Guaranteed Cost Control for T–S Fuzzy Neutral Systems with Time-Varying Delays

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In this paper, we focus on the guaranteed cost control for nonlinear neutral systems with time-varying delays represented by Takagi-Sugeno (T-S) fuzzy model. A linear quadratic cost function is considered as a performance index of the nonlinear neutral systems. The problem addressed that of designing a guaranteed cost control that guarantees the asymptotically stable of the nonlinear neutral systems. The sufficient stability conditions are derived via Lyapunov–Krasovskii functionals (LKF). When these conditions, which are given in terms of linear matrix inequalities (LMIs), are satisfied, the designed controller gain matrices can be obtained via a convex optimization. Furthermore, an illustrative example is provided to demonstrate the effectiveness of the proposed design procedure.

Index Terms—Neutral time-delay system, Takagi-Sugeno (T-S) fuzzy systems, linear matrix inequality (LMI)

I. INTRODUCTION

These years, the well-known Takagi-Sugeno (T-S) fuzzy model has been recognized as a popular and powerful tool in approximating and describing complex nonlinear systems. As a consequence, the study of T-S fuzzy systems has been gained much attention in the past decades. Nonlinear systems with time delay constitute basic mathematical models of real phenomena, for instance in biology, mechanics, and economics. Stability analysis and synthesis of retarded systems is an important issue addressed by many authors and for which surveys can be found in several monographs. A typical approach for the analysis and synthesis of nonlinear system with time delay is the local linearization approach. First a linearization model on the nominal operating point is gotten and then a linear feedback control is designed for this linear model. In particular, some delay-independent stability conditions and stabilization approaches have been proposed for these linear delay differential equations. Results are readily available in the literature.

The problems of stability and stabilization of neutral linear systems have been proposed [1,2,14,16]. For linear neutral systems, some delay-independent stability conditions were obtained. They were formulated in terms of matrix measure and matrix norm [6] or the existence of a positive definite solution to an algebraic Riccati matrix equation [16]. Delay-dependent stability criterion are obtained by employing Lyapunov-Krasovskii functional [1,14,2] and the Razumikhin theorem approach [13]. To date, however, the systems that the above papers consider are linear and no results on the problem of nonlinear neutral time-delay systems have been reported.

It has been shown that within the framework of T–S fuzzy model and PDC control design, conditions for the stability and performance of a system can be formulated into a LMI problem [5,6,7]. It is a straightforward idea to extend the T–S model to the nonlinear systems with time-delay. Recently, fuzzy systems with time delays have been introduced in [8,9,10], where several stability analysis results were also designed. Cao and Frank considered the analysis and synthesis problem for nonlinear retarded systems, which depend on only state delays, via PDC approach [8,9,10,11,13]. However, to the best of the author’s knowledge, there are no results on the problem of nonlinear neutral systems via T–S fuzzy model have been reported in the literature.

Motivated by the above observations, in this paper, we extend the above procedure to the nonlinear systems with time delay. In the proposed design procedure, first a given nonlinear retarded system is represented by the T-S model with time delay. This fuzzy modeling method is simple and natural. The system dynamics is captured by a set of fuzzy implications which characterize local relations in the state space. The main feature of the T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear state-space system model with time delay. The overall fuzzy model of the system is achieved by fuzzy “blending” of the linear delay models.

This paper is organized as follows. The T–S fuzzy neutral systems are presented in Section 2. In Section 3, delay-dependent stabilization criteria are derived. In Section 4, a numerical example is presented to show the effectiveness of the results. The paper is concluded in Section 5.

Notation: \( \mathbb{R}^n \) denotes the n-dimensional real Euclidean space; \( I \) is the identity matrix with appropriate dimension; the superscripts “\(^T\)” and “\(−1\)” stand for matrix transpose and inverse; \( R > 0 \) (\( R \geq 0 \)) means that \( R \) is real, symmetric and positive-definite(positive semidefinite); the star * denotes a block induced by symmetry.

II. PRELIMINARIES

Consider a nonlinear system with time-varying delays \( \tau(t) \) and \( g(t) \).

\[
\dot{x}(t) - f_1(z(t), \dot{x}(t - g(t))) = f_2(z(t), x(t), x(t - \tau(t)), u(t)) \tag{1}
\]

which can be described by the following T-S fuzzy neutral
model:

\[ R^i: IF \ z_i(t) (t) is \ \Gamma^i_1 \ and \ ... \ and \ \ z_p(t) (t) is \ \Gamma^i_p \]

\[ \quad \text{THEN } \dot{x}(t) - Cx(t - g(t)) = A_0x(t) + A_i x(t - \tau(t)) + B_i u(t) \]

(2)

where \( R^i, \ i \in I_r = \{1, 2, ..., r\} \), denotes the \( i \) th fuzzy rule, 
\( z_i(t), \ h \in I_\rho = \{1, 2, ..., p\} \), is the \( i \) th premise variable, \( \Gamma^i \), 
\( (i, h) \in I_r \times I_\rho \), is the fuzzy set of \( z_h(t) \) in \( R^i \), \( A_0, A_i \) and 
\( B_i \) are constant matrices with appropriate dimensions 

In this paper, the following fuzzy rule for the fuzzy controller is employed:

\[ R^i: \text{If } z_i(t) (t) is \ \Gamma^i_1 \ and \ ... \ and \ \ z_p(t) (t) is \ \Gamma^i_p \]

\[ \text{then } u(t) = K_i x(t). \]

Using the singleton fuzzifier, product inference engine, and center-average defuzzification, (2) and (3) are inferred as

\[ \dot{x}(t) = \sum_{i=1}^{r} \theta_i(z(t))(A_0x(t) + A_i x(t - \tau(t)) + B_i u(t) + Ci(t - g(t))), \]

(4)

\[ u(t) = \sum_{i=1}^{r} \theta_i(z(t))K_i x(t), \]

(5)

respectively, where \( \theta_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{r} \omega_i(z(t)) \), 

\( \omega_i(z(t)) = \prod_{h=1}^{p} \mu_{\Gamma^i_h}(z_h(t)), \mu_{\Gamma^i_h}(z_h(t)): U \rightarrow [0, 1] \)

is the membership function of \( z_h(t) \) on the compact set \( \Gamma^i_h \). 

Some basic properties are \( \theta_i(z(t)) \geq 0 \) and \( \sum_{i=1}^{r} \theta_i(z(t)) = 1 \). 

The initial condition of system (1) is given by

\[ x(t + \nu) = \phi(\nu), \quad \dot{x}(t + \nu) = \phi(\nu) \]

for all \( \nu \in [-\max g(t), \tau(t)], [0] \), where \( \phi(\cdot) \) is a continuous vector-valued initial function.

Given positive-definite symmetric matrices \( Q \) and \( R \), we consider the following linear quadratic cost function

\[ J = \int_{0}^{\infty} [\dot{x}^T(t)Qx(t) + u^T(t)Ru(t)]dt. \]

(6)

Associated with the cost function (6), the fuzzy guaranteed cost control is defined as follows.

Definition: Consider the T-S fuzzy system (4). If there exists a fuzzy controller (5) and a scalar \( J_o \) such that the closed-loop system is asymptotically stable and the closed-loop value of the cost function (6) satisfies \( J \leq J_o \), then \( J_o \) is said to be a guaranteed cost and the fuzzy controller \( u(t) \) is said to be a guaranteed cost controller for (4).

The objective of this paper is to design a guaranteed cost controller gain \( K_i \). The following lemma will play important roles in obtaining results in this paper. We show it as follows.

Lemma 1. Suppose that \( a \in R^n, \ b \in R^n \) and \( J \in R^n \times R^n \).

Then, for any matrices \( M = M^T \in R^n \times R^n, N \in R^n \times R^n \), and 
\( L = L^T \in R^n \times R^n \), the following inequality holds:

\[ -2a^TJb \leq \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} M & * \\ * & J \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \]

\[ \text{if } \begin{bmatrix} M & * \\ * & J \end{bmatrix} \leq 0. \]

Lemma 1 is straightforward and useful in this paper.

III. MAIN RESULTS

In the following, we present the delay-dependent stabilization conditions via state feedback fuzzy control.

Theorem 1. The fuzzy controller (4) with \( K_i = F_i W_i^{-1} \)

asymptotically stabilizes the equilibrium point of (2) if \( \sup_{t \in [0, \infty]} |z_1(t)| = 1 \) and there exist \( W_1 = W_1^T > 0, W_2, W_3, X = X^T > 0, y = Y^T > 0, Z = Z^T > 0 \),

\[ U_{1i} = U_{1i}^T, U_{2i} = U_{2i}^T, U_{3i} = U_{3i}^T, F_i \]

such that, for the prescribed scalars \( \epsilon_i \),

\[ \begin{bmatrix} \Pi_{1i} & * & * & * & * & * \\ \Pi_{2ij} & \Pi_{3i} & * & * & * \\ 0 & \Pi_{4i} & * & * & * \\ 0 & X C_i^T & 0 & -X & * \\ W_1 & 0 & 0 & 0 & -(1-\dot{\tau}(t))Z \\ W_2 & W_3 & 0 & 0 & 0 & -X \\ \tau_0 W_2 & \tau_0 W_3 & 0 & 0 & 0 & 0 & -\tau_0 Y \end{bmatrix} < 0 \]

(7)

for all \( (i, j) \in I_r \times I_r \) excepting the pairs \( (i, j) \) such that \( \theta_i(z(t))\theta_j(z(t)) = 0 \quad \forall t \), and

\[ \begin{bmatrix} U_{1i} & * & * \\ U_{2i} & U_{3i} & * \end{bmatrix} < 0 \]

(8)

for all \( i \in I_r \) where \( \Pi_{1i} = H c(W_2^{T}) + \tau_0 U_{1i} \).
\( \Pi_{2ij} = A_j W_1 + B_j F_j + e_i A_j W_1 - W_2 + W_2^T + r_0 U_{2i} \)

\( \Pi_{3j} = -He_3 W_3 T + r_0 U_{3j} \), and \( \Pi_{4i} = (1 - e_i) Z A_i^T \).

**Proof.** We omit proof here because of space limit.

When LMIs (7) are feasible, each guaranteed cost controller ensures the asymptotically stability of the resulting closed-loop system and an upper bound of the closed-loop cost function given by (4). In view of this, it is desirable to find an optimal guaranteed cost controller which minimizes the upper bound. This problem is dealt with in the following theorem.

**Theorem 2.** Consider the system (2) associated with cost function (4). Suppose the following optimization problem

\[
\min_{\alpha, W, X, Y, Z, M_1, M_2, M_3} \{ \alpha + \tau_r (M_1 + M_2 + M_3) \}
\]

subject to

1) LMIs (7);

2) \[
\begin{bmatrix}
-\alpha & \xi^T(0) \\
\xi(0) & -W
\end{bmatrix} \succ 0;
\]

3) \[
\begin{bmatrix}
-M_1 & \Omega_1^T \\
\Omega_1 & -X
\end{bmatrix} \succ 0;
\]

4) \[
\begin{bmatrix}
-M_2 & \Omega_2^T \\
\Omega_2 & -Y
\end{bmatrix} \succ 0;
\]

5) \[
\begin{bmatrix}
-M_3 & \Omega_3^T \\
\Omega_3 & -Z
\end{bmatrix} \succ 0;
\]

has solutions \( \alpha, W, X, Y, Z, M_1, M_2, M_3 \), where

\[
\Omega_1^T \xi_1 = \int_0^{\tau_0} x(s)x(s)^T ds, \quad \Omega_2^T \xi_2 = \int_0^{\tau_0} x(s)x(s)^T ds,
\]

\[
\Omega_3^T \xi_2 = \int_0^{\tau_0} x(s)x(s)^T ds.
\]

Then, the corresponding guaranteed cost controller in the form of (9) is an optimal guaranteed cost controller in the sense that under this controller the upper bound on the closed-loop cost function (4) is minimized.

**Proof.** We omit proof here because of space limit.

**IV. SIMULATION RESULTS**

In this section, we use an example to illustrate our LMI-based results. The solver used in LMI Toolbox in Matlab. The example is to show the merits and the use of our result in Theorem 1. Consider the nonlinear system proposed in [13] which is a nonlinear system with time delays expressed by the following T–S fuzzy model. The pendulum angle \( (x_1) \) and the angular velocity \( (x_2) \) satisfies

\[
x_1(t) = x_2
\]

\[
x_2(t) = \frac{g \sin(x_1) - am \dot{x}_2 \sin(2x_1)/2 - a \cos(x_1) u}{4l/3 - am \cos^2(x_1)} + w
\]

Where \( g = 9.8 \) is the gravity constant, \( w \) is the external disturbance which is assumed to be \( w = \cos(2 \pi t) \), \( 2l \) is the length of the pendulum, \( m \) is the mass of the pendulum, \( M \) is the mass of the cart, and \( a = \frac{1}{(m + M)} \).

To illustrate the proposed results on the above systems, we assume that the system \( x_1(t) \) and \( x_2(t) \) are perturbed by time-delay. The neutral system is given as

\[
\dot{x}_1(t) = x_2
\]

\[
\dot{x}_2(t) - \dot{x}_2(t+\tau(t)) = \frac{g \sin(x_1) - am \dot{x}_2 \sin(2x_1)/2 - a \cos(x_1) u}{4l/3 - am \cos^2(x_1)} + w
\]

where \( \tau(t) = \sin t \), \( g(t) = 0.1 \cos t \). In this example \( b = 0.5 \).

Then the nonlinear neutral system (11) can be exactly represented by the following T–S fuzzy models:

\[
A_1 = \begin{bmatrix} 0 & 1 \\ -17.2941 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix},
\]

\[
B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},
\]

\[
E_1 = \begin{bmatrix} -14.24 \\ -13.54 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -14.11 \\ -7.14 \end{bmatrix}.
\]

The membership functions as

\[
h_1(\theta) = (1 - \frac{1}{1 + \exp(-7(\theta + 0.25 \pi))}) \cdot \frac{1}{1 + \exp(-7(\theta + 0.25 \pi))},
\]

\[
h_2(\theta) = 1 - h_1(\theta).
\]

Obviously, the membership functions are continuous and piecewise continuously differentiable and, hence, the result of this thesis can be used to design the stabilizing controller for the neutral T–S fuzzy system.

Applying Theorem 1 and solving the associated LMI (7), the following controller gain matrices are obtained:

\[
F_1 = \begin{bmatrix} 2.555 \\ 0.514 \end{bmatrix},
\]

\[
F_2 = \begin{bmatrix} 1.3973 \\ 0.2154 \end{bmatrix}.
\]

With the fuzzy control applied, the state trajectories of the closed-loop systems are shown in Figures 1-2. The initial condition of the fuzzy system is in (1 1). The simulation results indicate that the designed fuzzy state-feedback
controllers can stabilize the nonlinear neutral systems. Obviously, the systems are stabilized.

![Fig. 1. Closed loop system response of state $x_1(t)$](image1)

![Fig. 2. Closed loop system response of state $x_2(t)$](image2)

V. CONCLUSION

In this paper, the new method for NCS has been proposed using the sampled-data fuzzy controller. The sampled-data fuzzy controller can be implemented digital computer to reduce the time and cost. LMI based sufficient stability conditions have been derived based on Lyapunov-Krasovskii functional. An application example has been given to show the effectiveness of the proposed method.

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