Algorithm to Elicit Utilities via the Uncertain Equivalence Method

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Abstract— An algorithm for the elicitation of single knots from the utility curve of real decision makers is proposed. The preferential equations of four utility elicitation techniques, namely probability equivalence, certainty equivalence, lottery equivalence and uncertain equivalence are derived from a generic theoretical setup of elicitation methods and a practical algorithm is elaborated for the application of the uncertain equivalence method to both monotonically increasing and decreasing preferences. The algorithm determines uncertainty intervals for the roots using triple bisection. Two example cases are discussed, and the preferences of the decision maker are elicited in one of them through a combination of uncertain equivalence – lottery equivalence algorithm.

Index Terms— Utility elicitation, decision analysis, uncertain equivalence, algorithm.

I. INTRODUCTION

The paradigms of utility theory [14] give grounds for the elaboration of techniques to elicit utilities in a lottery-based setup. Special attention is paid to the elicitation of single utility knots and the construction of the utility function. Among commonly used techniques to do that are probability equivalence (PE), certainty equivalence (CE), lottery equivalence (LE) or the recently proposed uncertain equivalence method (UE) [13].

All these elicitation schemes use the reference lottery, denoted by \(<x_{\text{best}}(p)x_{\text{worst}}\rangle\), which is a gamble between the most preferred \((x_{\text{best}})\) and the least wanted \((x_{\text{worst}})\) prize within a given set of prizes. In the lottery there is a probability \(p\) of receiving \(x_{\text{best}}\) and \((1-p)\) of ending up with \(x_{\text{worst}}\). PE proposes to the Decision Maker (DM) a prize and the reference lottery, and then changes \(p\) until the DM declares indifference, being this value of \(p\) the DM’s corresponding utility for the prize. CE offers a fixed-probability reference lottery and a prize, which is changed until indifference holds. LE eliminates the certainty effect that the other two methods lead to [3], by putting the certain prize in a 50/50 lottery. UE compares a reference lottery, with probability \(p\) times the value of the analyzed utility \(u_i\), and a lottery between the prize \(x_i\) and \(x_{\text{worst}}\) with probability \(p\); \(x_i\) is then varied until the indifference point, which defines the utility quantile value corresponding to \(u\).

For the case of CE and UE, monotonically increasing or decreasing preferences do not cause difficulties, unlike uni- or multi-modal preferences. Methods to elicit the extremes of the utility function are discussed in [11].

All the techniques generate a single estimate when they are applied to an ideal DM, that is, the one to whom utility axioms apply, and who exhibits infinite discriminating abilities. Real DMs, however, possess finite discriminating abilities, and do not follow some of the axioms. As a result, practical elicitation gives an uncertainty interval for the root. Fuzzy set theory can also be used in this case to express the DM’s degree of indifference and the degree of preference, and get a clearer insight to his/her opinion [8]-[10]. Whereas theoretical techniques use the bisection method looking only for the indifference point, triple bisection must be used in real DM cases, in order to estimate the uncertainty intervals [12]. It applies the bisection technique three times to find a point in the uncertainty interval, and its two boundaries.

The construction of the utility function over a continuous 1-D (one-dimensional) set of prizes and the objective, concept and formal preferential equations of the four equivalence methods are introduced in Section II. A practical algorithm for utility elicitation via the UE for the case of monotonically increasing and decreasing preferences and which uses the triple bisection method to find the real DM’s uncertainty interval of the root is proposed in Section III. The application of the method to two example cases of utility elicitation is presented in Section IV.

II. ELICITATION TECHNIQUES OVER MULTI-DIMENSIONAL SETS

Risky alternatives are modeled as lotteries – a number of excluding prizes \(x_i\) (consequences), and the probability \(p_i\) of receiving each: \(<x_1p_1x_2p_2...x_np_n>\) where \(\Sigma p_i=1\). The possible alternatives form the lottery set \(L\), whereas the prizes received from them form the prize set \(X\). Prizes in \(X\) can be analyzed as a whole, or can be represented as multi-dimensional vectors of attributes that measure the objectives of the decision [1]. If independence conditions hold [6], the
utility over the n-D (n-dimensional) consequences can be decomposed into n 1-D utility functions over each attribute, which requires the corresponding construction of the 1-D utility functions \( u() \). If the attribute is continuous, then the utility function over a continuous 1-D set of prizes has to be built (under the most common utility independence condition).

In the case of strictly increasing/decreasing preferences, the boundaries of \( X \) are the most \( \left(x_{\text{best}}\right) \) and the least \( \left(x_{\text{worst}}\right) \) preferred elements from the prize set. The utility curve can be approximated on several knots \( \left(x_u, u\right), \) for \( i=1,2,\ldots, M \), where \( x_u \) and \( u \) are an utility quantile and an utility quantile number. Thus, firstly it is necessary to elicit separate utility knots in such a way as to avoid biases in human cognition. At the second step, the elicited knots are used to approximate \( u() \). Some techniques succeed in the first stage, but fail to produce proper approximation due to an incorrect choice and/or insufficient number of elicited knots. [13] gives a general theoretical setup of the PE, CE, LE, and UE.

Classical methods to elicit utilities solve a preferential equation of the kind

\[
I_1 \sim I_2, \quad (1)
\]

where \( I_1=P_{I_1}, A_{I_1}; (1-P_{I_1}), B_{I_1}, I_2=P_{I_2}, A_{I_2}; (1-P_{I_2}), B_{I_2} \) (\( > \), \( > \), and \( \geq \) denoting strict preference, indifference and weak preference). Five of the parameters in (1) are fixed, and (1) is solved according to the sixth one during a dialog with the DM. When indifference holds

\[
P_1 \times u(A_1) + (1-P_1) \times u(B_1) = P_2 \times u(A_2) + (1-P_2) \times u(B_2). \quad (2)
\]

PE defines \( M \) utility quantiles \( x_{u_1}, x_{u_2}, \ldots, x_{u_i}, \ldots, x_{u_M} \), and assesses their corresponding utility quantile numbers \( \hat{u}_1, \hat{u}_2, \ldots, \hat{u}_i, \ldots, \hat{u}_M \). PE sets \( A_{1}=x_{\text{best}}, B_{1}=x_{\text{worst}}, A_{2}=x_{u_1}, \) and \( P_{1}=1 \), in (1) which leads to

\[
\langle x_{\text{best}}(P) \rangle_{x_{\text{worst}}} \sim x_{u_1}. \quad (3)
\]

Equation (3) is solved towards \( P_{1} \), and if the root is \( P^{*} \), then from (2) it follows that \( \hat{u}_i = u(x_{u_i}) = P^{*} \).

CE chooses \( M \) utility quantile numbers \( u_0=0, u_2, \ldots,u_{n-1}, u_n=1 \), and assesses the corresponding utility quantiles \( \hat{x}_{u_0}=x_{\text{worst}}, \hat{x}_{u_2}=x_{u_2}, \ldots, \hat{x}_{u_n}=x_{\text{best}} \). The substitutions \( A_{1}=x_{\text{best}}, B_{1}=x_{\text{worst}}, P_{1}=u_i \) and \( P_{2}=1 \) in (1) arrive at the CE equation

\[
\langle x_{\text{best}}(u_i) \rangle_{x_{\text{worst}}} \sim A_{2}. \quad (4)
\]

Equation (4) is solved towards \( A_{2} \), and if its root is \( A^{*} \), then from (2) it follows that \( \hat{x}_{u_i}=u(x_{u_i})=A^{*} \).

LE [7] chooses \( M \) utility quantiles \( x_{u_0}, x_{u_2}, \ldots, x_{u_i}, \ldots, x_{u_M} \), and finds the corresponding utility quantile numbers \( \hat{u}_1, \hat{u}_2, \ldots, \hat{u}_i, \ldots, \hat{u}_M \). By setting \( A_{1}=x_{\text{best}}, B_{1}=x_{\text{worst}}, A_{2}=x_{u_i}, B_{2}=x_{\text{worst}}, P_{2}=0.5 \) in (1) the LE equation is obtained

\[
\langle x_{\text{best}}(P) \rangle_{x_{\text{worst}}} \sim \langle 0.5,x_{u_i} \rangle_{0.5,x_{\text{worst}}} \quad (5)
\]

Equation (5) is solved towards \( P_{1} \), and if its root is \( P^{*} \), then from (2) it follows that \( \hat{u}_i = u(x_{u_i}) = 2 \times P^{*} \).

UE selects \( M \) utility quantile numbers \( u_0=0, u_2, \ldots,u_{n-1}, u_n=1 \), and an arbitrary probability \( p \). The corresponding quantiles \( \hat{x}_{u_0}=x_{\text{worst}}, \hat{x}_{u_2}, \ldots, \hat{x}_{u_n}=x_{\text{best}} \) are elicited. After substituting \( A_{1}=x_{\text{best}}, B_{1}=x_{\text{worst}}, B_{2}=x_{\text{worst}}, P_{1}=p \times u_i \) and \( P_{2}=p \) in (1), the UE equation is obtained

\[
\langle x_{\text{best}}(p \times u_i) \rangle_{x_{\text{worst}}} \sim \langle p,A_{2};1-p,x_{\text{worst}} \rangle. \quad (6)
\]

Equation (6) is solved towards \( A_{2} \), and if its root is \( A^{*} \), then from (2) it follows that \( \hat{x}_{u_i}=u\langle x_{u_i} \rangle = A^{*} \). \( p=2/3 \) is recommended, since it is a good compromise between the different requirements of the right hand side and left hand side for biased reduction in the perception of extreme probabilities [5].

The lotteries in (1), (3)-(6) can be visualized as a probability wheel with a winning sector, or as an urn of \( n \) balls, of which \( m=p \times n \) are white, and the rest are black. If the pointer of the probability wheel ends in the winning sector or if a white ball is drawn from the urn, the DM gets \( x_{\text{best}} \); otherwise she/he gets \( x_{\text{worst}} \). Since the ball urn is easier to be understood by DMs than the classical probability wheel [4], the arbitrary reference lottery is presented as \( \langle x_{\text{n/m}} \rangle_{x_{\text{best}}} \).

Equation (1), (3)-(6) have single roots only for ideal DMs, since real DMs, although rational, do not have infinite discriminating abilities and give an interval of indifference points as roots for the equations. Then a different formulation of the goals of elicitation is proposed for all the techniques.

If (3) is transformed into \( \langle x_{\text{best}}(m/n) \rangle_{x_{\text{worst}}} \sim x_{u_1} \), and the root is \( m \), then two things must be defined: 1) the greatest possible \( m=m_{\text{down}} \) for which the DM holds \( x_{u_1}<x_{\langle m_{\text{down}}/n \rangle x_{\text{best}}} \); 2) the smallest \( m=m_{\text{up}} \) for which the DM holds \( <x_{\langle m_{\text{up}}/n \rangle x_{\text{best}}} \). The root \( m_{\text{up}} \in (m_{\text{down}}/m_{\text{up}}) \), and the required utility \( u_i \in (m_{\text{down}}/n; m_{\text{up}}/n) \) is called uncertainty interval of the utility of \( x_{u_1} \). A point estimate could be chosen at \( \hat{u}_i = (m_{\text{up}}+m_{\text{down}}) \div 2n \). Triple bisection is used to elicit uncertainty intervals. It consists of three classical bisection phases, one finding a point in the interval, and the other two assessing its boundaries.

Similarly, in CE due to the finite discriminating abilities of the DM, (4) has an interval as the solution. The following has to be assessed for monotonically increasing preferences: 1) the greatest possible \( x_{\text{down}} \), where the DM holds \( \langle x_{\text{down}} < x_{\text{best}} \rangle \); 2) the smallest possible \( x_{\text{up}} \), where the DM holds \( x_{\text{up}} < x_{\text{best}} \). Then \( A^{*}=x_{\text{down}}; x_{\text{up}} \) \( (A^{*} = \hat{x}_{u_i}) \), whereas a point estimate can be \( A^{*}=x_{\text{up}}+x_{\text{down}} \).
If CE is applied to monotonically decreasing preferences, the following must be defined: 1) the greatest possible \( x_{\text{down}} \), where the DM holds \( x_{\text{down}} > \langle x_{\text{best}}(u_i) \rangle_{\text{xworst}} \); 2) the smallest possible \( x_{\text{up}} \), where the DM holds \( x_{\text{up}} < \langle x_{\text{best}}(u_i) \rangle_{\text{xworst}} \). Then \( A^* = (x_{\text{down}}, x_{\text{up}}) \) and \( A^* = (x_{\text{up}} + x_{\text{down}})/2 \).

For the LE case, \( \langle x_{\text{best}}(P) \rangle_{\text{xworst}} \) in (5) transforms into \( \langle x_{\text{best}}(m/n) \rangle_{\text{xworst}} \) and the root is \( m_i \). The following is required: 1) the greatest \( m_i = m_{\text{down}} \), where the DM holds \( <0.5, x_{\text{down}}; 0.5, x_{\text{worst}} > \); 2) the smallest \( m_i = m_{\text{up}} \), where the DM holds \( \langle x_{\text{down}}(m_i/n) \rangle_{\text{xworst}} > <0.5, x_{\text{down}}; 0.5, x_{\text{worst}} > \).

The root \( m_i = (m_{\text{down}} + m_{\text{up}}) \), whereas \( \hat{u}_i \in (2 \times m_{\text{down}}/n; 2 \times m_{\text{up}}/n) \), which is the uncertainty interval of the root. A point estimate is \( \hat{u}_i = (m_{\text{up}} + m_{\text{down}})/n \).

III. PRACTICAL ALGORITHMS TO ELICIT UTILITIES VIA UE

In the case of UE, the following is required for monotonically increasing preferences: 1) the greatest possible \( x_{\text{down}} \), where the DM holds \( x_{\text{down}} \geq \langle x_{\text{best}}(p \times u_i) \rangle_{\text{xworst}} \); 2) the smallest possible \( x_{\text{up}} \), where the DM holds \( <p, x_{\text{down}}(1-p), x_{\text{worst}} > \); 3) the greatest possible \( x_{\text{down}} \), where the DM holds \( \langle x_{\text{down}}(p \times u_i) \rangle_{\text{xworst}} > <p, x_{\text{down}}(1-p), x_{\text{worst}} > \) and \( A \geq 0 \) such that \( A \) is the smallest number, for which at an arbitrary \( A_i \in \{x_{\text{down}} - A \} \) the DM is not indifferent between the lotteries \( <0.5, A_i; 0.5, x_{\text{worst}} > \) and \( <p, A_i; 1-p, x_{\text{worst}} > \).

B. Practical algorithm for solving (with the triple bisection method) the UE preferential equation \( \langle x_{\text{best}}(p \times u_i) \rangle_{\text{xworst}} > <p, A; 1-p, x_{\text{worst}} > \) and go to step 11.

16. If \( x_{\text{up}} = x_{\text{up,ind}} \leq A \), then go to step 21.

17. Calculate \( A_1 = (x_{\text{up,ind}} + x_{\text{up}})/2 \).

18. Ask the DM for his preferences over the lotteries \( \langle x_{\text{best}}(p \times u_i) \rangle_{\text{xworst}} > <p, A_1; 1-p, x_{\text{worst}} > \) and go to step 14.

19. If \( <p, A_1; 1-p, x_{\text{worst}} > \), then put \( x_{\text{up}} = A_2 \) and go to step 20.

20. If \( \langle x_{\text{best}}(p \times u_i) \rangle_{\text{xworst}} > <p, A_1; 1-p, x_{\text{worst}} > \), then put \( x_{\text{up,ind}} = A_2 \) and go to step 21.

CE produces better distributed roots, which improves the approximation of the utility function compared to PE. The LE utility function is expected to show less risk-averse behavior for losses and less risk prone behavior for gains than the one of PE and CE since it avoids the certainty effect. UE is superior to CE for that same reason, and it also provides better estimates of the utility curve knots in the two lottery setup. UE performs better than LE in the distribution of the function knots and provides better utility approximation. Let’s define elicitation methods as PE-type (define prizes and find utility) and CE-type (define utilities and find prizes) ones. If applied over an arbitrary monetary interval, those would produce utility function approximation as showed on Fig.1. There are deviations from the typical form in both cases, but CE-type methods better capture the curvature of the function.

LE and UE require difficult comparisons. LE and PE have
wider application area than UE and CE, for they can assess the utility of any fixed consequence. For that reason in some practical cases, utility assessment can be improved if two methods are combined, as shown in one of the example cases in the next section.

IV. APPLICATION TO DOMESTIC HEATING SYSTEMS, AND DETERMINATION OF OPTIMAL SPARE PARTS FOR SHIP DIESEL ENGINES

UE is applied to two practical decision analysis examples. The first example is the assessment of the utility function over the NPV (Net Present Value) of average annual heating costs of the alternatives for a domestic heating system. The second example case discusses the definition of an optimal set of spare parts for ship diesel engine. Two types of consequences occur depending on the possibility of technical failures of the system. Two normalized utility functions are constructed and afterwards rescaled to form the complete utility function over the whole set of consequences.

A. Domestic Heating System Choice

Analyzed are several ways to heat an urban panel flat, with area of 80m², consisting of two bedrooms, a lining room, kitchen and service compartments, with a standard height of 2.5m, outward isolation of the north wall and PVC framing. Till now only electrical heaters were used. The possible heating alternatives include the installation of a coal-wood furnace system, electrical furnace system, air-conditioners, gas furnace system, or to keep the current heating system. The decision is made from the point of view of a typical Bulgarian household representative. The time horizon of the decision is 10 years (2006-2015). Eight objectives are outlined in the decision, namely minimize heating costs, minimize investment, minimize installation time, maximize accessibility to hot water during the year, minimize labor consumption, minimize inertia of the system, maximize space control, and heat service compartments. An 8-dimensional vector of consequence is formed with one coordinate for each objective. Minimizing heating costs in particular is measured by the continuous parameter NPV of average annual costs in BGN (Bulgarian Lev). All other objectives are analyzed by discrete (nominal) parameters. Analysis of inflation and price changes of energy resources showed that for all alternatives this parameter varies in the interval \([\text{NPV}_{\text{min}}; \text{NPV}_{\text{max}}]\) \([1000; 7000]\). The utility function \(u_c(\cdot)\) over the NPV of average annual costs is built in the interval \([1000; 7000]\). Obviously, \(u_c(1000)=1\), \(u_c(7000)=0\). Nine values of utility are chosen with a step 0.1. The UE algorithm for monotonically decreasing preferences is applied to assess the uncertainty intervals of the NPVs that have those utilities. The point estimates of the utilities are: \(u_c(5615.1)=0.1\), \(u_c(4784.9)=0.2\), \(u_c(4197.2)=0.3\), \(u_c(3730.4)=0.4\), \(u_c(3324.6)=0.5\), \(u_c(2943.2)=0.6\), \(u_c(2557.7)=0.7\), \(u_c(21381)=0.8\) and \(u_c(1643)=0.9\) as shown in Fig. 2. The utility curve reflects typical risk attitude of DMs over monetary prizes [2].

B. Application to Determine the Optimal Spare Parts for a Ship Diesel Engine

Let the set of spare parts of a vessel diesel engine contains \(q\) different types of elements in each of the \(n_k\) cylinders. The set is presented as \(q\)-dimensional vector \(\mathbf{R}=(m_1, m_2, \ldots, m_q)\). If the price of the \(m_k\)-th element of \(k\)-th type is \(c_k(m_k)\) (thousand USD), then the price \(y\) of the whole set is \(y=\sum_{k=1}^{q} c_k(m_k)\). The minimal and maximal number of elements of \(k\)-th type are \(m_k^{\text{min}}\) and \(m_k^{\text{max}}\). The probability for the \(k\)-th
element to be repaired is $p^{pp}_k$. If it is not repaired or if a spare part for the element is not available, the engine is not operational. The set of spare parts shall not be renewed within the next $T$ hours of machine time. Let’s denote $P_k(m_k)$ for $m_k = m^{\min}_k, \ldots, m^{\max}_k$, and $k = 1, 2, \ldots, q$ the probability that there will be no technical failure for $T$ hours due to the $k$-th type of element if the spare parts were initially $m_k$. Then the probability of not having technical failure for time $T$ with a set of spare parts $\tilde{R}$ is $P(\tilde{R}) = \prod_{k=1}^q P_k(m_k)$. The cheapest possible set is $y_{\min} = \sum_{k=1}^q c_k(m^{\min}_k) = 39$, whereas the most expensive is $y_{\max} = \sum_{k=1}^q c_k(m^{\max}_k) = 120.2$. For any sum $c \in [y_{\min}; y_{\max}]$, it is possible to identify an optimal set $\tilde{R}^{opt}(c) = (m_1^{opt}, m_2^{opt}, \ldots, m_q^{opt})$ with the maximal probability $p(c)$ for flawless work for time $T$ among all the sets whose price does not exceed the allocated sum $c$. If several sets have equal $p(c)$, the cheapest is the optimal one.

The sum for spare parts $c \in [39; 120.2]$. The DM faces the consequence $x_{13}(c)$, i.e. $y$ expenses and flawless work for time $T$ with probability $p_1(c) = p(c)$, or the consequence $x_{23}(c)$, i.e. $y$ expenses and at least one technical failure for time $T$ with probability $p_2(c) = 1 - p_1(c)$. The utility functions over the consequences can be divided into two parts:

$$u_{[x_{13}(c)]=\begin{cases} v(y(c)) = v(y), \text{ for } i=1 \\ w(y(c)) = w(y), \text{ for } i=2. \end{cases}$$ (7)

A combined approach is applied to assess the utility function, using both UE and LE:

1. Assess the two normalized functions $v^N(\cdot)$ and $w^N(\cdot)$ using UE over the whole interval of possible prices $[39; 120.2]$. 
2. Use LE to elicit the utility of paying the maximum sum and experience no failure $x_{13,120.2}$, and of paying the lowest possible sum and have at least one technical failure $x_{23,39}$. 
3. Rescale the normalized utility functions assessed at 1) with the utility knots estimated at 2) to form the utility function (7) over the consequences of the problem, and calculate the uncertainty intervals of the knots.

Step 1: $p = 2/3$ is set in (6) and $M=9$ utility quantile numbers $v_i^N$ and $w_i^N$ (see columns 2 and 9 of Table I) are chosen. The utility quantiles are assessed via UE algorithm at monotonically decreasing preferences. The elicited utility quantiles are shown shaded in columns 4, 5, 11, and 12 of Table I. Calculated point estimates $\hat{y}_i$ and $\hat{y}_n$ are given in columns 3 and 10 of Table I. The interpolated normalized functions with the uncertainty intervals of the knots are depicted on the upper part of Fig. 3.

Step 2: The uncertainty interval $(\hat{v}_i^d; \hat{v}_i^u)$ of the utility quantile number $v_i(x_{13,29})$ is assessed via LE. The results are shown in bold in the last row of columns 14 and 15 of Table I. The same is given in columns 7 and 8 of Table I for the uncertainty interval $(\hat{v}_i^d; \hat{v}_i^u)$ of the utility quantile number $v_i(x_{23,29})$.

Step 3: The results from step 1 and 2 are used to rescale the normalized utility functions by the formulae

$$\hat{v}_i = v_i^N(1 - \hat{v}_i) + \hat{v}_i, \quad \hat{v}_i^d = \hat{v}_i - 0.5 \times (\hat{v}_i - \hat{v}_i^d) (\hat{y}_i - \hat{y}_n) (\hat{y}_n - \hat{y}_i), \quad \hat{v}_i^u = \hat{v}_i + 0.5 \times (\hat{v}_i - \hat{v}_i^u) (\hat{y}_i - \hat{y}_n) (\hat{y}_n - \hat{y}_i), \quad \hat{w}_i = w_i^N \times \hat{y}_i, \quad \hat{w}_i^d = \hat{w}_i - 0.5 \times (\hat{w}_i - \hat{w}_i^d) (\hat{y}_i - \hat{y}_n) (\hat{y}_n - \hat{y}_i), \quad \hat{w}_i^u = \hat{w}_i + 0.5 \times (\hat{w}_i - \hat{w}_i^u) (\hat{y}_i - \hat{y}_n) (\hat{y}_n - \hat{y}_i).$$ (8-13)

The uncertainty intervals $(\hat{v}_i^d; \hat{v}_i^u)$ of the utility quantile numbers for the rescaled function $v_i(\cdot)$ are shown in columns 7 and 8 of Table I, whereas the point estimates $\hat{V}_i$ are in column 6. The uncertainty intervals $(\hat{w}_i^d; \hat{w}_i^u)$ of the utility quantile numbers for the rescaled function $w_i(\cdot)$ are shown in columns 14 and 15 of Table I, whereas the point estimates $\hat{W}_i$ are in column 13. The utility function $u(\cdot)$ is depicted in the lower part of Fig. 3. The use of both LE and UE result in errors in the elicited utility knots in both coordinates. When only LE is used the estimates err only in one coordinate, but the
uncertainty of estimates does not decrease, and the quality of approximation is much lower due to the flaws of LE compared to UE.

V. CONCLUSION

The paper discussed four techniques for utility elicitation, namely PE, CE, LE and UE. UE is emphasized and algorithms for monotonically increasing and decreasing preferences over 1-D continuous prize set are proposed. Both use the triple bisection method to define the uncertainty interval of the root. Special cases are also discussed for CE. Proper choice of knots, adequate elicitation and good approximation of the utility function are of importance when comparing utility techniques. As Fig. 1 showed, CE-type techniques, although being in most cases difficult to apply, give better overall results than PE-type methods.

Theoretical comparisons showed that CE surpasses PE in the approximation precision. LE is an improved version of PE, whereas UE is to be preferred over LE due to more adequate estimates and accurate approximation of the utility function.

The UE method was applied to two decision analysis problems of different complexity and structure. As shown in the second example case, because of their complexity, UE and LE could be applied simultaneously in the utility analysis, in which a better overall performance is achieved.

In addition to being meaningful, a numerical statement defined during elicitation should also be related to a statement well understood by the DM. Due to the complicated comparisons both the LE and UE fail to meet this requirement. That is why results from LE and UE should be subjected to the “quick and dirty” correction procedures, based on Kahneman and Tversky’s prospect theory.

Uncertainty intervals are useful within quantitative decision analysis for two reasons. One is that interval of values for the root is the best that people can do. Finding the whole interval allows to properly define a point estimate for further calculation procedures, and thus improve the quality of the decision analysis results. On the other hand, uncertainty intervals allow testing the sensitivity of the decision to human imprecision. Simulation techniques might be a good approach to this problem.

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