Consensus with Dual Fallible Wormhole Network

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Abstract - A wormhole network may suffer from various types of hardware failure. In order to enhance the fault-tolerance and reliability of the wormhole networks, we revisit the consensus problem in the wormhole network in this paper. The proposed protocol is called as the DWAP (Dual Wormhole Agreement Protocol) which can make each correct node reach an agreement value to cope with the faulty component in the wormhole network. Keywords: Byzantine Agreement, Consensus, Asynchronous Transfer Mode, Wormhole Network

I. INTRODUCTION

The fault-tolerant is an important topic in the distributed system. To cope with the influence from faulty components, reaching a common agreement in the presence of faults before performing some special tasks is essential. When a common agreement is achieved, some special tasks can then be achieved. Some examples are clock synchronization problem and landing tasks controlled by a flight path finding system. The characteristic of wormhole routing network technology is different the IP (Internet Protocol) that lead to more application on wormhole routing network [12]. However, few researches discussed the influence generated by network communication technology before [8].

To achieve a common value among correct nodes was first studied by Lamport [7] in 1982, called the Byzantine Agreement (BA). In the BA problem, there is an initial value needs to be set in a source node before reaching a common value among correct nodes. Another related sub-problem is called consensus problem [10,11]. In the consensus problem, each correct node has its own initial value and then each correct node communicates with each other to reach a common value among correct nodes. The protocol that we proposed attempts to solve the consensus problem if it satisfies the following constraints:

(Agreement): All correct nodes agree on the same value.

(Validity): If the initial value of all nodes is v, then all correct nodes shall agree on v.

In previous works, many results in the BA or consensus problem are based on the assumption of node failure in a fail-safe network. Based on this assumption, a link fault is treated as a node fault, regardless of the validity of an innocent node; hence, an innocent node does not involve a consensus. This assumption contradicts the definition of the BA or consensus problem, which stipulates that all correct nodes reach a consensus. Therefore, it is not reasonable. In this study, we will propose a new protocol to solve the consensus problem under unreliable link. The symptom of a faulty link can be classified into two types: arbitrary (also called malicious or Byzantine faults) and dormant (such as crash or stuck-at). The receiver can identify the dormant fault of a link always if the transmitted message was encoded appropriately (i.e. by NRZ-code, Manchester code) before transmission. Besides, the arbitrary faulty links are unanticipated. In previous works [12], only have probing into arbitrary faulty links. However, in the actually network evolution the link fault not only arbitrary faulty, it maybe crash or stuck-at. Therefore, we revisit consensus problem to increase the fault tolerant capability by permit both dormant faults and arbitrary faults exist in the system simultaneously.

The proposed DWAP (Dual Wormhole Agreement Protocol) can tolerate \(\lceil(c + f_d + 1) / 2\rceil\) faulty links where \(c\) is the connectivity in the wormhole network, \(f_d\) is the dormant faulty links in the wormhole network, by the way, there \(f_d\) is less or equal \(c-2\). In addition, the protocol requires only two rounds of message exchange, where the common term round [10] is used to denote each interval of exchanging messages. The amount of necessary information exchange is only \(O(n^2)\), where \(n\) is the number of nodes in a wormhole network.

The rest of this paper is organized as follows. Section 2 introduces the wormhole network. Section 3 presents in detail a new protocol DWAP and gives an example of executing DWAP. Section 4 proves the correctness and demonstrates the complexity of the new protocol. Section 5 canvassed the impossible cases of an unreliable distributed system and the optimization of the protocol. Finally, Section 6 concludes this paper.

II. WORMHOLE NETWORK

The wormhole routing network structure is different from general network structure. Under the past network structure, each node is only a pure network node, which communicate the outside world through a router; however, under the wormhole routing structure, each node contains a processor and a router device [8].

In the wormhole routing, the packets are segmented into fixed-size flits; flit sizes have usually ranged from 4 to 16 bytes. The routing decision is taken as soon as the packet header has been received [8]. If a possible output link is free, the header is transmitted and the rest of the packet is sent directly from the input to the output without being stored. If all the possible outputs are unavailable, the packet waits until a possible output link is free. The data transmission way to take shapes the characteristic of “pipeline”. There are so differences with the “Store-and-Forward” of the data transmission way [8]. In the Fig.1 shows the data transmission way of wormhole.

In the Fig.1 shows the data transmission way of wormhole. When source node wants to submit data to the destination node, data will be split into fixed-size called flits. Then, the processor of source node through its router will transmit the data. The router of source node will search the routing table with destination address and transmit the data. When destination node received the data, the router of destination node will pass the data to the processor of
destination node.

In the Fig.2, shows the example of wormhole network structure of un-fully connected network in this paper, and below the example for solve consensus problem will use the wormhole network structure. There are ten nodes in this network, and each node denotes as ni (i=1, …,10). In addition, the connectivity of this wormhole network is five. However, every node can be guarantee received the data that other node transmitted even if the interaction between nodes is disconnection in a wormhole network structure.

### III. THE PROPOSED PROTOCOL

This paper proposes a new protocol, named DWAP (Dual Wormhole Agreement Protocol), to solve the consensus problem due to faulty link, to concern the system to achieve a consensus in a wormhole network. The faulty link include two kind of model, the kind of one is arbitrary fault that may send wrong messages and the other kind of faulty model is dormant faulty that lead message cannot be transmission or transition bit always the same. As the previous studies [1,6-9,11], the DWAP protocol, like other consensus protocols, consists of two phases and needs two rounds of message exchange to solve the consensus problem. In the first round of the message exchange phase, each node ni multicasts its initial value vi through known connected links and then receives the initial value of other nodes as well. In DWAP, each node may receive at most c (c is the connectivity degree of wormhole network) messages and constructs a vector from the received messages. For simplicity, any value not identified or received within the predefined time limit or stuck-at the receiver, assuming a dormant fault, will set a constant value, to λ. In the second round, each node ni acts as the sender, sending the vector received in the first round, and constructs a matrix by the procedure MATRIX shown in Fig. 3, called the MATi, where 1≤i≤n. Finally, the decision making phase will reach a consensus among the nodes. The proposed protocol DWAP is presented in Fig. 4. In a synchronous network, the bounds of delay for each fault-free component are finite [3,4,11]. The assumptions and parameters of our protocols:

- The underlying network is synchronous.
- Each node in the network can be identified uniquely.
- Let c be the connectivity of the general network.
- Let fλ be the maximum number of arbitrary faulty link.
- Let f0 be the maximum number of dormant faulty link.
- A node does not know the faulty status of link.
- The communication links in the network are assumed fallible.
- Every node has initially value and transmission to the other node in the network.
- The message transmission routing would not take dead lock.

Subsequently, an example of executing the DWAP protocol based on the network configuration shown in Fig. 5(a) is illustrated as follows. In addition, assume the links between node ni and nj is arbitrary faulty link, node ni and nj, node ni and nj are dormant faulty links. The ten nodes have their own initial value. The initial value of ni is 1 (where 1 ≤ i ≤ 5), otherwise, it is 0 (where i is equal to 10).

In the message exchange phase, the node transmits the message by using the concept of virtual channel, and the virtual channel can let an un-fully connected network work just like a fully connected network. Because, each node in the network has the common knowledge of the graphic information of the entire general network and there are at least c (c > 2fλ + f0) node-disjoint paths to each node.

However, in the first round of message exchange, each node ni multicasts its initial value vi through connected links to all other nodes, where 1 ≤ i ≤ n, and receives the initial value of other nodes as well. Then, each node uses the received message to construct vector Vi. In the second round of message exchange, each node multicasts its vector Vi and receives the vectors from other nodes to construct the matrix MAT, by the procedure MATRIX shown in Fig. 3. Finally, the decision making phase each node removes all the λ’s in MATi to reduce the influence of faulty behaviors and takes the majority value of MATi to construct the matrix MAJi, and achieves the common value by MAJI.

In this example, the vector Vi received by each node ni for 1 ≤ i ≤ n in the first round is shown in Fig. 5(b). The corresponding MATi is shown in Fig. 5(c) in the second round. Because the links between node ni and nj is arbitrary faulty, the vectors Vi and Vj may have changed arbitrary, moreover, the links between node ni and nj and nj are dormant faulty, the vectors Vj, Vj and Vj will be replace the value with λ. The majority value of each row is shown in Fig. 5(d).

In the decision making phase, each node removes all the λ’s in MATi to reduce the influence of dormant faulty behaviors. Each node has the MAJI = ? and Vj = vi so the DECi is equal to ↓, for 1 ≤ i ≤ n, is shown in Fig. 5(e).

In the example, all correct nodes have the same DEC value and are equal to ↓, so all correct nodes can reach a consensus value, and the consensus problem is done.

### IV. THE CORRECTNESS AND COMPLEXITY OF DWAP

The following lemmas and theorems are used to prove the correctness and complexity of DWAP. Subsequently, we will show the fault tolerant capability in DWAP.

**Lemma 1**: If there is a majority value = ¬vi in MATi, then mean at least one node with an initial value is discordant vi in the network.

**Proof**: If the k-th row of MAT, the majority value is ¬vi then that means there are at least \(\lfloor (n-d)/2 \rfloor + 1 \) ¬vi’s in the k-th row, where d is the number of dormant faulty links and c is connectivity in the wormhole network. In the network the maximum number of arbitrary faulty links is at most \(\lfloor (c-d+1)/2 \rfloor - 1\) and \(\lfloor (n-d)/2 \rfloor + 1 \leq \lfloor (c-d+1)/2 \rfloor - 1\) \(\leq \lfloor (n-c+3)/2 \rfloor\). Therefore, there exists at least one value ¬vi received through a perfect link. In other words, there is a node which has a disagreeable initial value ¬vi.

**Lemma 2**: Let the initial value of source node ni be vi and the link between the source node ni and node nj is correct or dormant, then the majority value MAJ should be vi.

**Proof**: Case1: Since link between source node ni and node nj is correct, node nj will receive vi from source node ni in the first round. Meanwhile, the value vi of source node ni will be broadcasted to the other nodes. There are at most \(\lfloor (c-f_0 + 1) / 2 \rfloor - 1\) arbitrary faulty links in the network. In the second round, node nj receives at least \(\lfloor (c-f_0 + 1) / 2 \rfloor - 1\) in the ith row of MATi, where d is the number of which will be eliminated during
the voting of majority. Hence, there are at least \( \lfloor (2n - f_d + 1) / 2 \rfloor \) \( v_i \)'s in the i-th row, and the majority value in the i-th row should be equal to \( v_i \).

**Case2-1:** Link \( ij \) is dormant and \( c \) is an even number, the node \( j \) will receive \( v_i \) from node \( i \) in the first round and \( v_i = \lambda \) in MAT\(_i\). Meanwhile, the value \( v_i \) of node \( i \) will be broadcasted to the other nodes. There are at most \( \lfloor (c - f_d + 1) / 2 \rfloor \) - 1 arbitrary faulty links and \( f_d \) dormant links in the network. After the second round, node \( j \) receives at least \( (f_d + 1) \) \( \lambda \)'s and at least \( n - (f_d - 1) - \lfloor (c - f_d + 1) / 2 \rfloor \) \( v_i \)'s in the i-th row of MAT\(_j\), where \( f_d \) is the number of \( \lambda \)'s which will be eliminated during the voting of majority. Hence, there are \( n - (f_d - 1) \) non-\( \lambda \)'s and at least \( (2n - f_d - c - 1) \) \( v_i \)'s in the i-th row, so, the majority value in the i-th row should be equal to \( v_i \).

**Case2-2:** Link \( ij \) is dormant and \( c \) is an odd number, the node \( j \) will receive \( \lambda \) from node \( i \) in the first round and \( v_i = \lambda \) in MAT\(_i\). Meanwhile, the value \( v_i \) of node \( i \) will be broadcasted to the other nodes. There are at most \( \lfloor (c - f_d + 1) / 2 \rfloor \) - 1 arbitrary faulty links and \( d \) dormant links in the system. After the second round, node \( j \) receives at least \( (f_d + 1) \) \( \lambda \)'s and at least \( n - (f_d + 1) - \lfloor (c - f_d + 1) / 2 \rfloor \) \( v_i \)'s in the i-th row of MAT\(_j\), where \( f_d \) is the number of \( \lambda \)'s which will be eliminated during the voting of majority. Hence, there are \( n - (f_d + 1) \) non-\( \lambda \)'s and at least \( (2n - f_d - c - 1) \) \( v_i \)'s in the i-th row, so, the majority value in the i-th row should be equal to \( v_i \).

**Lemma 3:** If the initial value of node \( i \) is \( v_i \), whether or not link \( ij \) is perfect or dormant, the majority value at the i-th row in MAT\(_j\) is correct. If the initial value of node \( k \) was \( v_k \), then DEC\(_k\) \( \neq \perp \). It is a contradiction; hence, all correct nodes \( k \) \( \neq \perp \).

**Proof:** By Lemma 2, when link \( ij \) is perfect or dormant, the majority value of the i-th row in node \( j \) is \( v_i \), for \( 1 \leq j \leq n \). When link \( ij \) is under the influence of arbitrary fault, we consider the following two cases after running the first round.

**Case 1:** \( v_j = v_i \)

Since there are at most \( \lfloor (c - f_d + 1) / 2 \rfloor \) - 1 arbitrary faulty links connected with node \( j \), there are at most \( \lfloor (c - f_d + 1) / 2 \rfloor - 1 \) value that may be \( \perp \). In the second round. The number of \( v_i \)'s is \( (n - f_d - 1) - \lfloor (c - f_d + 1) / 2 \rfloor - 1 \) = \( (n - f_d - 1) / 2 \) in the i-th row where \( d \) is the number of \( \lambda \) which will be eliminated during the voting of majority; therefore, the majority value of the i-th row in MAT\(_j\) is \( v_i \).

**Case 2:** \( v_j = \perp \)

There are at most \( \lfloor (c - f_d + 1) / 2 \rfloor - 1 \) arbitrary faulty links. Therefore, in the second round, the total number of \( \perp \)'s is no greater than \( \lfloor (c - f_d + 1) / 2 \rfloor - 1 + 1 = \lfloor (c - f_d + 1) / 2 \rfloor \) and number of \( v_i \)'s is at least \( (n - f_d - 1) - \lfloor (n - f_d) / 2 \rfloor + 1 = \lfloor (n - f_d - 1) / 2 \rfloor \). If \( n - d \) is an even number, then \( \lfloor (n - f_d - 1) / 2 \rfloor \) = \( \lfloor (n - f_d - 1) / 2 \rfloor \) of the majority of the i-th row in MAT\(_j\) is \( v_i \). If \( n - d \) is odd number, then \( \lfloor (n - f_d - 1) / 2 \rfloor \) = \( \lfloor (n - f_d - 1) / 2 \rfloor \), hence the majority of the i-th row in MAT\(_j\) is \( v_i \).
Theorem 3: The amount of information exchange by DWAP is $O(n^2)$.

Proof: In the first round, each node sends out $(n-1)$ copies of its initial value to other nodes. In the second round, an n-element vector is sent to the other n-1 nodes in the network; therefore, the total number of message exchange is $(n-1) + (n * (n-1))$. This result implies that the complexity of information exchange is $O(n^2)$.

V. IMPOSSIBILITY

In this section, some impossibility of the consensus problem is presented for the case of all perfect nodes on an unreliable message communication system. First, we show that the completeness of consensus by using less than two rounds of message exchange is impossible. Next, when the number of faulty links is greater than $(f_a + f_d)$ where $f_a \leq \lceil (c - f_d + 1) / 2 \rceil - 1$, it is impossible to reach a consensus. Based on these results, we can show that protocol DWAP is optimal in the sense that it uses the minimum number of rounds and can tolerate the maximum number of faulty components by the following theorems.

Theorem 4: One round of message exchange to achieve consensus is impossible.

Proof:

Part1: Message exchange is necessary.

Without message exchange, a node cannot know whether or not a disagreeable value exists in other nodes; hence, consensus achievement is impossible.

Part2: One round message exchange is not enough to achieve consensus.

If node i is connected with node j by faulty link ij. Node i may not know the initial value in node j by using only one round of message exchange. Therefore, it is impossible to achieve consensus by using only one round of message exchange.

Theorem 5: If the total number of the faulty links $t > f_a + f_d$, where $f_a \leq \lceil (c - f_d + 1) / 2 \rceil - 1$, achieving consensus is impossible.

Proof: When $t > f_a + f_d$, n is an even number and each node has c links, c is odd number, in the system. It is possible that a node has more arbitrary faulty links than perfect link even if the influence of d dormant faults was eliminated. Regardless of the number of rounds of message exchange, this node will always be confused by the messages transferred through those arbitrary faulty links. The decision making by the node may conflict with other nodes. In this case, consensus achievement is impossible.

Theorem 6: Using the minimum number of rounds, DWAP can tolerate the maximum number of faulty links in a perfect node, un-fully connected wormhole network.

Proof: From Theorems 2, 4 and 5, the theorem is proved.

VI. CONCLUSION

The consensus problem is a fundamental problem in the distributed environment. The problem has been studied by various kinds of network model in the past. According to previous studies, the network topology plays an important role in this problem. Therefore, this paper proposes a new protocol DWAP for the wormhole network and finds the bound on allowable arbitrary faulty links and dormant faulty links such that the previous protocols cannot adapt to it. The DWAP protocol redefines the consensus problem in a wormhole network and can achieve a common value if the condition $c > 2f_a + f_d$ is satisfied. In addition, the maximum number of allowable faulty links is $\lceil (c + f_d + 1) / 2 \rceil - 1$.

REFERENCE


Procedure $\text{MATRIX}$ (for node $i$ with initial value $v_i$)

Step 1: Transmit $v_i$ to all other nodes and receive the initial value $v_i$ from node $j$, for $1 \leq j \leq n$.

Step 2: Construct the vector $V_i$ = [$v_1$, $v_2$, ..., $v_k$, ..., $v_n$], $1 \leq k \leq n$. If a dormant link was found then $v_i = \lambda$.

Step 3: Transmit $V_i$ to all other nodes and receive the vector $V_j$ from node $j$, $1 \leq j \leq n$.

Step 4: Construct an $\text{MAT}_i$ (Setting the vector $V_j$ in column $j$, for $1 \leq j \leq n$), if a dormant link was found then $V_i$ = [$\lambda$,$\lambda$,...,$\lambda$].

Fig. 3. The procedure for setting $\text{MAT}_i$ on each node $i$.

Protocol $\text{DWAP}$ (for each node $n_i$ with initial value $v_i$)

Definition:

$\neg$: the complement operator, e.g., $\neg(1) = 0$.

$v_i$: the initial value of node $n_i$.

$V_i$: the vector of node $n_i$.

$\bot$: the default value.

$\lambda$: replace the value of received message by dormant faulty link.

$v_k$: the $k$-th element in vector $V_i$.

$\text{MAT}_i$: all vectors from other nodes received by $n_i$.

$\text{MAJ}_i$: a majority function used to remove the influence of a faulty links on the messages stored in the vector $V_i$ of $\text{MAT}_i$.

$\text{DEC}_i$: the decision value of node $n_i$.

Message Exchange Phase:

Round 1: Multicast the initial value $v_i$ of node $n_i$ through $c$ links to other nodes.

Receive values $v_j$ from other nodes $n_j$ connecting to $n_i$ via links.

Construct vector $V_i$ from $v_j$'s.

Round 2: Multicast the vector $V_i$ to other nodes.

Receive the vectors $V_j$ sent by other nodes $n_j$.

Construct $\text{MAT}_i$ from $V_j$'s.

Decision Making Phase:

Step 1: Remove all $\lambda$ and take the majority value of each row $k$ of $\text{MAT}_i$, as $\text{MAJ}_i$.

Step 2: If ($\exists \text{MAJ}_i = \neg v_i$), then $\text{DEC}_i = \bot$; else if ($\exists \text{MAJ}_i = ?$) AND ($v_i = v_i$), then $\text{DEC}_i = \bot$, else $\text{DEC}_i = v_i$ and halt.

Fig. 4. The proposed protocol $\text{DWAP}$.

(a) A wormhole network with ten nodes.

(b) The vector received in the first round

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\begin{array}{cccccccccc}
V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 & V_{10} \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
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(c) The corresponding MAT

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Majority value of each row of MAT_{\gamma} = 0

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(d) The majority value of each row

DEC_i=⊥ (for MAJ_i=0=v_i & v_i=1); i=1 to 5
DEC_i=⊥ (for MAJ_i=1=v_i & v_i=0); i=6 to 10

The result of DWAP, all nodes agree on the same value ⊥.

(e) The Consensus value

Fig. 5 An example of ten nodes wormhole network reaching consensus (Cont’d)