An Application of Tabu Search for Multi-Dimensional Competitive Facility Location Problem

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Abstract—Competitive facility location problem has been studied in many literatures, which were dealt with problems considering location of some competitive facilities on at most two-dimensional space. In this study, we formulate competitive facility location problem in the situation that a new facility is located on a multi-dimensional space which has been already located several facilities. In order to find an optimal solution for the problem efficiently, first we show that one of optimal solutions is found by a combination problem, and then we apply tabu search algorithm for 0-1 programming problems to find approximate solution of the problem. In order to illustrate the efficiency of our proposing algorithm, results for numerical experiments are shown.

I. INTRODUCTION

Competitive facility location problem (CFLP) is an optimal location problem for commercial facilities which involve competitiveness with the same type of other facilities. Mathematical studies of CFLP were started by Hotelling [5]. He considered CFLP in the situation that all facility locaters being competitive mutually can change sites of facilities at any times, and that objective of each of them is to maximize sum of buying power (BP) obtaining from customers who exist on a line segment uniformly. Wendell et al. [18] considered CFLP with assumption that any customer exists on one of nodes in a network and facilities can be located on any nodes. For such a network location model, Hakimi [3] assumed that there are two types of facilities: one is the type of facilities which has already located, and the other is which will be located in the future. Drezner [1] considered CFLP by assumption that in a plain, any customer exists on one of points, defined as “demand points” (DP), and facilities can be located on any site of the plain.

In the above CFLPs, customers choose their using facilities by estimating distance from them to facilities. On the other hand, Huff [6], [7] assume that an attractive power of a facility to a customer is estimated as a combination of quality level of facility and to the distance. Karkazis [8] considered CFLP for the location model of Hakimi with estimating both quality level of facilities and distance to customers parallelly. Uno [14], [15], [16] considered CFLP for the location model of Drezner with the assumption of Huff.

Studies of CFLP are applied for other areas of decision making, for example, Kishimoto et al. [9] proposed to use Hotelling’s model [5] for election analysis. In former CFLP, facilities can be located on at a space with most two dimensions or a network. However, there are usually many decision variables in other areas of decision making. This means that in order to apply CFLP to other areas of decision making, we need to consider multi-dimensional location of facilities. Uno et al [17] considered multidimensional CFLP for a location of one facility by extending Uno’s model [16], which was dealt with two dimensional location. Uno et al showed that the formulated multidimensional CFLP can be found by solving a combinatorial optimization problem, and applied genetic algorithm for nonlinear 0-1 programming problem proposed by Sakawa et al.[11]. However, in order to solve this CFLP efficiently, this algorithm need to be more improved on both its computational time and precision.

In this paper, we propose a new efficient solving algorithm for multi-dimensional CFLP. As one of efficient solving algorithm for 0-1 programming problem, Hanafi et al. [4] proposed tabu search algorithm based upon oscillation strategy. We applied tabu search algorithm proposed by Hanafi et al. to solve this combinatorial optimization problem. In order to verify efficiency of this solving algorithm, we do numerical examination for a given example of CFLP.

The construction of this paper is as follows. In Section II, we formulate multi-dimensional CFLP. For the formulated problem, we show that an optimal solution for this CFLP can be found by solving a combinatorial optimization problem in Section III. In order to solve its combinatorial optimization problem, in Section IV, we introduce tabu search algorithm for 0-1 programming problems and apply the algorithm to the problem. Computational results are given in Section V, and finally we summarize conclusion and future studies of this study in Section VI.

II. FORMULATION OF MULTI-DIMENSIONAL CFLP

Let $m$ demand points (DPs) be given on the plane $R^n$, and let $I \equiv \{1, \cdots , m\}$. With each DP $i \in I$, its site denoted by $u_i \in R^n$ and buying power (BP) denoted by $w_i > 0$ are

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\begin{align*}
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\text{With each DP } i \in I, \text{ its site denoted by } u_i \in R^n \text{ and buying power (BP) denoted by } w_i > 0 \text{ are}
\end{align*}
associated. Let \( u \equiv (u_1, \ldots, u_m) \).

Let \( k \) facilities exist on the plane, and let \( J_A \equiv \{1, \ldots, k\} \) be set of indices of these facilities. In this CFLP, we consider the location of one new facility on the plane. Let \( 0 \) be index of the new facility, and \( J \equiv J_A \cup \{0\} \) be set of all facilities. With each facility \( j \in J \), its site is denoted by \( v_j \in R^n \), and its qualitative value is given and denoted by \( q_j \geq 0 \). In this CFLP, the decision vector is \( v_0 \in R^n \).

We assume that customers on demand points use only one facility according to the following criterion. For DP \( i \in I \) and facility \( j \in J \), distance from \( v_i \) to \( u_j \) is represented as Euclid norm, denoted by \( ||v_i - u_j|| \). We represent the attractive power of facility \( j \) to customers on DP \( i \) as the following definition suggested by Huff [6], [7]:

\[
c_i(v_j) \equiv \begin{cases} 
q_j / \sqrt{||v_j - u_i||^2}, & \text{if } ||v_j - u_i|| > \varepsilon, \\
q_j / \varepsilon, & \text{if } ||v_j - u_i|| \leq \varepsilon.
\end{cases}
\]

(1)

Here, \( \varepsilon \) is a positive number meaning the upper of distance that all customers think as no troubles about movement to facilities. With equation (1), the maximum attracting power in existing facilities is represented as follow:

\[
c_A^i \equiv \max_{j \in J_A} c_i(v_j)
\]

(2)

It is assumed that customers on DP \( i \) use one of new facilities if \( c_i(v_0) > c_A^i \) and then the locater of new facilities obtains BP \( w_i \). Then, we denote the following 0-1 variable in order to represent which of type of facilities are used by customers on each DP:

\[
\theta_i(v_0) = \begin{cases} 
1, & \text{if } c_i(v_0) > c_A^i, \\
0, & \text{otherwise}.
\end{cases}
\]

(3)

Let \( \theta(v_0) \equiv (\theta_1(v_0), \ldots, \theta_k(v_0)) \). Then, for location \( v_0 \), sum of BP that the locater of new facilities obtains is represented as follows:

\[
f(v_0) \equiv \theta(v_0) \cdot w',
\]

(4)

where, \( w' \) is a transpose vector of \( w \).

Therefore, CFLP is formulated as the following maximizing problem for obtaining BP:

\[
\text{maximize } f(v_0) \quad \text{subject to } v_0 \in R^n
\]

(5)

Next, we introduce difficulty to solve the above CFLP with using some figures. We consider an example of CFLP for one new facility, where \( (k, m, n) = (2, 6, 2) \) and sites of DPs and facilities having already located are given in Fig.1. In this example, we give the situations that all DPs have the same BP, denoted by \( \bar{w} \), and that all facilities, including the new facility, have the same quality level. Then, from the latter situation, all customers at DPs use the nearest facility. In Fig.1, before locating new facility, facility 1 obtains \( 2\bar{w} \) from DP 1, 4 and facility 2 obtains \( 4\bar{w} \) from DP 2, 3, 5, 6.

One of optimal solutions of CFLP in this example is shown in Fig.2. In Fig.2, a new facility obtains \( 4\bar{w} \) from DP 2, 3, 4, 6.

To solve CFLP analytically is generally difficult because CFLP has the following characters:

- Objective value of CFLP increases or decreases discontinuously. In the above example, an objective value for any location is either \( 0, \bar{w}, \ldots, 4\bar{w} \).
- In general CFLP, region of optimal solution is very small because optimal locations are sites which are well-balanced near to many DPs. In Fig.2, if a new facility is shifted with a few distance, the facility can only obtains no more than \( 3\bar{w} \).

These characters mean that it is difficult to apply general solving methods for non-linear programming problem, e.g. Steepest descent method, to use Kuhn-Tucker conditions, etc to CFLP. In the next section, we reformulate CFLP to a combinatorial optimization problem in order to solve CFLP more easily.

III. REFORMULATION OF CFLP TO A COMBINATORIAL OPTIMIZATION PROBLEM

From equation (1) and (2), a total set of DPs with customers who never use a new locating facility for any location is represented as follows:

\[
I_0 \equiv \{i \mid \sqrt{q_i/c_A^i} \leq \varepsilon, i \in I\}.
\]

(7)

Let \( I_0 \equiv I \setminus I_0 \). For subset \( \bar{I} \subseteq I_0 \), we consider the following maximization problem \( (P(\bar{I}))\):

\[
\text{maximize } \quad r_0^2
\]

(8)

\[
\text{subject to } \quad ||v_0 - u_i|| - \sqrt{q_i}\eta_i r_0 \leq 0, \quad \text{for } i \in \bar{I},
\]

(9)

\[
v_0 \in R^n, \quad r_0 \geq 0
\]

(10)

Here, \( r_0 \) is a variable introduced by the following theorem:
Theorem 1: For subset $\bar{I} \subseteq I_0$, let $(v_0^i, r_0^i)$ denote an optimal solution of $P(\bar{I})$. Then, the following two conditions hold:

- if $r_0^i \geq 1$, a new facility cannot obtain BP from all DPs in $\bar{I}$ for any location, and
- if $r_0^i < 1$, a new facility can obtain BP from all DPs in $\bar{I}$ by being located on $v_0^i$.

Proof: Constraint inequality (9) is transformed as follows:

$$\tilde{c}_i^A \leq \frac{q_0}{||v_0 - u_i||^2} \cdot r_0^i, \text{ for } i \in \bar{I}. \quad (11)$$

From equation (1) and (3), the above relation means that the locator can obtain BP from DP $i$ if $v_0$ holds that $r_0 < 1$. Therefore, a new facility cannot obtain BP from all DPs in $\bar{I}$ for any location if $r_0^i \geq 1$. Moreover, if $r_0^i < 1$, a new facility can obtain BP from all DPs in $\bar{I}$ by being located on $v_0^i$. ■

From Theorem 1, it is shown that $v_0^i$ is a candidate of optimal solution for CFLP. We define that $v_0^i$ is “a candidate point (CP)” for $\bar{I}$. Moreover, if $r_0^i < 1$, we define that $v_0^i$ is “an effective CP”. From Theorem 1, the following theorem about CFLP is shown:

Theorem 2: An optimal solution for CFLP is given by locating each facility on an effective CP for its facility.

From Theorem 2, an optimal solution for CFLP can be found by solving the following problem combination problem ($P_C$):

$$\begin{align*}
\text{maximize} & \quad f(v_0^i) & \quad \text{(12)} \\
\text{subject to} & \quad \bar{I} \in 2^{I_0} & \quad \text{(13)}
\end{align*}$$

Next, we think about computational time for solving $P_C$. In order to solve $P_C$, we need to find CP for each subset of $I_0$, that is to solve $P(I)$. Because $P(I)$ is a convex programming problem, we can use SQP (Sequential Quadratic Programming) method [13] to find an optimal solution for $P(I)$ efficiently. Let $T$ be computational time for solving $P(I)$ for each subset of $I_0$. Then, computational time for solving $P_C$ is estimated by $O(2^m \cdot T)$. Therefore, in cases that the number of locating facilities is large, to find strict optimal solution for problem $PC$ need enormous computational time and cost. In the next section, we propose an efficient algorithm in order to find an approximate optimal solution for $P_C$.

IV. APPLICATION OF TABU SEARCH ALGORITHM FOR CFLP

In this section, we introduce tabu search algorithm and apply the algorithm to problem ($P_C$). For details of tabu search, the reader can reed the literatures of Glover [2] and Reeves [10]. In order to represent each subset $\bar{I} \subseteq I_0$ as 0-1 expression, we denote the following variables for each DP $i \in I$:

$$y_i = \begin{cases} 
1, & \text{if } i \in \bar{I}, \\
0, & \text{if } i \not\in \bar{I}
\end{cases} \quad \text{(14)}$$

Let $y(\bar{I}) \equiv (y_1, y_2, \cdots, y_m)$ be 0-1 $n$-tuple, which represents $\bar{I}$. Let $e^i \equiv (1, 0, 0, \ldots, 0), e^0 \equiv (0, \ldots, 0, 1)$ be $n$-tuples. Then, the following theorem plays an important role for solving problem $P_C$.

Theorem 3: For DP $i \in I_0$, let $\bar{I}_1, \bar{I}_2 \subseteq I_0$ be subsets such that $y(\bar{I}_1) = y(\bar{I}_1) + e^i$. Then, the following relation is satisfied:

$$r_0^{I_1} \leq r_0^{I_2} \quad (15)$$

Proof: From equation (14), $\bar{I}_1 \subseteq \bar{I}_2$. Therefore, equation (15) is satisfied from problem $P(\bar{I})$. ■

For the above $\bar{I}_1, \bar{I}_2$, although it is not always satisfied that $f(x^{I_1}) \leq f(x^{I_2})$, this relation usually holds if $r_0^I < 1$. Moreover, the following theorem plays an important role for solving problem $P_C$.

Theorem 4: Let $I^* \subseteq I_0$ be subset such that $v_0^{I^*}$ is an optimal solution for problem $P_C$ and

$$f(v_0^{I^*}) = \sum_{i \in I^*} w_i. \quad (16)$$

Then, for any DP $i$ such that $y_i = 0$ for $I^*$, it holds that $r_0^I + e^i > 1$.

Proof: From equation (16), the set of DPs which are obtained their BPs by locating on $v_0^{I^*}$ is $I^*$. Because $v_0^{I^*}$ is an optimal solution for problem $P_C$, the new facility cannot obtain BP from other DPs not in $I^*$ adding to all DPs in $I^*$. This means that $r_0^I + e^i > 1$ for any DP $i$ such that $y_i = 0$ for $I^*$. ■

From Theorem 3 and 4, it follows that the new facility had better be located such that $r_0^I$ is nearly one. Hanafi et al [4] proposed tabu search algorithm for multi-dimensional 0-1 knapsack problem based upon oscillation strategy, which can find good approximate optimal solution efficiently by searching near boundary of its feasible set intensively. In this paper, based on Hanafi’s tabu search algorithm, we propose an efficient solving algorithm for problem $P_C$ by searching locations such that $r_0^I \approx 1$ intensively.

Tabu search is one of local search methods. Let $S_M \equiv \{\pm e^i \mid i = 1, \cdots, n\}$, each of whose elements is called “move”. We define neighbor region of subset $I$ as $N(I) \equiv \bar{I} \cup S_M$. In local search, searching point moves from current solution to a new solution which is best about an estimate function, usually objective function. In tabu search, moves chosen in past search are not chosen in a certain term since these moves were chosen. Such moves are called “tabu”, and such a term is called “tabu term”, denoted by $T > 0$. Memory which reserves tabu term for each move is called “tabu list”.

Next, we apply tabu search algorithm based upon oscillation strategy proposed by Hanafi et al [4] to problem $P_C$. Let $N_C$ denote number of times of searching locations such that $r_0^I \approx 1$ intensively at each iteration of algorithm. Let $P_{max}, P_{min} > 0$ denote parameters about oscillation strategy, and $N_C$ denote number of iteration for tabu search algorithm. Then, applied tabu search algorithm based upon oscillation
strategy is described as follows:

**Tabu search algorithm based upon oscillation strategy**

Step 0. (Initialization) Initialize tabu list, and set $c = 0$, $s = TRUE$, $z = 0$. Set a location $v_0 \in R^n$ randomly. Give $I$ by the set of DPs which are obtained their BPs by locating on $v_0$, and then find $v_0^I$ by solving problem $P(I)$. Note that $r_0^I < 1$ because a new facility can obtain BP from all DPs in $I$ by locating $v_0$.

Step 1. (TS_ADD) Add one DP to $I$. Move current solution $v_0(I)$ to new solution by the most improving move about problem $P_C$ in all non-tabu moves with subsets in $N(I)$. This step is repeated as long as $r_0^I < 1$.

Step 2. (COMPLEMENT) If $c > N_c$, go to Step 3. Otherwise, set $c \leftarrow c + 1$. For $y(I)$, if $r_0^I < 1$, return to Step 1, otherwise go to Step 5.

Step 3. (Terminal judgment and Oscillation) Set $c = 0$ and $z \leftarrow z + 1$. If $z > N_z$, this algorithm is terminated and approximate optimal solution for problem $P_C$ is the best solution in all searched solutions. Otherwise, if $s = TRUE$, set $s = FALSE$ and go to Step 4, otherwise set $s = TRUE$ and go to Step 6.

Step 4. (TS_INFEASIBLE_ADD) Add one DP to $I$. Move current solution $v_0(I)$ to new solution by the most improving move about problem $P_C$ in all non-tabu moves with subsets in $N(I)$. This step is repeated as long as $r_0^I$ is less than a given positive number, denoted by $R^{max}$.

Step 5. (TS_PROJECT) Delete one DP in $I$. Move current solution $v_0(I)$ to new solution by the smallest increasing move about $r_0^I$ in all non-tabu moves with subsets in $N(I)$. This step is repeated as long as $r_0^I < 1$. Return to Step 2.

Step 6. (TS_DROP) Delete one DP in $I$. Move current solution $v_0(I)$ to new solution by the smallest increasing move about $r_0^I$ in all non-tabu moves with subsets in $N(I)$. This step is repeated as long as $r_0^I$ is more than a given positive number, denoted by $R^{min}$. Return to Step 1.

In the above algorithm, to search all moves with subsets in $N(I)$ demands many computational times. Then, we suggest restricting searched moves for the following two cases in the algorithm.

(i) In the cases of TS_ADD and TS_INFEASIBLE_ADD: To solve problem $P(I \cup \{i\})$ for all $i \in I \setminus I$ occurs many computational times. Then, we suggest more easy estimation for DP $i \in I \setminus I$. In order to increase objective value of problem $P_C$ larger, it is desirable that DP adding to $I$ has large BP. In order to increase objective function of problem $P(I \cup \{i\})$ smaller, it is desirable that DP adding to $I$ is near by $v_0^I$. Moreover, it is undesirable to choose the same DP at many times. Therefore, we define the following estimate function in order to choose one DP in $I \setminus I$:

$$W(I, i) = \frac{w_i}{||v_0^I - u_i||^2} \cdot \phi(e^I)$$

where $\phi : \{e^1, \ldots, e^n\} \rightarrow R$ is a penalty function about frequency of choosing moves, where it is a non-decreasing function for frequency.

(ii) In the cases of TS_PROJECT and TS_DROP: From problem $P(I)$, value of $r_0^I$ is decreased by deleting one DP in $I$ such that it is an active constraint 9 in the problem. Therefore, we memorize DPs with active constraints in problem $P(I)$ for current $I$, and only search such DPs.

V. NUMERICAL EXPERIMENTS

In this section, we apply tabu search algorithm to example of CFLP and verify its efficiency. For dimension of locating space, we set $n = 7$. In this paper, we give five examples of CFLP setting that $m = 60$. For DPs, we set their sites to $u_i \in [0, 10]^n$ randomly, and their BP to $w_i \in \{1, \ldots, 20\}$ randomly. For competitive facilities existing in $R^n$, we set $k = 15$, their sites to points in $[0, 10]^n$ randomly, and their quality value to $q_i = \{4, \ldots, 9\}$ randomly. For new locating facility, we set their quality value to $q_0 = 8$.

About parameters of tabu search algorithm, we set $N_C = 50$, $R^{max} = (3, 0.3)$, and $N_z = 50$.

In order to verify results obtained by tabu search algorithm, we implement genetic algorithm for 0-1 programming problem to problem $P_C$, for details about this genetic algorithm, the reader can see the book of Sakawa et al. [12]. About fitness function and generation gap, we set $\beta = 0.2$ and $G = 0.9$ respectively. About probability of crossover, mutation, and inversion, we set $p_c = 0.9$, $p_m = 0.01$, $p_s = 0.03$, respectively. About scaling constraint, we set $c_{mult} = 1.8$. Population size and terminal generation for each example of CFLP are $S_G = 100$ and $T_G = 2000$, respectively.

Results of implementation for tabu search algorithm and genetic algorithm at 20 times is given in TABLE I. Here, CPU times in these tables mean computational times for all implementations of tabu search algorithm with using DELL Optiplex GX260 (CPU: 2.33 GHz, RAM: 512MB).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Best</th>
<th>Mean</th>
<th>Worst</th>
<th>CPU times (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tabu search</td>
<td>208</td>
<td>205.7</td>
<td>185</td>
<td>323.98</td>
</tr>
<tr>
<td>Genetic algorithm</td>
<td>208</td>
<td>208.0</td>
<td>208</td>
<td>358.67</td>
</tr>
</tbody>
</table>

From TABLE I, from the points of view of both precision and speed of computation, tabu search is superior to genetic algorithm for problem $P_C$. This shows efficiency of tabu search algorithm in the previous section for multi-dimensional CFLP.
VI. Conclusion

In this paper, we have proposed an efficient algorithm for multi-dimensional CFLP. We applied tabu search algorithm for multi-dimensional 0-1 knapsack problem based upon oscillation strategy proposed by Hanafi et al [4] to multi-dimensional CFLP by utilizing characteristic of the problem. It was shown efficiency of tabu search algorithm by solving example of multi-dimensional CFLP with comparison to genetic algorithm.

We considered location of one facility in a multi-dimensional space in this paper. For location of two or more facilities, we can also utilize all theorems in Section III and IV. Therefore, it is expected that tabu search algorithms can find good approximate solutions for location of two or more facilities efficiently.

References