Problems of Neutrino Mass Matrix

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We have discussed the lepton mass matrices with the \(U(1)\) flavor symmetry, which leads to the large mixing angle MSW solution of solar neutrinos. We also comment on the non-abelian flavor symmetry.

Super-Kamiokande has almost confirmed the neutrino oscillation in atmospheric neutrinos, which favors the \(\nu_\mu \to \nu_\tau\) process with \(\sin^2 2\theta_{\text{atm}} \geq 0.88\) and \(\Delta m^2_{\text{atm}} = (1.5 \sim 5) \times 10^{-3}\text{eV}^2\) \(^{[1]}\). For the solar neutrinos \(^{[2]}\), the 1117 days data in Super-Kamiokande favors the LMA-MSW solution \(^{[3]}\), which is \(\sin^2 2\theta_\odot = 0.65 \sim 0.97\) and \(\Delta m^2_\odot = 10^{-5} \sim 10^{-4}\text{eV}^2\). However, there are still allowed four solutions, the small mixing angle (SMA) MSW \(^{[4]}\), the large mixing angle (LMA) MSW, the low \(\Delta m^2\) (LOW) and the vacuum oscillation (VO) solutions \(^{[5]}\).

There are a lot of ideas for the single large mixing angle in the neutrino mixing matrix (MNS matrix) \(^{[6]}\). However, it is not easy to get the nearly bi-maximal mixings \(^{[7]}\) with the LMA-MSW mass scale in the GUT models \(^{[8]}\). Therefore, it is important to search for the texture of the lepton mass matrix with the LMA-MSW solution.

At first, we study how to get the LMA-MSW solution with the help of the \(U(1)\) flavor symmetry. In the \(U(1)\) flavor symmetry, Vissani has already shown the texture of the neutrino mass matrix with the LMA-MSW solution \(^{[9]}\). Recently, Sato and Yanagida have also studied it numerically \(^{[10]}\). We discuss the texture of the quark-lepton mass matrix with the \(U(1)\) flavor symmetry in the SU(5) GUT. We also study another texture with the \(U(1) \times Z_2\) symmetry.

Assuming that oscillations need only account for the solar and the atmospheric neutrino data, we consider the LMA-MSW solution, in which the mixing matrix and the neutrino masses are given as

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If neutrino masses are hierarchical, we expect the neutrino mass ratio $m_{\nu 2}/m_{\nu 3} = \lambda^2 \sim \lambda$ with $\lambda \simeq 0.2$, which is similar to the charged lepton mass hierarchy.

Let us consider the $U(1)$ flavor symmetry [11, 12], in which fermions carry $U(1)$ charges, $U(1)$ is spontaneously broken by the VEV of the electroweak singlet with $U(1)$ charge $-1$, and Yukawa couplings appear as effective operators through Froggatt-Nielsen mechanism [13]. We discuss the LMA-MSW solution in the SU(5) GUT. Taking the $U(1)$ charges of $5^*$ and $10$ fermions for the (1st, 2nd, 3rd) family [14]

$$\Psi_{5^*} \sim (2, 0, 0), \quad \Psi_{10} \sim (3, 2, 0),$$

we obtain the quark mass matrices as follows:

$$M_U \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad M_D \sim \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix}.$$

These mass matrices are consistent with the experimental values of the quark masses and the CKM mixing matrix [15], except for the value of the lightest u-quark mass.

The lepton mass matrices are given as follows:

$$M_E \sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad M_{\nu} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}.$$

The left-handed mixings of the charged lepton and the neutrino between the second and the third family are almost maximal $s_{23}^E \simeq s_{23}^\nu \simeq 1/\sqrt{2}$. Then, the (2-3) family mixing in the MNS matrix is written as:

$$U \equiv L_E^\dagger L_\nu \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}^T \begin{pmatrix} 1 & e^{i\alpha} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & e^{i\beta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix},$$

where we taken into account phases, which are factored out from the mass matrices. If we choose $\alpha = \pi/2$, $\beta = 0$, we get $U_{\mu 3} = i/\sqrt{2}$, in which the maximal mixing is still kept. Thus, maximal mixings of both sectors lead to the maximal mixing of the MNS matrix without cancellation if we take account of the phases.
On the other hand, the left handed mixings between the first and the second (third) family should be carefully examined. The mixings in the charged lepton sector are found to be
\[ s_{12}^E \simeq s_{13}^E \simeq \lambda^2, \]  
by using the general formula in the ref.[16]. How large is the (1-2) family mixing in the neutrino sector? In order to answer this question, we discuss the following neutrino mass matrix as discussed by Vissani [9]:
\[ M_{\nu} \sim \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & 1 & 1 \\ \epsilon_{13} & 1 & 1 \end{pmatrix}, \]  
where $\epsilon_{ij}$'s are the same order except for factors of order one. At first, diagonalizing the (2-3) submatrix by $s_{23}^\nu = 1/\sqrt{2}$, we get the mass matrix in the new basis:
\[ \tilde{M}_{\nu} \sim \begin{pmatrix} \epsilon_{11} & \tilde{\epsilon}_{12} & \tilde{\epsilon}_{13} \\ \tilde{\epsilon}_{12} & \delta & 0 \\ \tilde{\epsilon}_{13} & 0 & 1 \end{pmatrix}, \]  
where
\[ \epsilon_{11}^2 = \frac{1}{2} \epsilon_{11}^2, \quad \tilde{\epsilon}_{12} = \frac{\epsilon_{12} - \epsilon_{13}}{2\sqrt{2}}, \quad \tilde{\epsilon}_{13} = \frac{\epsilon_{12} + \epsilon_{13}}{2\sqrt{2}}, \quad \delta = \frac{m_{\nu_2}}{m_{\nu_3}}. \]  
The mass matrix in eq.(8) gives
\[ \sin^2 2\theta_\odot = \frac{4\tilde{\epsilon}_{12}^2}{(\delta - \epsilon_{11}^2)^2 + 4\tilde{\epsilon}_{12}^2}, \]
\[ \Delta m_{\odot}^2 = (\delta + \tilde{\epsilon}_{11}^2)\sqrt{(\delta - \epsilon_{11}^2)^2 + 4\tilde{\epsilon}_{12}^2} \Delta m_{\text{atm}}^2, \]  
where $\theta_{12} = \theta_\odot$ is taken. After eliminating the unknown parameter $\delta$, we obtain
\[ \frac{\Delta m_{\odot}^2}{\Delta m_{\text{atm}}^2} \simeq 4\frac{\tilde{\epsilon}_{12}^2}{\sin^2 2\theta_\odot} (\cos 2\theta_\odot \pm \frac{\tilde{\epsilon}_{11}}{\tilde{\epsilon}_{12}} \sin 2\theta_\odot). \]  
Since $\epsilon_{12}$ and $\epsilon_{13}$ are complex in general, $\tilde{\epsilon}_{12}$ is expected to be comparable to them from eq.(9). Let us consider the case of $\epsilon_{12} \simeq \epsilon_{13} \simeq \lambda^2$, which corresponds to the model in eq.(4). Taking $\tilde{\epsilon}_{11} \simeq \tilde{\epsilon}_{12} \simeq \lambda^2$, we show the allowed region on the $\sin^2 2\theta_\odot - \Delta m_{\odot}^2$ plane in Fig.1, where we take $\tilde{\epsilon}_{11} = 0.05$ and $\tilde{\epsilon}_{12} = 0.03 \sim 0.08$ in practice. As seen in Fig.1, our mass matrices are consistent with only the LMA-MSW solution.

However, if the phase of $\tilde{\epsilon}_{12}$ is equal to the one of $\epsilon_{13}$, $\tilde{\epsilon}_{12}$ could be supressed from eq.(9). When we take $\tilde{\epsilon}_{11} \simeq \lambda^2$ and $\tilde{\epsilon}_{12} \simeq 0.1 \times \lambda^2$, we can get another allowed region as seen in Fig.1, where we take $\tilde{\epsilon}_{11} = 0.05$ and $\tilde{\epsilon}_{12} = 0.003 \sim 0.008$. This case is consistent.
with both the SMA-MSW and the LOW solutions, and may be also consistent with the VO solution. It is helpful to comment on the magnitude of \( U_{e3} \). Since \( U_{e3} \) is given approximately as \( \max\{s_{12}^F, s_{13}^F, s_{12}^s \} \), its expected magnitude is \( \lambda^2 \), which is consistent with the CHOOZ data [17].

Since the parameter \( \delta \) is give as

\[
\delta \simeq 2\epsilon_{12} \cot \theta_\odot + \epsilon_{11}^2,
\]

the LMA-MSW solution corresponds to \( \delta \simeq \lambda^2 \). However, there is no principle to fix \( \delta \) in advance within the framework of the \( U(1) \) flavor symmetry. In other words, the lepton mass matrices in eq.(4) are consistent with the LMA-MSW, SMA-MSW, LOW and VO solutions. Thus, these mass matrices cannot predict the solar neutrino solution because \( \delta \) is the unknown parameter.

Now we go beyond the \( U(1) \) flavor symmetry to give the unique solution of the solar neutrino oscillation. Let us consider another mass matrix in \( U(1) \times Z_2 \) symmetry, which has already discussed in ref.[18]. The effective Yukawa couplings of the lepton sector are given by introducing two singlet Higgs \( S_1 \) and \( S_2 \) as follows [19].

Giving the \( U(1) \) and \( Z_2 \) charges to the doublet leptons \( L_i \) and the singlets \( \xi_j \) as follows:

\[
L_1(1,0), L_2(1,0), L_3(0,1); \quad \ell_1^c(5,0), \ell_2^c(2,0), \ell_3^c(0,0),
\]

we obtain

\[
M_E \sim \begin{pmatrix}
\epsilon_1 \epsilon_1^6 & f_1 \epsilon_1^3 & g_1 \epsilon_1 \\
\epsilon_2 \epsilon_1^6 & f_2 \epsilon_1^3 & g_2 \epsilon_1 \\
\epsilon_3 \epsilon_2 \epsilon_2 & f_3 \epsilon_2 \epsilon_2 & g_3 \epsilon_2
\end{pmatrix}, \quad M_\nu \sim \begin{pmatrix}
\epsilon_1^2 & \epsilon_1^2 & \epsilon_1 \epsilon_2 \\
\epsilon_1^2 & \epsilon_1^2 & \epsilon_1 \epsilon_2 \\
\epsilon_1 \epsilon_2 & \epsilon_1 \epsilon_2 & 1
\end{pmatrix},
\]

where \( \epsilon_i, f_i \) and \( g_i \) are supposed to be real coefficients of order one. Taking \( \epsilon_1 = \epsilon_2 = \lambda \), which is realized in the supersymmetric vacuum [20], we obtain \( m_{\nu_2}/m_{\nu_3} = \lambda^2 \), which is consistent with the mass scale of the LMA-MSW solution. It is found that the large (2-3) family mixing of the MNS matrix originates from \( M_E \). On the other hand, the large (1-2) family mixing comes from both \( M_E \) and \( M_\nu \). In these lepton mass matrices, an important problem is the magnitude of \( U_{e3} \). In order to investigate \( U_{e3} \), we make \( M_E M_E^\dagger \)

\[
M_E M_E^\dagger \sim \lambda^2 \begin{pmatrix}
g_1 \\
g_2 \\
g_3
\end{pmatrix} \begin{pmatrix}
g_1 & g_2 & g_3 \\
f_1 & f_2 & f_3 \\
f_1 & f_2 & f_3
\end{pmatrix} + \lambda^6 \begin{pmatrix}
f_1 \\
f_2 \\
f_3
\end{pmatrix} \begin{pmatrix}
f_1 & f_2 & f_3 \\
f_1 & f_2 & f_3 \\
f_1 & f_2 & f_3
\end{pmatrix}
\]

\[\text{In this calculation, we take account of } s_{12}^E \text{ with the condition } \alpha - \beta \simeq \pm \pi/2, \text{ which is required to keep the maximal MNS mixing as seen in eq.(5).}
\]

\[\text{As far as } \alpha - \beta \simeq \pm \pi/2, \text{ the cancellation never occur among } s_{12}^E, s_{13}^E \text{ and } s_{13}^s.
\]
which is the sum of three rank-one matrices. Therefore the magnitude of $U_{e3}$ is not small in general. However, as far as the following relation is kept:

$$\frac{g_1}{g_2} = \frac{f_1}{f_2} \quad \text{and} \quad \frac{g_2}{g_3} \neq \frac{f_2}{f_3} \quad \text{or} \quad \frac{g_1}{g_3} \neq \frac{f_1}{f_3},$$

$U_{e3}$ is suppressed. We have checked numerically that the predicted value of $U_{e3}$ is below the CHOOZ bound ($U_{e3} < 0.16$) if these relations are satisfied with the accuracy of 10%. Since the lepton doublets $L_1$ and $L_2$ have same $U(1)$, $Z_2$ charges as seen in eq.(13), these relations are probable one in our model. Thus, we get the preferable lepton mass matrices, which lead to the LMA-MSW solution.

The non-abelian flavor symmetry also leads to the LMA-MSW solution. In this case, three neutrino masses are degenerated. For example, we have a numerical result

$$m_1 \simeq 0.030\text{eV}, \quad m_2 \simeq 0.033\text{eV}, \quad m_3 \simeq 0.058\text{eV}.$$

Can we accept degenerated neutrino masses? Fortunately, $\beta\beta_0$ experiments can test $\langle m_\nu \rangle_{ee} \sim 0.03\text{eV}$.

Thus, the solar neutrino solution makes a big impact on the lepton mass matrix with the flavor symmetry. We expect that the LMA-MSW solution will be checked in KamLAND as well as SNO in the near future.

References


Fig.1: The allowed region on the $\sin^2 2\theta_{12} - \log_{10} (\Delta m^2_{12} / eV^2)$ plane. The upper region corresponds to $\bar{\xi}_{12} = 0.03 \sim 0.08$ and the lower one to $\bar{\xi}_{12} = 0.003 \sim 0.008$. The black rectangle regions show the LMA-MSW, the SMA-MSW and the LOW solutions approximately.