Chaotic Inflation in Supergravity∗

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We propose a chaotic inflation model in supergravity. In the model the Kähler potential has a
Nambu-Goldstone-like shift symmetry of the inflaton chiral multiplet which ensures the flatness of
the inflaton potential beyond the Planck scale. We show that the chaotic inflation naturally takes
place by introducing a small breaking term of the shift symmetry in the superpotential. This may
open a new branch of model building for inflationary universe in the framework of supergravity.
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1. INTRODUCTION

Among various types of inflation models proposed so
far, chaotic inflation model is the most attractive since it
can realize an inflationary expansion even in the presence
of large quantum fluctuations at the Planck time. So, in
this talk, we propose a natural chaotic inflation model
in supergravity where the form of Kähler potential is
determined by a symmetry. With this Kähler potential the
inflaton ϕ may have a large value ϕ ≫ M_G to begin the
chaotic inflation. Our models, in fact, need two small
parameters for successful inflation. However, we empha-
size that the smallness of these parameters is justified by
symmetries and hence the model is natural in 't Hooft's
sense.

Our model is based on the Nambu-Goldstone-like
shift symmetry of the inflaton chiral multiplet Φ(x, η).
Namely, we assume that the Kähler potential K(Φ, Φ∗)
is invariant under the shift of Φ,

Φ → Φ + iCM_G, (1)

where C is a dimensionless constant. Thus, the
Kähler potential is a function of Φ + Φ∗, K(Φ, Φ∗) =
K(Φ + Φ∗). It is now clear that the supergravity effect
eK(Φ+Φ∗) discussed above does not prevent the imagi-
nary part of the scalar components of Φ from having a
larger value than M_G. We identify it with the inflaton
field ϕ. We also stress that the present model overcomes
so-called the η problem and it is an alternative to other
inflation models such as D-term inflation models and run-
nning inflaton mass models. However, as long as the shift
symmetry is exact, the inflaton ϕ never has a potential
and hence it never causes the inflation. Therefore, we
have to introduce a small breaking term of the shift sym-
metry in the theory. The simplest choice is to introduce
a small mass term for Φ in the superpotential,

W = mϕ^2. (2)

Then, we have the potential,

V = e^K \left\{ \left( \frac{\partial^2 K}{\partial \Phi \partial \Phi^*} \right)^{-1} D_\Phi W D_\Phi^* \right\} - 3|W|^2, (3)

with

D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K(\Phi + \Phi^*)}{\partial \Phi} W. (4)

Here, Φ denotes the scalar component of the superfield
Φ and we have set M_G to be unity. We easily see that
V → -∞ as |ϕ| → ∞ with Φ + Φ∗ = 0 and the chaotic
inflation does not take place, where ϕ = -(Φ - Φ*)/\sqrt{2}.

In this talk, we propose instead the following small
mass term in the superpotential introducing a new chiral
multiplet X(x, η),

W = mXΦ. (5)

Notice that the present model possesses U(1)_R symmetry
under which

X(θ) → e^{-2iθ}X(θe^{iθ}),

Φ(θ) → Φ(θe^{iθ}), (6)

and Z_2 symmetry under which

X(θ) → -X(θe^{iθ}),

Φ(θ) → -Φ(θe^{iθ}). (7)

The above superpotential is not invariant under the shift
symmetry of Φ. However, we should stress that the
present model is completely natural in 't Hooft's sense,
since we have an enhanced symmetry (the shift symmetry)
in the limit m → 0. That is, we consider that the
small parameter m is originated from small breaking of
the shift symmetry in a more fundamental theory. We
consider that as long as m << O(1), the corrections from
the breaking term eq.(5) to the Kähler potential are negli-
gibly small.∗ Then, we assume that the Kähler potential

∗The Kähler potential may have also the induced breaking
terms such as K ∼ |mϕ|^2 + · · ·. However, these breaking terms
are negligible in the present analysis as long as |ϕ| ≲ m^{-1}.

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II. DYNAMICS OF INFLATION

The Lagrangian density \( L(\Phi, X) \) is now given by

\[
L(\Phi, X) = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi^* + \partial_\mu X \partial^\mu X^* - V(\Phi, X),
\]

with the potential \( V(\Phi, X) \) given by

\[
V(\Phi, X) = m^2 e^K \left[ |\Phi|^2 (1 + |X|^4) + |X|^2 \left( 1 - |\Phi|^2 + |(\Phi + \Phi^*)|^2 (1 + |\Phi|^2) \right) \right],
\]

where we have neglected higher order terms in the Kähler potential eq.(9) whose effects will be discussed later. Here, \( X \) denotes also the scalar component of the superfield \( X \). Now, we decompose the complex scalar field \( \Phi \) into two real scalar fields as,

\[
\Phi = \frac{1}{\sqrt{2}} (\eta + i\varphi).
\]

Then, the Lagrangian density \( L(\eta, \varphi, X) \) is given by

\[
L(\eta, \varphi, X) = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \partial_\mu X \partial^\mu X^* - V(\eta, \varphi, X),
\]

with the potential \( V(\eta, \varphi, X) \) given by

\[
V(\eta, \varphi, X) = m^2 \exp \left( \eta^2 + |X|^2 \right) \times \left[ \frac{1}{2} (\eta^2 + \varphi^2) (1 + |X|^4) + |X|^2 \left( 1 - \frac{1}{2} (\eta^2 + \varphi^2) + 2\eta^2 \left( 1 + \frac{1}{2} (\eta^2 + \varphi^2) \right) \right) \right].
\]

Note that \( \eta \) and \( |X| \) should be taken as \( |\eta|, |X| \leq \mathcal{O}(1) \) because of the presence of \( e^K \). On the other hand, \( \varphi \) can take a value much larger than \( \mathcal{O}(1) \) since \( e^K \) does not contain \( \varphi \). For \( \eta, |X| \ll \mathcal{O}(1) \), we can rewrite the potential as

\[
V(\eta, \varphi, X) \approx \frac{1}{2} m^2 \eta^2 (1 + \eta^2) + m^2 |X|^2.
\]

At around the Planck time, we may have a region where

\[
\dot{\varphi}^2 \sim (\nabla \varphi)^2 \sim V(\varphi) \sim 1.
\]

(initial chaotic situation)

Here the dot represents the time derivative. In this region the classical description of the \( \varphi \) field dynamics is feasible because of \( |\varphi| \gg \mathcal{O}(1) \) though quantum fluctuations are \( \delta \varphi \gg \mathcal{O}(1) \). Then, as the universe expands, the potential energy dominates and the universe begins inflation.

Since the initial values of the inflaton \( \varphi(0) \) is determined so that \( V(\varphi(0)) \approx \frac{1}{2} m^2 \varphi(0)^2 \sim 1 \), \( \varphi(0) \sim m^{-1} \gg 1 \). (Notice that one has only to demand \( \varphi(0) \gtrsim 15.0 \) in order to solve the flatness and horizon problems.) For such large \( \varphi \) the effective mass of \( \eta \) becomes much larger than \( m \) and hence it quickly settles down to \( \eta = 0 \). On the other hand, the \( X \) field has a relatively light mass \( \sqrt{2}m \) and slowly roll down toward the origin \( (X = 0) \). With \( \eta = 0 \), the potential eq.(15) is written as

\[
V(\varphi, X) \approx \frac{1}{2} m^2 \varphi^2 + m^2 |X|^2.
\]

Since \( \varphi \gg 1 \) and \( |X| < 1 \), the \( \varphi \) field dominates the potential and the chaotic inflation takes place. The Hubble parameter is given by

\[
H \approx \frac{m \varphi}{\sqrt{3}}.
\]

During the inflation both \( \varphi \) and \( X \) satisfy the slow roll condition \( (|V''(\varphi)| / V(\varphi) \ll 1, |V''(X)| / V(X) \ll 1 \) where the dash represents the derivative of \( \varphi \) or \( X \) and hence the time evolutions are described by

\[
3H \frac{d\varphi}{dt} \approx -m^2 \varphi,
\]

\[
3H \frac{dX}{dt} \approx -m^2 X.
\]

Here and hereafter, we assume that \( X \) is real and positive making use of the freedom of the phase choice. From the above equations we obtain

\[
\left( \frac{X}{X(0)} \right) \approx \left( \frac{\varphi}{\varphi(0)} \right),
\]

where \( \varphi(0) \) and \( X(0) \) are the initial values of \( \varphi \) and \( X \) fields. Therefore, \( X \) decreases faster than \( \varphi \). At the end of the inflation, i.e. \( \varphi \approx 1 \) \( (|V''(\varphi)| \sim 1, |V''(X)| \sim 1) \), \( X \) is given by

\[
X \lesssim m,
\]

where we have used \( X(0) \lesssim 1 \) and \( \varphi(0) \sim m^{-1} \). We see that the \( X \) field becomes much smaller than 1 \( (m \sim 10^{-5}) \) as shown below. The density fluctuations produced by this chaotic inflation is estimated as

\[
\frac{\delta \rho}{\rho} \approx \frac{1}{5 \sqrt{3} \pi} \left( \frac{\varphi^2 + X^2}{2} \right).
\]

Since \( X \ll \varphi \), the amplitude of the density fluctuations is determined only by the \( \varphi \) field and the normalization at the COBE scale \( (\delta \rho / \rho \approx 2 \times 10^{-5} \text{ for } \varphi \text{COBE} \approx 14) \) gives \(^1\)

\(^1\) The spectral index \( n_s \approx 0.96 \) for \( \varphi \text{COBE} \approx 14 \).
\[ m \approx 10^{13} \text{ GeV}. \] (24)

After the inflation ends, an inflaton field \( \varphi \) begins to oscillate and its successive decays cause reheating of the universe. In the present model the reheating takes place efficiently if we introduce the following superpotential:

\[ W = \lambda X H \bar{H}, \] (25)

where \( H \) and \( \bar{H} \) are a pair of Higgs doublets whose R-charge are assumed to be zero and \( \lambda \) is a constant.\(^1\) Then, we have the coupling of the inflaton \( \varphi \) to the Higgs doublets as

\[ L \approx \lambda \varphi H \bar{H}, \] (26)

which gives the reheating temperature

\[ T_R \approx 10^9 \text{ GeV} \left( \frac{\lambda}{10^{-5}} \right) \left( \frac{m}{10^{3} \text{GeV}} \right)^{1/2}. \] (27)

In order to avoid the overproduction of gravitinos, the reheating temperature \( T_R \) must be lower than \( 10^9 \text{GeV} \), which requires the small coupling \( \lambda \lesssim 10^{-5} \). The small coupling \( \lambda \) is naturally understood in 't Hooft's sense provided that \( H \bar{H} \) is even under the \( Z_2 \) symmetry in eq. (7).

So far we have taken the minimal Kähler potential and neglected higher order terms like \((\Phi + \Phi^*)^4, |X|^4, \) and \( \cdots \). Here, we make a comment on the higher terms in the Kähler potential. Since the leading quadratic terms make the expectation values of \( \eta \) and \( X \) fields less than 1, the inflation dynamics is almost unchanged in the presence of the higher terms. The only relevant difference comes from the \( \zeta |X|^4 \) (\( \zeta \): constant) term which induces the effective mass of \( X \) given by

\[ m_X^2 = 2m^2 - 2\zeta m^2 \varphi^2 \approx -2\zeta m^2 \varphi^2. \] (28)

Thus, \( \zeta \) should be negative to ensure the positiveness of \( m_X^2 \). If \( |\zeta| \gtrsim 1 \), the effective mass becomes larger than the Hubble parameter and the \( X \) quickly settles down to \( X = 0 \) without slow-roll.

III. CONCLUSION

We have shown that a chaotic inflation naturally takes place if we assume that the Kähler potential has the Nambu-Goldstone-like shift symmetry of the inflaton chiral multiplet \( \Phi \) and introduce a small breaking term of the shift symmetry in the superpotential eq.(5). Unlike other inflation models the chaotic inflation model has no initial value problem and hence it is the most attractive. However, it had been difficult to construct a natural chaotic inflation model in the framework of supergravity because the supergravity potential generally becomes very steep beyond the Planck scale. Therefore, the existence of a natural chaotic inflation model may open a new branch of inflation-model building in supergravity. Furthermore, the chaotic inflation is known to produce gravitational waves (tensor metric perturbations) which might be detectable in future astrophysical observations.

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[2] See also references in the above reference.