( C_r(\rho_i, q) \) and the strict properness of \( T_d(q) \), we see that \( C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q) \) is invertible. Moreover, from \( G(q) C_e(\rho_i, q) \neq -1 \), we also see that \( S(\rho_i, q) \) is invertible. Thus, we see that \( C_r(\rho_i, q) + T_d(q) C_e(\rho_i, q) S(\rho_i, q) \) is also invertible. Together with the invertibility of \( G(q) C_e(\rho_i, q) \) in the first experiment, the rational function of Eq.(15) is also invertible. This observation enables us to see that \( \tilde{e}(\rho_i) = 0 \) implies \( e(\rho_i) = 0 \) for \( t = 1, \ldots, N \). Hence, \( \sum_{t=1}^{N} \| e(\rho_i) \|^2 = 0 \) implies \( \sum_{t=1}^{N} \| \tilde{e}(\rho_i) \|^2 = 0 \).

**Remark 1.** As for the assumption \( G(q) C_e(\rho_i, q) \neq -1 \), we can not check this condition because \( G(q) \) is unknown. However, since we assume \( G(q) \) is strictly proper as stated in the previous section, fixing only the coefficient of the highest degree term of the denominator of \( C_e(\rho, q) \) enables us to guarantee that this condition is satisfied.

**Remark 2.** By the way, the assumptions in the theorem are required to guarantee the invertibility of the rational function in Eq.(15). Suppose that our FRIT method is used in discrete time. Together with the feature that

\[\text{[Appendix: Details of computations]} \]
our method can be performed off-line, applying the shift
operator \( q \) to the denominator or the numerator of this
rational function yields to have no problem on this point
under the situation in which these assumptions are not
satisfied. □

The above theorem guarantees that the minimization of
\( J_e(\rho) \) by using fictitious reference in the off-line computa-
tion yields the optimum parameter \( \rho^* \) in the real closed
loop. From the practical points of view, if one can find the
optimum parameter, then there are many cases in which it is
to possible to regard that the sum of the squared error is
almost equal to zero. Thus, the above theorem gives one
of the practical solutions for the question on whether the
minimizations of the fictitious cost and that of the real
cost are almost equivalent. Of course, it is preferable to
guarantee that the minimization of the real cost function
is equivalent to that of the fictitious one, which is one of
our future studies.

Remark 3. At first glance, our proposed method
"FRIT" seems to be similar to "VRFT" proposed by
Campi et.al. in 3) and 4) in the sense that we require
only one-shot experimental data and the fictitious or vir-
tual signals are used for the parameter tuning in the off-
line. However, in VRFT, the virtual signal is computed
with respect to the input while our fictitious signal is com-
puted with respect to the output. Thus, VRFT is some-
what indirect in the sense that the aim of the controller
is to let the output follow the desired output. Moreover,
VRFT requires the power density spectrum of the input
signal with all-pass characteristics over the whole of fre-
cuencies, e.g., well known white Gaussian signal, in the
first experiment. This means that the first experiment in
VRFT must be open-loop experiment. The reason for this
is that it is very difficult to obtain such a power density
spectrum of the input in the closed loop data. Thus, it
is difficult for VRFT to be applied to an unstable system
or an already-operated closed loop systems in the real in-
dustries. On the contrary to VRFT, our method FRIT
can be applied to the case in which the first experimental
data is obtained in the closed loop system. Hence, our
method FRIT has advantages in these senses. □

3.2 Closed loop identification using FRIT

Consider again the two-degree of freedom control
scheme. It is preferable that the feedforward controller
is designed as

\[
C_r(\rho_r, q) = G(q)^{-1}T_d(q),
\]

if we have a nominal model \( G(q) \). By changing tracks,
we focus on this point for deriving our closed-loop system
identification method in the case of that the dynamics of
a plant \( G(q) \) is unknown.

Now, we suppose that \( G(q) \) is unknown, and our aim is
to obtain \( G(q) \). For this purpose, if we apply IFT or our
FRIT, the eventual feedforward controller of non-linear
optimization can be parameterized so as to be described by
\( G(q)^{-1}T_d(q) \) from the experimental data. Moreover, our
FRIT requires only one-shot experiment for arriving at the
optimal parameter while IFT requires many experiments
for the same purpose. Hence, our FRIT for two-degree of
freedom control scheme with unknown plant enables us to
obtain the inverse model \( G(q)^{-1} \), which means that the
model \( G(q) \) can also be obtained, by using only one-shot
experimental data.

As a result, combining the role of the feedforward con-
troller in the two-degree of freedom control scheme and
the concept of FRIT, we can obtain one of the identifi-
cation methods of a system embeded in the closed loop
so as to enable us to reduce a lot of costs for the identifi-
cation experiments. The algorithm of closed loop system
identification with FRIT in Fig.1 can be summarized as
follows:

**Algorithm:**

(a).Initial setting: Give a desired transfer function \( T_d(q) \)
from \( r \) to \( y \) and the initial controllers \( C_e(\rho^0_e, q) \) and
\( C_r(\rho^0_r, q) \) (the relative degree is zero) with the initial
parameter \( \rho^0 = (\rho^0_e^T, \rho^0_r^T)^T \).

(b).The first experiment: Perform one-shot experiment
and obtain the finite input/output data \( u(\rho^0) \) and \( y(\rho^0) \).

(c).The off-line tuning: Compute the following Gauss-
Newton method by using \( u(\rho^0) \) and \( y(\rho^0) \).

\[
\begin{align*}
\text{(1). Set } i &= 0. \\
\text{(2). Compute the fictitious reference signal } \tilde{r}(\rho^i) \text{ in Eq.}(4). \\
\text{(3). Compute the update equation Eq.(9) by using Eq.(8) and Eq.(11). } \\
\text{(4). Check } &\|\rho^{i+1} - \rho^i\|^2 < \epsilon \text{ (where } \epsilon \text{ is a sufficiently }
\text{small positive real number).} \\
\text{*No: } i &= i+1 \text{ and go back to (2).} \\
\text{*Yes: } \rho^* := \rho^i \text{ and go to (d).} \\
\text{(d).Calculate the plant model by using } \\
G(q) &= T_d(q)C_r(\rho^*, q)^{-1} (17)
\end{align*}
\]

with the eventual optimal parameter \( \rho^* \).

Remark 4. In the above algorithm, there may be a
situation in which \( \rho \notin \mathcal{P} \). Similary to Remark 1, fixing
some of the parameters (for example, the coefficient of the
higest degree terms of the denominator or the numerator
in \( C_r(\rho_e, q) \) or \( C_e(\rho_e, q) \)) is one of the methods so as to
Remark 5. The obtained model depends on $T_d(q)$, because it determines the frequency band in which we can obtain the accurate mathematical model. Moreover, since the initial experimental data reflects on not only the plant dynamics but also the feedback loop property, the initial feedback controller also plays a crucial role in this algorithm (this is also a problem for IFT, VRFT, and so on). This is also one of the further studies.

Remark 6. The initial input and output data generated by the reference signal $r$ should be with a required information on the plant in the closed loop, so the selection of $r$ is also one of the important factors which affect the result of the proposed identification method. One of the reasonable lines is to apply $r$ including frequency components on which a system is to be identified. The intuitive reason is that such a reference signal could excite the dynamics on the desired frequency components indirectly. However, quantitative effects of $r$ should be clarified. Thus, the issue on $r$ is also one of the further studies.

Remark 7. In this method, by identifying a controller yielding $T_d(q)r$, we identify a plant indirectly. Thus, roughly speaking, it might be regarded as one of the indirect approaches to closed loop system identification (e.g., 6, 12, 13, 18).

Remark 8. In the standard approach, the identifiability is clarified as the conditions on the degree of the controller, PE characteristics of the noise, and the property of the reference signals, and so on, explicitly. In our approach, the identifiability of $G(q)$ depends on whether $p_i^r$ is the desired local minimum or not, which is related to the issues on numerical computations of non-linear optimization like Gauss Newton method.

Remark 9. In the case in which the physical structure of a plant e.g., a mechanical system, an electrical circuit, and so on, has already been known, it is convenient to treat controllers $C_r$ and $C_e$ as the continuous time transfer functions. In such a case, we first consider a controller described by a continuous time transfer function

$$C_r(p,s) = \rho_r(n+1)s^n + \cdots + \rho_r(n+3)s + \rho_r(n+2) \left/ \rho_r(n+1)s^n + \cdots + \rho_r(2)s + \rho_r(1) \right. ,$$

Next, by transforming it into the discrete time transfer function, we can rewrite

$$C_r(\rho_r, q) = \frac{f_{n2}(\rho_r)q^{n2} + \cdots + f_{n+1}(\rho_r)q + f_{n+2}(\rho_r)}{f_{n+1}(\rho_r)q^{n+1} + \cdots + f_2(q) + f_1(\rho_r)}$$

by using appropriate coefficients $f_i(\rho_r)$, $i = 1, \cdots, n'$ which are scalar functions of the parameter vector $\rho_r$. Since Eq. (19) is also parameterized by $\rho_r$, we can apply the same discussion in this case.

4. Experimental result

In this section, we give an experimental result for showing the validity of our approach. The system we address here is described by Fig. 2. The cart is attached to the belt and the belt is moving by the rolling of the servo motor. The location $y$ (output) from the initial position of the cart is measured by the potentiometer and sent to the personal computer (PC). And the servo motor is driven by the voltage $u$ (input) from PC. The controllers are

$$C_r^e(p_r, s) = \rho_r^e(1) + \frac{\rho_r^e(2)}{s} ,$$

and

$$C_r^e(p_r, s) = \frac{\rho_r^e(5)s^5 + \rho_r^e(4)s^4}{\rho_r^e(3)s^3 + \rho_r^e(2)s + \rho_r^e(1)}$$

(The closed loop controller is a well known P.I. Controller). We have no information on the dynamics of this system a priori except the fact that the plant has one integrator. This is the reason why we take $C_r(p_r, s)$ with a zero at the origin. We give the desired response model as

$$T_d^s(s) = \left( \frac{1}{0.01s + 1} \right)^2 .$$

We use the reference signal as the sum of sinusoidal waves described by $r = 0.05 \sum_{i=1}^{3} \sin(10^{-1}t)$. Moreover, $\Delta$ is the sampling time 0.01[sec], the experimental time is 10[sec] (i.e., $N = 1000$), and $\gamma = 1.0 \times 10^{-6}$. We set the initial parameter $\rho_r^e(k) = 1.0$ for $k = 1, 2$ and $\rho_r^e(k) = 1.0$ for $k = 1, \cdots, 5$. Firstly, we obtained the initial experimental data $y(\rho_r^0)$ and $u(\rho_r^0)$ described by Fig. 3 and Fig. 4, respectively. Next, rewrite $T_d^s(s)$, $C_r^e(p_r, s)$ and $C_r^e(p_r, s)$ as the discrete time transfer functions $T_d(q)$, $C_r(p_r, q)$ and $C_r(p_r, q)$ by using an appropriate digitization technique, we perform our FRIT algorithm described by the previous section. As a result, we obtain the optimal controllers

$$C_r^e(\rho_r^*, s) = \frac{0.5294s^2 + 0.2806s + 1}{0.0230s^2 + 1.9847s + 1.0415}.$$
and
\[ C_0^*(\rho^*, s) = 2.0873 + \frac{1.0553}{s}. \]

In Fig. 5, we draw the output with the obtained optimal parameter \( y(\rho^*) \), the desired output \( T_d(q)r \) and the initial output \( y(\rho^0) \), as the real line, the dotted line and the chained line, respectively. From this figure, we see that the obtained parameter \( \rho^* \) yield the property such that \( y(\rho^*) \) tracks \( T_d(q)r \). As a result, we obtain a model of the plant described by
\[ \hat{G}(s) = \frac{0.0230s^2 + 1.9847s + 1.0415}{0.5294s^2 + 0.2806s}. \]

(23)

We perform the validation test in terms of the step response (see Fig. 6) in the closed loop implementing only P.I. controller (the proportional gain \( K_P = 1.0 \), the integrator gain \( K_I = 1.0 \) in the continuous time case). Moreover, we also perform the open loop validation test in terms of the sum of some sinusoidal waves \( u = 0.05\sin(10t) + 0.005\sin(t) \) (see Fig. 7). From Fig. 6 and Fig. 7, we can observe that the obtained model which well describes the dynamics (particularly, in the low-frequency).
By the way, for the purpose of the system identification, it is enough to tune only $\rho'_1$ with fixing $\rho'_2$. Indeed, by taking such a way for this example (we fix $\rho'_2 = [1 1]$), we obtain

$$ C_r(\rho'_1, s) = \frac{0.5242s^2 + 0.2661s}{0.0229s^2 + 1.9630s + 0.9887} $$

which yields almost similar results to Fig. 5, Fig. 6, and Fig. 7 (we omit the detailed figures here).

5. Conclusions

In this paper, we have expanded FRIT into the two-degree of freedom control scheme and we have provided a new and facile method of closed loop system identification from the practical points of view.

Of course, the study in this paper is the first step of the research topic on fictitious reference iterative tuning and its applications to closed loop system identification. The difference between FRIT and VRFT by Campi et al. need to be clarified. Moreover, the effectiveness of the noise we disregard in this paper should also be considered in order to develop our method as one of the practical tools for system identification.

References

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Appendix A. Computations of Eq.(13)

Eq.(13) can be computed as follows.

$$ e(\rho) = y(\rho) - T_d(q)r $$
$$ = \left( \frac{G(q)C_r(\rho_r, q) + G(q)T_d(q)C_r(\rho_r, q)}{1 + G(q)C_r(\rho_r, q)} \right) r - T_d(q)r $$
$$ = \frac{G(q)C_r(\rho_r, q) - T_d(q)r}{1 + G(q)C_r(\rho_r, q)} $$
$$ = S(\rho_r, q)(G(q)C_r(\rho_r, q) - T_d(q)r) $$

(A.1)

Appendix B. Computations of Eq.(14)

Eq.(14) can be computed as follows.

$$ \bar{e}(\rho) = y(\rho^\theta) - T_d(q)\bar{r}(\rho) $$
$$ = \left( \frac{G(q)C_r(\rho^\theta_r, q) + G(q)T_d(q)C_r(\rho^\theta_r, q)}{1 + G(q)C_r(\rho^\theta_r, q)} \right) r $$
$$ - \frac{T_d(q)C_r(\rho_r, q) + T_d(q)C_r(\rho_r, q)\bar{u}(\rho^\theta)}{C_r(\rho_r, q) + T_d(q)C_r(\rho_r, q)} $$
$$ = \frac{G(q)C_r(\rho^\theta_r, q) + G(q)T_d(q)C_r(\rho^\theta_r, q)}{1 + G(q)C_r(\rho^\theta_r, q)} $$
$$ - \frac{T_d(q)(C_r(\rho^\theta_r, q) + T_d(q)C_r(\rho^\theta_r, q))}{(C_r(\rho_r, q) + T_d(q)C_r(\rho_r, q))(1 + G(q)C_r(\rho^\theta_r, q))} $$
\[-\frac{T_a(q)C_e(\rho_a, q)(C_r(\rho_c, q)G(q) + T_d(q)C_e(\rho_c, q)G(q))}{(C_r(\rho_c, q) + T_a(q)C_e(\rho_c, q))(1 + G(q)C_e(\rho_c, q))} \] 

\[ \frac{1}{(C_r(\rho_c, q) + T_d(q)C_e(\rho_c, q))(1 + G(q)C_e(\rho_c, q))} \] 

\[ \{ (G(q)C_r(\rho_c^0, q) + G(q)T_d(q)C_e(\rho_c^0, q)) \] 

\[ \times (C_r(\rho_c, q) + T_d(q)C_e(\rho_c, q)) \] 

\[ - T_d(q)(C_r(\rho_c, q) + T_d(q)C_e(\rho_c, q)) \] 

\[ - T_d(q)C_e(\rho_c, q)(C_r(\rho_c^0, q)G(q) + T_a(q)C_e(\rho_c^0, q)G(q)) \} \] 

\[ S(\rho_c^0, q) \] 

\[ \frac{S(\rho_c^0, q)}{(C_r(\rho_c, q) + T_d(q)C_e(\rho_c, q))} \] 

\[ \{ G(q)C_r(\rho_c^0, q)C_r(\rho_c, q) + T_d(q)C_e(\rho_c^0, q)C_r(\rho_c, q)G(q) \] 

\[ - T_d(q)C_r(\rho_c^0, q) - T_a(q)C_e(\rho_c^0, q) \} \] 

\[ = \frac{S(\rho_c^0, q)}{(C_r(\rho_c, q) + T_d(q)C_e(\rho_c, q))} \] 

\[ (C_r(\rho_c^0, q) + T_d(q)C_e(\rho_c^0, q))(G(q)C_r(\rho_c, q) - T_a(q)) \]