Nonperturbative studies of the fuzzy spheres in a matrix model with the Chern-Simons term

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We have studied a three dimensional bosonic matrix model with a Chern Simons term

\[ S = N \text{Tr} \left( -\frac{1}{4} [A_\mu, A_\nu]^2 + \frac{2i\alpha}{3} \varepsilon_{\mu\nu\rho} A_\mu A_\nu A_\rho \right) \]

where \( A_\mu : N \times N \) hermitian matrix, \( \mu, \nu, \cdots = 1, 2, 3 \). This model has \( SO(3) \) rotational, \( SU(N) \) gauge symmetry. Fuzzy \( S^2 \) Sphere ( \( A_\mu = \alpha L_\mu \), \( L_\mu \in SU(2) \) Lie algebra) is a typical solution of this model. We have analysed this model through the heat bath algorithm of the Monte Carlo simulation. This approach is nonperturbative unlike the perturbative approaches used in two-loop diagrammatic calculation (hep-th/0303120,0307007) or in first order calculation of the Gaussian expansion (hep-th/0303196).

Nonperturbative Stability of the Fuzzy Sphere

We have found the nonperturbative Stability of the Fuzzy Sphere solution of this model. For this we start the Monte Carlo simulation from the fuzzy sphere initial configuration \( A^{(0)}_\mu = \alpha L_\mu \). And plot the eigenvalue distribution of the Casimir \( Q = A^2_\mu \). If the fuzzy sphere solution is stable then we see a lump consistently at \( \rho^2 = \frac{3}{4} (N^2 - 1) \). If the solution is not stable then the lump moves to some other location.

In our simulation we have seen that the fuzzy sphere solution is stable for \( \alpha > \alpha_{cr}^{(I)} \). This can also be reconfirmed from the dynamical evolution of the different variables and the eigenvalues.

First Order Phase Transition

To probe into more detail we measure the action (\( \langle S \rangle \)), the spacetime extent (\( \langle \frac{1}{N} \text{Tr} A^2_\mu \rangle \)), the bosonic Yang-Mills term (\( \langle \frac{1}{N} \text{Tr} F^2_\mu \rangle \)) and the Chern-Simons term (\( \langle \frac{1}{3N} \text{Tr} \varepsilon_{\mu\nu\rho} A_\mu A_\nu A_\rho \rangle \)) for different \( \alpha \). In each case, we have clearly seen a first order transition at lower critical point \( \alpha_{cr}^{(I)} = 2.1 \). When we start our simulation from initial configuration \( A_\mu = 0 \), the phase transition occurs at upper critical point \( \alpha_{cr}^{(u)} = 0.66 \), which is independent of \( N \). There is a strong hysteresis loop for large \( N \).

Yang Mill Phase

In small \( \alpha \) phase the quantum effect is large. We can see the value of action and the space time extent remains unaltered with \( \alpha \) similar to the bosonic IIB model study in hep-th/9811220. This indicates this phase to be the Yang Mill phase.

Fuzzy Sphere Phase

In large \( \alpha \) phase the value of the observables agree well with the one loop perturbative results obtained around a single fuzzy sphere. This shows this phase to be the Single Fuzzy Sphere phase. The quantum correction in this phase is suppressed.

We define the width \( \sigma \) of the eigenvalue distribution of Casimir \( Q \) as \( \sigma^2 = \langle \frac{1}{N} \sum_{i=1}^N \lambda_i^2 \rangle - (\langle \frac{1}{N} \sum_{i=1}^N \lambda_i \rangle)^2 \). \( \sigma \) turns out to behave as \( \sigma^2 \sim 2 \log N - 0.8 \).

Meta-stability of multi-fuzzy spheres

As multi-fuzzy spheres are also valied solutions of this Chern Simons bosonic matrix model, we have also study the multi-fuzzy sphere solutions. To do that we start our simulation from \( A_\mu = 0 \) and plotted various observables and eigenvalues of the casimir against the sweep number. The system goes through many multi fuzzy sphere phases before it finally reaches to stable single fuzzy sphere phase.

We have also seen that the life time \( \tau \) of \( k \) coinciding fuzzy spheres follows an \( N \) independent power law \( \tau \sim \alpha^4 k^{-3} \) over a huge range of \( \alpha (\alpha = 50 \sim 20000) \).