Assessment of Coronary Artery Bypass Grafting based on the Dynamics Characteristics of the Vascular System: Verification by Computer Simulation

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Abstract—The Coronary Artery Bypass Grafting (CABG) has been established as one of the most effective treatments for the ischemic heart disease. Graft flow (blood flow in graft) measurement is used for the intraoperative assessment. However, graft flow is influenced by various factors that are unrelated to graft. Therefore, a method that only depends on the graft failure factors is required to improve the reliability of the intraoperative assessment. That method can be designed by using a mathematical model based on the dynamics characteristics of the vascular system underwent CABG. The purpose of this study is to develop a mathematical model of the vascular system underwent CABG and a method of the parameter identification for the intraoperative assessment. By regarding a bypassed vascular system as single circulatory loop, the mathematical model can be simplified. Parameters in the model represent the dynamics characteristics of the vascular system underwent CABG that can be used to the intraoperative assessment. Furthermore, the model can represent the influence of the contraction of the cardiac muscle by changing its parameters. In order to avoid the influence of the cardiac muscle during the identification process, we developed a method that limits identification to the diastolic phase. We verified developed model and method by simulating graft flow and identifying parameters in the model from simulated data with computer. As a result of the computer simulation, graft flow increased during the diastolic phase and backflow was observed. Parameters were identified with small error (maximum 3%), but only if the developed method was applied. The results showed the model could represent features of the vascular system underwent CABG and the identification method could avoid the influence of the cardiac muscle. We confirmed that the developed model was feasible for representing the dynamics characteristics of the vascular system underwent CABG and the developed method was appropriate to identify parameters in the model avoiding the influence of the cardiac muscle.

I. INTRODUCTIONS

Ischemic heart disease is a serious health problem because of high risk of death or disease complication. A main cause of ischemic heart disease is blockage of blood flow in the artery of the heart. The artery of the heart is called coronary artery and supplies blood to the cardiac muscle. However, when the coronary artery is narrowed (stenosed) or blocked (occluded), the cardiac muscle start to die due to the lack of blood. As for surgical treatments for the ischemic heart, the coronary artery bypass grafting (CABG) has been established as one of the most effective treatments. In CABG, surgeons connect blood vessels that are called “graft” to stenosed coronary arteries. Commonly, grafts are harvested from other parts of the body and used to bypass the blocked arteries. Thus blood flow can be recovered by the surgical operation.

For effective and safe CABG, long-term graft patency is required. Long-term graft patency indicates that the bypass function is kept for a long time. Graft patency is threatened by a graft failure after the surgery. In the case of graft failure, the graft or the connected area (anastomotic site) is stenosed or occluded, and the bypass function is lost. In order to avoid the graft failure, assessment of graft patency during surgery (intraoperative assessment) is required. By the intraoperative assessment, surgeons can revise the graft if graft failure is predicted. In contrast, the probability of success of the procedure increases if graft failure is not predicted. Therefore, effective and safe CABG can be performed by the intraoperative assessment.

For the intraoperative assessment, blood flow measurement in graft is generally used. The reason of the assessment using blood flow is that factors of graft failure generate the change of blood flow. Many factors of graft failure are investigated [1]. The three main factors that can be revised during surgery are anastomotic characteristics, vasospasm and atherosclerosis. Those factors change blood flow because the dynamics characteristics are changed. To measure blood flow, Transit Time Flow Measurement (TTFM) has been introduced [2-6]. TTFM enables surgeon to measure blood flow by ultrasonic technology. However, blood flow is influenced by factors that are not directly related to the graft failure such as blood pressure and peripheral circulation. Therefore, a method that only depends on factors of graft failure is needed to improve the reliability of the intraoperative assessment. That method can be realized by analyzing the dynamics characteristics of the vascular system underwent CABG.

Analysis of the dynamics characteristics of the vascular system is required a mathematical model. Many mathematical model of the vascular system underwent CABG have been proposed [7-9]. However, those models are not practical for assessment of in CABG each patient because the included parameters are too complex to identify during the operation. As for the mathematical model of the vascular system, a
The identification method also lied to the case below:

The purpose of this study is to develop a mathematical model that can represent the vascular system underwent CABG and a method of the parameter identification for the intraoperative assessment. However, two points need to be improved because of change of the model. Firstly, a mathematical model that can represent the vascular system underwent CABG need to be developed because the model in previous research represents only ordinary systemic circulation. Secondly, special features of coronary circulation must be considered because peripheral circulation is significantly affected by the cardiac muscle. After improving those two points, the model can be applied to the intraoperative assessment. The identification method also needs to be improved because of change of the model.

The purpose of this study is to develop a mathematical model of the vascular system underwent CABG and a method of the parameter identification for the intraoperative assessment.

II. MATHEMATICAL MODEL AND PARAMETER IDENTIFICATION

A. The Mathematical Mode of the Vascular System underwent CABG

In order to develop a mathematical model that can represent the vascular system underwent CABG a model of blood vessel was introduced in the same manner as previous research [14]. The model of blood vessel is described as below:

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} &= \begin{bmatrix} \frac{-L+RaRpC}{Ra+Rb} & \frac{1}{Rb+C} \\ \frac{1}{Rb+C} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\
Q(t) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}
\end{align*}
\]

where \( P(t) \) is Aortic Pressure (AoP) that measured at radial artery or carotid artery and \( Q(t) \) is the blood flow volume that measured at a graft (graft flow). \( P(t) \) is the input of the model and \( Q(t) \) is the output of the model. Thus, the complex vascular system underwent CABG is dealt as a linear and lumped model. These parameters in the model adjust the hemodynamics and reflect on the physiological behavior.

Ra corresponds to the vascular resistance at both the graft and the anastomotic site. Ra is increased by anastomotic error or vasospasm because the grafts and the anastomotic site are narrowed. C corresponds to the compliance at both graft and anastomotic site. C is decreased by vasospasm or atherosclerosis because elasticity of vascular wall is lost. L corresponds to the mass of blood and \( Rp \) corresponds to peripheral vascular resistance.
Fig. 2. A new concept for the intraoperative assessment based on the dynamics characteristics of the vascular system underwent CABG consists of two axes: those are vascular resistance at both graft and anastomotic site (Ra), and compliance at graft and anastomotic site (C). These two parameters are linked to the three main factors of graft failure that can be revised: that is, anastomotic characteristics, vasospasm and atherosclerosis. If both Ra and C are high, anastomotic error is indicated. If Ra is high but C is low, vasospasm is indicated. If both Ra and C are low, atherosclerosis is indicated.

These parameters in the model represent the dynamics characteristics of the vascular system underwent CABG that can be used to the intraoperative assessment. A concept for the intraoperative assessment based on the dynamics characteristics of the vascular system underwent CABG is designed as shown in Fig. 2. The concept has two axes: Ra and C. These two parameters are linked to the three main factors of graft failure that can be revised: that is, anastomotic characteristics, vasospasm and atherosclerosis. If both Ra and C are high, anastomotic error is indicated. If Ra is high but C is low, vasospasm is indicated. If both Ra and C are low, atherosclerosis is indicated.

We developed the simple mathematical model of the vascular system underwent CABG as above. Furthermore we designed a new concept for the intraoperative assessment based on the dynamics characteristics of the vascular system underwent CABG.

B. Considering Influence of the Cardiac Muscle

Figure 3 shows a pattern of a blood flow in the left coronary artery (LCA). The LCA is grafted the most commonly in CABG. Therefore, the feature of blood flow of the LCA is strongly related to graft flow and is important in the intraoperative assessment. There are two special and major features of blood flow in the LCA. Firstly, more blood flow is observed during the diastolic phase and two upward peaks are observed in the single cardiac cycle. That phenomenon is called diastolic dominant [4, 5]. Secondly, backflows of arterial blood flow are observed at beginning of the systolic phase. That phenomenon is called slosh phenomenon [15, 16]. In fact, this phenomenon is result from constriction in peripheral circulation by contraction of the ventricular cardiac muscle. Those phenomena are also observed in the vascular system underwent CABG.

In order to include the contraction of cardiac muscle to the developed model, peripheral vascular resistance, Rp, increase during the systolic phase. In regard to backflow, negative flow resistance was introduced. The “negative” means inverse direction of blood flow. Parameters in the model change dramatically, rapidly and periodically. Thus, the model can represent the influence of contraction of cardiac muscle by changing its parameter.

C. The Parameter Identification for the Intraoperative Assessment

Figure 4 shows the outline of the parameter identification. The parameters in the developed model are identified by system identification method. As deriving the identification algorithm is described almost same as previous research [10-14], only the final formula of online least square is shown as bellow:

$$\delta \theta(t) = \frac{1}{T_1} \frac{D(t-l)\psi(t)}{1+\psi(t)^2} [Q(t) - q(t)^T \theta(t)]$$

$$\delta D(t-l) = - \frac{1}{T_0} \frac{D(t-l)\psi(t)\psi(t)^2 D(t-l)}{1+\psi(t)^2} [Q(t) - q(t)^T \theta(t)]$$

where

$$q(t) = \begin{bmatrix} \delta Q(t) \\ \delta^2 + e^2 \delta + e_0 \end{bmatrix}$$

$$Q(t) = \begin{bmatrix} \delta^2 + e^2 \delta + e_0 \\ \delta^2 + e^2 \delta + e_0 \\ \delta^2 + e^2 \delta + e_0 \end{bmatrix}$$

$$\delta Q(t) = \frac{Q(t+T_0)-Q(t)}{T_0}$$
Fig. 4. Outline of the parameter identification consists of real system, mathematical model, parameter identifier block and assessment of graft patency block. Parameters in the developed model are identified by system identification method. The least square method is used to the developed model. By using online least square method, parameters are calculated from measured AoP and graft flow online.

\[
\theta(t) = \left[ e_1 - \frac{L + RaRPCL}{RPCL} e_0 - \frac{Ra + Rp}{RPCL} \right] \delta \\
\left( \frac{2RPCL - RaRPCTs}{2RPCL} \right) + \left( \frac{2L - RTs - RPCTs}{2RPCL} \right) \right]^T.
\]

$\delta$ is the operator for describing the discrete time model, both $e_1$ and $e_2$ are the coefficient of a static filter and $Ts$ is the sampling period.

However, using the conventional method, it is difficult to identify parameters because the dynamics characteristics of the vascular system underwent CABG are influenced by the cardiac muscle before the convergence of identified parameters. As a result, identified parameters converge to wrong values or do not converge. In order to overcome this problem, we performed the identification separately between the systolic phase and the diastolic phase. Furthermore, the diastolic phase is considered to be appropriate to identify since the influence of the cardiac muscle can be minimized.

In summary, a method of the parameter identification that limits identification to the diastolic phase was developed. Then, we can identify parameters in the model in spite of the influence of contraction of the cardiac muscle.

III. VERIFICATION BY COMPUTER SIMULATION

A. Protocol of Computer Simulation

In order to verify the developed model and method, we performed the computer simulation and parameter identification as shown in Fig. 5. Firstly, the developed mathematical model of vascular system underwent CABG with preset parameters were constructed in a computer. Secondly, we input data that resembled AoP to the mathematical model and calculated the graft flow as output. Finally, we identified parameters in the prepared model from both AoP and graft flow data that were simulated and verified the results of identification.

![Computer Simulation](image)

**Fig. 5.** Outline of verification of the developed model and method consists of computer simulation step and identification step. Firstly, the developed mathematical model of vascular system underwent CABG with preset parameters were constructed in a computer. Secondly, we input data that resembled AoP to the mathematical model and calculated the graft flow as output. Finally, we identified parameters in the prepared model from both AoP and graft flow data that were simulated and verified the results of identification.

<table>
<thead>
<tr>
<th>Cardiac Phase</th>
<th>$Ra$ [mmHg min/ml]</th>
<th>$C$ [ml/mmHg]</th>
<th>$L$ [mmHg min/ml]</th>
<th>$Rp$ [mmHg min/ml]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systolic</td>
<td>0.4</td>
<td>0.1</td>
<td>0.03</td>
<td>-20</td>
</tr>
<tr>
<td>Diastolic</td>
<td>0.4</td>
<td>0.1</td>
<td>0.03</td>
<td>2.0</td>
</tr>
</tbody>
</table>

During the systolic phase, $Rp$ was set to negative value so that the backflow could be represented. And, the absolute value of $Rp$ was raised so that the contraction of the cardiac muscle could be represented. $Ra$, $C$ and $L$ were not changed between the systolic phase and the diastolic phase.
B. Results

Figure 6 shows the result of the computer simulation. The solid lines show AoP data that were input to the mathematical model and dashed lines show graft flow data that were calculated. Graft flow during the systolic phase increased only a little in spite of increase of AoP. By contrast, graft flow during the diastolic phase increased. Consequently, more graft flow was observed during the diastolic phase and two upward peaks were observed in the single cardiac cycle. Backflows of graft flow were observed at the beginning of the systolic phase (a). The computer simulation was continued for 20 [s] (b).

Figure 7 shows the identified parameters by using conventional method (a) and the identified parameters by using the developed that limits identification to the diastolic phase (b). Solid lines show $Ra$, dashed lines show $Rp$, dash-and-dotted lines show $C$ and dotted lines show $L$. In the case of identification using conventional method, parameters did not converge to preset values (a). On the other hand, in the case of identification using the developed method, parameters converged to the preset value in the computer simulation (b).

As a result of computer simulation, more graft flow was observed during the diastolic phase and two upward peaks were observed in the single cardiac cycle. We confirmed that diastolic dominant waveform was represented by the developed model. Backflows of graft flow were also observed at the beginning of the systolic phase. We confirmed that slosh phenomenon was represented by the developed method. Since the two major special features [4, 5, 15, 16] were observed, the ability of the developed model that can represent the hemodynamics characteristics of the vascular system underwent CABG was confirmed. Therefore, this experiment yields that the developed mathematical model can represent the dynamics characteristics of the vascular system underwent CABG appropriately.

Parameters in the model could be identified only if the developed method that limits identification to the diastolic phase was applied. Furthermore, the maximum error of identification (3.00%) was acceptable since conventional assessment method by flow meter was allowed the ± 15% (Transonic Systems. Inc., USA). Therefore, these experiment yield that the developed identification method can avoid the influence of the cardiac muscle and can identify parameters more accurately.

C. Discussions

In this study, we developed a simple model of the vascular system underwent CABG and a method the parameter identification that limits identification to the diastolic phase. We verified the developed model and method by simulating graft flow and identifying parameters from simulated data with computer.
IV. CONCLUSIONS

In this study, we developed a mathematical model of the vascular system undergone CABG and a method of the parameter identification for the intraoperative assessment. The developed model represented the dynamics characteristics of the vascular system undergone CABG. In the developed method, identification was limited to diastolic phase. As a result of computer simulation that parameters in the model were identified in enough accurate. We confirmed that the developed model was feasible for representing the dynamics characteristics of the vascular system undergone CABG and the developed method was appropriate to identify parameters in the model avoiding the influence of the cardiac muscle. The developed model and method were enough appropriate for the intraoperative assessment. In future work, the developed model and the method will be applied to \textit{in-vivo} and clinical study. Our study will help to improve intraoperative assessment for efficacy and safety of CABG.

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REFERENCES