Processing of the outliers included in a ship's position obtained from GPS via Particle Filter

Daisuke Terada
National Research Institute of Fisheries Engineering, Fisheries Research Agency
Hasaki7620-7, kamisu, Ibaraki, 314-0408
Email: dterada@frc.affrc.go.jp

Abstract— A new technique for an estimation of a ship's position is introduced. In general, since an observed position signal with GPS has a large noise component, the probability density function of the observation noise does not satisfy the assumption of Gaussian sequence. In order to solve this problem, sequential data assimilation by using Monte Carlo Filter (MCF) which is a kind of a Particle Filter is introduced. Moreover, in this study, we assume Cauchy distribution as the probability density function of the observation noise. The verification of the proposed technique is carried out by numerical experiments and onboard experiments. From the comparison of the proposed procedure and the procedure by using Kalman Filter, it is confirmed that the proposed procedure can remove the influence of the large noise component.

I. INTRODUCTION

Recently, in the field of the maritime science, the research to realize an advanced control of the ship of the course tracking control etc. is carried out aggressively. In this case, the highly accurate estimation of the ship’s position is most important point.

As a current previous work, there is a navigation filter with the Kalman Filter (KF) developed by Fukuda et al. [1]. In this method, the estimation of the ship’s position is realized by using the observed signal with GPS. And, since the calculation algorithm is very simple, it is widely used as a practicable ship’s position estimation method [2]. However, the problem of this method is in the point assumed to be able to explain the observational noise of the observed signal with GPS by the linear Gaussian model. Because of the influence of the switch timing etc. of the GPS satellite, the ship's position obtained from GPS includes the outliers [2]. It is difficult for the linear Gaussian model to explain such a phenomenon.

In this study, to solve this problem concerning the position estimation of the ship, the nonlinear non-Gaussian model is introduced as the ship’s position estimation method. Traditional method and this model have two different points. First, as to a system model in a state-space model, a simulation model for calculation due to computer which is an approximation of the physical model [3] is used. On the other hand, as to an observation model in the state-space model, the Cauchy distribution [4] is assumed as the probability density function of the observation noise.

In order to examine the proposed procedure, numerical simulation and onboard experiments are carried out. As the results, from the comparison of the proposed procedure and the procedure by using KF, since it is confirmed that the proposed procedure can remove the influence of the large noise component, we conclude that the proposed procedure realizes the highly accurate estimation of the ship’s position.

II. TRADITIONAL METHOD BY KALMAN FILTER FOR THE ESTIMATION OF SHIP’S POSITION [1]

Consider that the coordinate system for ships maneuvering model is defined by Figure 1. And, let a measured ship’s position \([x’,y’]\) with GPS, then a state space model for the dynamics of the ship motions can be expressed as follows:

\[
\begin{align*}
x(n) &= 1 0 \Delta t 0 0 \frac{1}{2} \Delta t^2 0 \\
y(n) &= 0 1 0 \Delta t 0 0 \frac{1}{2} \Delta t^2 \\
v_x(n) &= 0 0 1 0 \Delta t 0 \\
v_y(n) &= 0 0 0 1 0 \Delta t \\
\alpha_x(n) &= 0 0 0 0 1 \\
\alpha_y(n) &= 0 0 0 0 0 \\
\end{align*}
\]

\[
\begin{align*}
x(n-1) \\
y(n-1) \\
v_x(n-1) \\
v_y(n-1) \\
\alpha_x(n-1) \\
\alpha_y(n-1) \\
\end{align*}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} w_x(n) \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} w_y(n)
\]

Fig.1 Coordinate system for ships maneuvering model
Here, $x(n)$ is a position for X axis that is removed the influence of observation noise, $y(n)$ is a position for Y axis that is removed the influence of observation noise, $v_x(n)$ is a velocity for X axis, $v_y(n)$ is a velocity for Y axis, $a_x(n)$ is an acceleration for X axis, $a_y(n)$ is an acceleration for Y axis, $\Delta t$ is a sampling interval of the observation, $w_x(n)$ is a system noise adding $a_x(n)$ that is a white noise with mean zero and variance $\tau$, $w_y(n)$ is a system noise adding $a_y(n)$ that is a white noise with mean zero and variance $\tau$, and $[u_x(n), u_y(n)]^T$ is an two dimensional observation noise that is a Gaussian white noise sequence with mean vector and variance-covariance matrix $\mathbf{I}_2$. For simplicity of the expression, Equation 1 and 2 are expressed by the following vector representation

\[
\begin{bmatrix}
x(n)
y(n)
v_x(n)
v_y(n)
a_x(n)
a_y(n)
\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 
\end{bmatrix} \begin{bmatrix} x(n) \\
y(n) \\
v_x(n) \\
v_y(n) \\
a_x(n) \\
a_y(n) 
\end{bmatrix} + \begin{bmatrix} u_x(n) \\
u_y(n) 
\end{bmatrix}.
\tag{2}
\]

Since Equation 3 is the linear Gaussian state-space model, we can use the Kalman Filter for the state estimation. The algorithm is as follows:

\[\begin{align*}
\text{[One step ahead prediction]} \\
x[n|n-1] &= Fx[n-1|n-1] \\
V[n|n-1] &= FV[n-1|n-1]F^T + \frac{\tau^2}{\sigma^2}GG^T, \\
\text{[Filter]} \\
K(n) &= V[n|n-1]H(n)^T(H(n)V[n|n-1]H(n)^T + \sigma^2I)^{-1} \\
x[n] &= x[n|n-1] + K(n)(y(n) - H(n)x[n|n-1]) \\
V[n] &= (I - K(n)H(n))V[n|n-1].
\end{align*}\tag{4}
\]

Here,

\[\begin{align*}
x[n|n-1] &= \mathbb{E}[x(n)|y(1), \ldots, y(n-1)] \\
V[n|n-1] &= \mathbb{E}[(x(n) - x[n|n-1])(x(n) - x[n|n-1])^T].
\end{align*}\tag{5}
\]

III. ESTIMATION METHOD OF SHIP’S POSITION BASED ON SEQUENTIAL DATA ASSIMILATION BY USING MONTE CARLO FILTER

In this study, we introduce three statistical techniques for the state estimation with respect to Equation 3. First, as to the observation noise in the observation model, the Cauchy distribution as the probability density function is assumed because the ship’s position obtained from GPS includes the outliers by the influence of the switch timing etc. of the GPS satellite. This distribution can express a heavier-tailed distribution. Two dimensional Cauchy distribution is defined as follows [5]:

\[
C(x, y|\sigma) = \frac{1}{2\pi\sigma^2} \cdot \left[1 + \left(\frac{x}{\sigma}\right)^2 + \left(\frac{y}{\sigma}\right)^2\right]^{-\frac{1}{2}}.
\tag{6}
\]

Here, $\sigma^2$ is the dispersion parameter and $\nu$ is the parameter to guarantee the characteristic of the probability density function.

Second, assume that the characteristics of the noise in the state-space model indices vary from hour to hour. Then as the one of the model we can consider the following equation;

\[
\theta(n) = \theta(n-1) + \epsilon(n),
\tag{7}
\]

where, $\theta(n) = [\log\sigma^2(n), \log\nu^2(n)]^T$, $\epsilon(n) \sim N(0, \Sigma)$ is the two-dimensional white noise sequence vector and the elements of the variance-covariance matrix can be expressed by using the assumption of the independency relationship between variables as follows:

\[
\Sigma = \text{diag}(\eta_1^2, \eta_2^2).
\tag{8}
\]

Note that the notation T means the transpose of the vector. In this case, $\eta_1^2$ and $\eta_2^2$ are a hyper-parameter that controls the trade-off between the good fit to the data and the smoothness of the parameter change in the model.

Consider that it is including the parameter vector $\theta(n)$ in the state vector $x(n)$. Then, we can define the following extended state vector $x(n)$ and noise vector $v(n)$ and $u(n)$,

\[
\begin{align*}
x(n) &= \begin{bmatrix} x(n)^T \theta(n)^T \end{bmatrix} \\
v(n) &= \begin{bmatrix} Gw(n)^T \epsilon(n)^T \end{bmatrix} \\
u(n) &= \begin{bmatrix} u(n)^T \theta(n)^T \end{bmatrix}.
\end{align*}
\tag{9}
\]

Here, $\theta$ is the row vector which is all elements zero.

From Equation 9, we can extend the state-space model into the following self-organizing state-space model (SOSS) [6]

\[
\begin{align*}
\dot{x}(n) &= f(z(n-1), v(n)) \\
y(n) &= h(x(n), u(n)).
\end{align*}
\tag{10}
\]

Final, the SOSS model is the nonlinear non-Gaussian model. Therefore, in order to implement the state estimation of Equation 9, we apply a Monte Carlo filter (MCF) was proposed by Kitagawa [7] which is effective to the nonlinear non-Gaussian state-space modeling. In this part, note that for simple expression the time step replace from $n$ to subscript $n$. In this method, each probability density function that is the predictor $p(x(n|Y_{n-1})$ and the filter $p(x(n|Y_n))$; where $Y_n$ is the set
of observations \{y_1, \ldots, y_n\}, is approximated by \( j \) particles, which can be regarded as independent realizations from that distribution. According to Kitagawa [7], it can be shown that these particles can be recursively given by the following MCF algorithm:

[Step 1] Generate a \( k + l \) dimensional random number \( f_0^{(i)} \sim p_0(z) \) for \( i = 1 \sim j \).

[Step 2] Repeat the following steps for \( n = 1 \sim N \).

(a) Generate an \( l \) dimensional random number \( u^{(i)} \sim q(u) \) for \( i = 1 \sim j \).

(b) Compute the following equation:

\[
p^{(i)}_n = F(f^{(i)}_{n-1}, u^{(i)}).
\]

(c) Compute the likelihood function as follows:

\[
\alpha^{(i)}_n = \frac{1}{2\pi\sigma^2} \left\{ 1 + \left[ \frac{y_n - \begin{bmatrix} H & \mathbf{0} \end{bmatrix} p^{(i)}_n}{\sigma} \right]^2 \right\}^{\frac{3}{2}}.
\]

(d) Generate \( f^{(i)}_n \) according the following probability for \( n = 1 \sim N \) by the resampling of \( p^{(i)}_n \sim p^{(i)}_n \).

\[
\Pr(f^{(i)}_n = p^{(i)}_n) = \frac{\alpha^{(i)}_n}{\alpha^{(i)}_1 + \cdots + \alpha^{(i)}_j}.
\]

(e) Return to (a).

### IV. NUMERICAL EXPERIMENTS

In this study, the sample model is the training ship *Shioji Maru* shown in Figure 2 that belongs to Tokyo University of Marine Science and Technology. The principal particulars of *Shioji Maru* are summarized in Table 1.

<table>
<thead>
<tr>
<th>Principal Particulars of Shioji Maru</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (P.P.)</td>
</tr>
<tr>
<td>Breadth (M.LD)</td>
</tr>
<tr>
<td>Depth (M.LD)</td>
</tr>
<tr>
<td>Draught (M.LD)</td>
</tr>
<tr>
<td>Displacement</td>
</tr>
</tbody>
</table>

The data for the numerical experiments was represented by the MMG model [8]. In this case, the ship speed is 12 knots, the rudder angle is 35 degrees. The pseudo observation data is made by adding the noise according to the Cauchy distribution. The true and the pseudo observation data are shown in Figure 3.

The results of state estimation are shown in Figure 4 and 5. In this figure, the left hand side shows the results of the trajectory of ship, the right hand side is the results of the deviation of x axis and the y axis from above, respectively. In this figure, the black thin line shows the true value, the red thin line shows the results of the state estimation with the MCF and the thick line shows the results of the state estimation with the KF, respectively. In the state estimation with the MCF, the number of particle is \( 10^6 \). In the state estimation with the KF, the ratio of variance \( \tau^2/\sigma^2 \) is \( 10^{-6} \). From these figures, it can be seen that the deviation of results with the MCF is small with respect to x axis and y axis, although the deviation of results with the KF is large with respect to x axis and y axis. It is considered that the effect of the assumption that the probability density function of the observation noise is the Cauchy distribution.

![Fig. 2 Photo of Shioji Maru](image)

![Fig. 3 Simulated ship trajectory and pseudo observation data for turning motion with rudder angle 35 degrees](image)

![Fig. 4 Results of numerical experiments for port side turning with rudder angle 35 degrees](image)
V. ONBOARD EXPERIMENTS

The onboard experiments were carried out Tateyama off of Chiba Prefecture on October 10, 2007. As the rudder angle 35 degrees, the data was consecutively measured in order of the starboard side and the port side. In this case, the sampling interval of the data is 1.0 seconds, and the total time of the experiment was 380 seconds. The experimental conditions are shown in Table 2. The obtained time series are shown Figures 6. From this figure we can see that the influence of external disturbance is very large. That is, the ship trajectory is moved into the lee side with winds.

The results of state estimation are shown in Figure 7. In this figure, the black thin line shows the observation data, the red thin line shows the results of the state estimation with the MCF and the thick line shows the results of the state estimation with the KF, respectively. Note that the maneuvering motion changes rapidly, the simulation model is not appropriate. Therefore, in the state estimation with the MCF, the number of particle is $10^7$ because of the stable state estimation. In the state estimation with the KF, the ratio of variance $\tau^2/\sigma^2$ is $10^{-6}$, as well as numerical experiments. From this figure, it can be seen that the proposed procedure can remove the influence of the large noise component, although the results with the KF has the large difference between the observation data. From this fact, in the proposed procedure the stable state estimation is possible by increasing the number of particles in the MCF even if the maneuvering motion changes rapidly. Therefore, we conclude that the proposed procedure can estimate the position of the ship to be highly accurate compared with the traditional procedure by using the KF.

![Fig. 6 Trajectory of Shioji-Maru](image)

![Fig. 7 Comparison estimates for each procedure and observation data](image)

VI. CONCLUSIONS

In this study, in order to estimate the position of the ship in high accuracy, the method based on sequential data assimilation is proposed. Numerical experiments and onboard experiments are carried out to verify the domination of the proposed method against the traditional method with the Kalman Filter. Main conclusions are summarized as follows:

(a) From the results of numerical experiments, it can be seen that even the estimated result of the ship’s position by the proposed method is to be included a large observational error locally compared with the estimated result with the Kalman Filter, the accuracy is high. As for this, it is considered that the effect of the assumption that the probability density function of the observation noise is according to the Cauchy distribution.

(b) From the results of numerical experiments and onboard experiments, since the used simulation model is based on the linear uniform motion, when the maneuvering motion changes rapidly, the simulation model is not appropriate.

<table>
<thead>
<tr>
<th>TABLE 2 EXPERIMENTAL CONDITIONS</th>
</tr>
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<tbody>
<tr>
<td>Items</td>
</tr>
<tr>
<td>Rudder angle</td>
</tr>
<tr>
<td>Initial speed</td>
</tr>
<tr>
<td>Initial course</td>
</tr>
<tr>
<td>Wind conditions</td>
</tr>
</tbody>
</table>
However, when it can be to increase the number of particles in the Monte Carlo Filter, then the estimation calculation is possible.

We conclude that the proposed procedure can estimate the position of the ship to be highly accurate compared with the traditional procedure by using the Kalman Filter.

ACKNOWLEDGEMENTS

This work was supported by the Sasagawa Scientific Research Grant from Japan Science Society (19-741M) and the Fundamental Research Developing Association for Shipbuilding and Offshore (REDAS). The author expresses their gratitude to the above organizations. And the author heartily thanks Prof. Genshiro Kitagawa, Institute of Statistical Mathematics, and Prof. Toshio IKEKI, Associate Prof. Tadatsugi Okazaki, and the captain and all the crews of the training ship Shioji Maru, Tokyo University of Marine Science and Technology.

REFERENCES


APPENDIX

Here, we show how to introduce with respect to Equation 1. For simplicity, we consider the case of one-dimension focused on the X axis in Figure 1. Let

\[ \frac{dx(t)}{dt} = Ax(t) \]  \hspace{2cm} (A-2)

is obviously satisfied. Thus, the general solution of Equation A-2 is as follows:

\[ x(t) = \exp[A \Delta t] \cdot x(0), \]  \hspace{2cm} (A-3)

where, \[ x(t) \] is the position at time \[ t \], \[ v(t) \] is the speed at time \[ t \] and \[ a(t) \] is the acceleration at time \[ t \], respectively. Then, as

\[ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \]

Equation A-3 becomes

\[ x(t) = \begin{bmatrix} 1 & \Delta t & \frac{1}{2} \Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \cdot x(0) \]  \hspace{2cm} (A-4)

Moreover, since the expansion into power-series of \[ \exp[A \Delta t] \] is

\[ \exp[A \Delta t] = I + A \Delta t + \frac{(A \Delta t)^2}{2!} + \frac{(A \Delta t)^3}{3!} + \cdots, \]  \hspace{2cm} (A-5)

by substituting the matrix \[ A \] into Equation A-8, we obtain:

\[ \exp[A \Delta t] = \begin{bmatrix} 1 & \Delta t & \frac{1}{2} \Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \]  \hspace{2cm} (A-9)

Therefore, by setting zero into the initial value of the acceleration, we get the simulation model of the linear uniform motion as follows:

\[ x(n+1) = \begin{bmatrix} 1 & \Delta t & \frac{1}{2} \Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \cdot x(n). \]  \hspace{2cm} (A-10)

And then, by adding the noise into the term of the acceleration, Equation 1 is obtained.