Neutrino Masses in Extended Supersymmetric Unified Model.

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Abstract

We investigate the neutrino masses and mixings extending the supersymmetric standard model (MSSM) by adding one pair of extra families (ESSM). We show that an $SO(10)$ grand unified is possible to reproduce the neutrino masses and mixings consistent with the present indications with an additional $SU(2)_L$ singlet neutrino with very light mass and which naturally has large mixing with muon neutrino. This explains the atmospheric neutrino anomaly, and the tau neutrino is provided as a candidate for the hot dark matter. Also we shortly address the bottom tau ratio problem which in the usual MSSM prohibits the the existence of the Majorana mass with its mass less than $10^{12-13}$ GeV. We argue that ESSM is free from the bottom tau problem. Finally we argue the $U(1)$ assignment of our texture of the fermion and Higgs fields.

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1 Introduction

The recent experimental results has stimulated us very much by not only their confirmation of the neutrino oscillations but also their strong suggestion of the existence of large mixing. Also if we take seriously the hot matter candidates, it almost difficult to preserve the parallelism between quarks and leptons to neutrino sectores in the simple GUT frameworks of $SO(10)$.

Here we make a quick review of the present status of the neutrinos non-vanishing masses and mixings. The observed solar neutrino deficits can be explained in terms of the matter induced resonant oscillation with the parameters $\Delta m^2 \simeq (0.4 - 1.1) \times 10^{-5} \text{eV}^2$ and $0.003 \sim \sin^2 2\theta_{\odot} \sim 0.012$. The atmospheric neutrino anomaly has been confirmed which indicates the neutrino oscillation $\nu_\mu \leftrightarrow \nu_{\tau,\bar{\nu}}$ with $\Delta m^2 \sim 10^{-(2-3)} \text{eV}^2$ and $0.8 \sim \sin^2 2\theta_{\mu\tau} \sim 1$. Thirdly a hint of the neutrino masses and mixings comes from the astrophysics and cosmology and neutrinos are natural candidates for the hot dark matter component which are needed to explain the large scale fluctuation of the cosmic microwave background radiation, It requires the neutrino masses to be a few eV.

Within the known three neutrino framework, the only solution which can explain the above experimental results requires three almost degenerate mass eigenstates with masses $\simeq O(\text{eV})$. However, this requires fine-tunings or very hierarchical right-handed neutrino Majorana masses. Together with the large 2-3 mixing, this seems to be in contrast to the character of the quark and leptons. Thus, the simultaneous explanation of the solar, the atmospheric and the hot dark matter neutrino within the three generation scenario seems to contradict with the $SO(10)$ like GUT frameworks.

We here introduce the extended standard model (ESSM) with extra vector-like fermion families. The model is motivated by the possibility of the dynamical gauge bosons with asymptotically nonfree standard gauge theory. We found that ESSM solves the problem by introducing extra neutrinos of $SU(2)_L \times U(1)_Y$ singlets state. However if one considers that the gauge unification or the left-right symmetry may be realized in nature, it should be pursued to understand this neutrino spectrum.
from the relations in some GUT framework. Then, the large mixing may originate from the mixing with sterile neutrinos other than the ordinary three generations since it is expected that the mixings are small between the ordinary neutrinos.

Based on the SO(10) gauge group we analyse ESSM in which an extra light neutrino is included and naturally has large mixing with the ordinary neutrinos. In this model, we add a pair of extra vector-like generations from which a sterile neutrino arises in addition to the ordinary three ones. The important feature of the model is that due to the existence of the extra generations (hereafter, we describe them as $4$ and $\bar{4}$ generations), all the gauge couplings become asymptotically non-free while preserving gauge coupling unification.

In section 2, we summarize the importance of the infrared fixed points (IRFP), and with this property we can determine the texture of the quark and lepton mass matrices. The most characteristic feature of this texture is that only the second generation strongly couples to the extra generations. This fact indicates that the muon neutrino may have a large mixing with the extra generations which gives the origin of the atmospheric neutrino anomaly. In section 3, we shall see that, using the GUT relations for Yukawa couplings, we can also fix the Majorana mass matrix of the right-handed neutrinos. Then it is interesting to see how the light neutrinos can be provided and their mass matrix is predicted in this $SO(10)$ model. The neutrino masses and mixings are calculated with this model with a full use of the texture which is determined from the quark lepton mass matrix. Finally we address how the intermediate right-handed neutrino masses affect on the bottom tau ratio in section 4.

## 2 Infrared Fixed Points in ESSM

We consider the following ‘extended’ supersymmetric standard model (ESSM) with 5 generations; MSSM (3 generations) + 1 extra vector-like family ($4 + \bar{4}$). The matter contents of this model is

$$Q_i, u_i, d_i, L_i, e_i, \quad i = 1, \ldots, 4$$

(1)
In addition to $H, \tilde{H}$ which are $SU(2)_W$ doublet Higgs fields, we have $\Phi$, a (standard gauge group) singlet Higgs which yields masses for extra vector-like family, which we assume to get vacuum expectation value of the order of TeV scale.

A remarkable fact of ESSM is that due to the ANF gauge couplings the the Yukawa couplings are determined almost insensitively to their initial values fixed at GUT scale $M_{GUT}$, because these Yukawa couplings reach to their fixed points. This is physically significant because we can get important information on the physical parameters for which we can not derive from strong dynamics responsible to produce dynamical composite particles. Maybe the infrared structure is something to do with the structure of the dynamics at the scale above which the low energy effective theory does no more work because of the strong interactions. Thus I would stress that RGE analysis is the most effective way to get the physical parameters instead solving strong dynamics. Of course we should notice that we cannot get less important small parameters which may come from less important higher dimentional operators. Actually we found in our case that the IRF structure constraints the mass texture almost uniquely, leaving very small (less than the order of $\epsilon^3 = 0.008$) yukawa couplings.

3 Texture of Yukawa Couplings in ESSM

The texture is determined in the following way:

(1) The forms of $5 \times 5$ fermion mass matrices are generally written as,

$$m_U = \begin{pmatrix} \begin{array}{ccc} u_{1R} & \cdots & u_{4R} \\ u_{1L} & Y_{u} v_u & Y_{Q_1} V \\ \vdots & & \vdots \\ u_{4L} & Y_{u} V & Y_{d} v_d \end{array} \end{pmatrix}$$

(4)
where $u_{il}, d_{il}, (u_{4R})^C$ and $(d_{4R})^C$ are fermionic components of $SU(2)_W$ doublet quarks $Q_i, \bar{Q}_i$, and $(u_{iR})^C, (d_{iR})^C, u_{4L}$ and $d_{4L}$ are those of $SU(2)_W$ singlet quarks $u_i, d_i, \bar{u}$ and $\bar{d}$, respectively.

(2) Since the realistic texture should yield typical hierarchical structures, we can fix the dominant part of the matrices (for simplicity $m, \tilde{m}$ and $M$ are used symbolically to represent $Y_{f_{ij}} v_{ud}, Y_f v_{ud}$ and $Y_f V (f = u, d, e)$ respectively since in our classification only the order of their masses are important ($m \ll M$)).

(3) The Higgs fields are required to be 10 and 126 rep of $SO(10)$ group for those coupled to up and down sector, respectively. This is because of the enormous QCD enhancement which requires us to adopt the unification condition of 126-Higgs ($R_b/\tau_{GUT} = -1/3$, allowing $R_b/\tau$ enhanced by a factor of $5 \sim 6$ to reproduce the experimental value of $R_b/\tau = 1.6 \sim 1.8$.

\[
Y_i(M_{GUT}) = Y_b(M_{GUT}) = \frac{1}{3} Y_\tau(M_{GUT}) \quad \rightarrow \quad R_{b/\tau}(M_Z) \sim \frac{5}{3} \sim 6.
\]  

Note that 126 Higgs also couples to right-handed neutrino majorana type coupling.

(4) We finally found the mass texture for quark and lepton at GUT scale which is the only possible one to reproduce the low-energy experimental values. We use
the Higgs field of 126 (45) representation of $SO(10)$ ($SU(5)$) so that the boundary conditions in the down/lepton sector may correctly reproduce the observed $b/\tau$ (or $s/\mu$) ratio as noted in the previous section. In our texture, all quark Yukawa couplings except for $Y_{d,3}$ converge to their IRFP independently of their initial values at GUT scale. The full mass matrix including quark mixing angles is obtained by introducing hierarchically very small Yukawa couplings. This time it is not necessary to introduce any small mixing parameter other than $\epsilon$. and after all, there is a reasonable $5 \times 5$ GUT-scale texture which explains the experimental values of the CKM mixing angle and we can see that this texture is actually the only possibility left under this situation.

$$m_U \simeq \begin{pmatrix}
1 & 2 & 3 & 4 & 4 \\
0 & 0 & \epsilon^4m & 0 & 0 \\
0 & 0 & 0 & m & 0 \\
\epsilon^4m & 0 & m & 0 & 0 \\
0 & m & 0 & 0 & M \\
0 & 0 & 0 & M & \bar{m}
\end{pmatrix}, \quad (9)$$

$$m_D \simeq \begin{pmatrix}
1 & 2 & 3 & 4 & 4 \\
0 & \epsilon^4m & 0 & 0 & 0 \\
\epsilon^4m & 0 & \epsilon^3m & m & 0 \\
0 & \epsilon^3m & \epsilon m & 0 & 0 \\
0 & m & 0 & m & M \\
0 & 0 & 0 & M & 0
\end{pmatrix}, \quad (10)$$

$$m_E \simeq \begin{pmatrix}
1 & 2 & 3 & 4 & 4 \\
0 & 3\epsilon^4m & 0 & 0 & 0 \\
3\epsilon^4m & 0 & 3\epsilon^3m & 3m & 0 \\
0 & 3\epsilon^3m & 3m & 0 & 0 \\
0 & 3m & 0 & 3m & M \\
0 & 0 & 0 & M & 0
\end{pmatrix}, \quad (11)$$

For the obtained results for all the fermion masses and mixings I will not address here, and would like to summarize our findings: 1) We can understand the charm
quark mass as well as the top quark mass in terms of their infrared fixed point values. It is interesting that the hierarchical factor of the top and charm ratio comes from the existence of the 4th and 4th generations. Also we should like to note that the Yukawa couplings of $Y_{24}(Y_{42})$ reach their infrared fixed points with considerable strength. This indicates that the second generation is strongly coupled with the extra generations. 2) Though the masses of the other lighter quarks are not related to the infrared structure of the ANF character, we can determine their mass texture almost uniquely by introducing only one small parameter. It is interesting that this small parameter happens to be equal to the Cabibbo mixing angle. In the down-quark texture the resultant strange-quark eigenvalue is suppressed by the existence of the extra generations in spite of the appreciable large induced Yukawa coupling $Y_{22}$. 3) As for the lepton masses, they are reproduced quite successfully by assuming that the relevant Higgs field belongs to 126 representation of $SO(10)$. This is in a remarkable contrast to the case of the MSSM in which, as is seen from the Georgi-Jarlskog type of texture, we have to assume that the relevant Higgs field must be the mixture of 10 and 126 representations. It was essential for us to understand the heavier fermion masses as their IR fixed point values that not only the SUSY breaking scale but also the invariant masses of the extra generations are of the order of TeV scale.

4 Neutrino Mass and Mixing

Here I want to convince you that using the GUT relations, we can obtain neutrino mass matrix naturally in which one of the $SU(2)_L$ singlet (sterile) neutrino is very light and has large mixing with muon neutrino. This can explain the atmospheric neutrino anomaly, and the hot dark matter neutrino as well. Now we have fixed all the textures of quarks and leptons in our ESSM, we now proceed to the neutrino sector, which can be analysed quite in parallel with the above textures. As I stated already, in ESSM, we have add a pair of extra vector-like generations from which a sterile comes into play.

Let us determine the neutrino mass texture, $m_\nu^D$ (Dirac) and $m_\nu^R$ (right-handed
Majorana) in accordance to \(SO(10)\) relation to the texture of quark and charged lepton. This time we have one more scale of the right-handed neutrino Majorana mass \(M_R\) in addition to \(M\) and \(w\), among which a large hierarchy exists; \(w < M \ll M_R\).

Let us forget for the moment about the first generation which is responsible for the solar neutrino problem, since the Cabibbo angle is enough to provide the corresponding MNS 1-2 mixing angle to reproduce the MSW small angle solution. Then we can consider the \(4 \times 4\) neutrino mass matrices. From the quark texture (11), we can get the following texture for neutrinos;

\[
\begin{bmatrix}
2 & 3 & 4 & 4 \\
2 & 3 & 4 & 4 \\
3 & w & 1 & M \\
4 & w & M & w \\
4 & w & M & w \\
\end{bmatrix}, \quad \begin{bmatrix}
2 & 3 & 4 & 4 \\
2 & 3 & 4 & 4 \\
3 & \epsilon M_R & 1 & 1 \\
4 & M_R & M_R & 1 \\
4 & M_R & M_R & 1 \\
\end{bmatrix}, \quad (12)
\]

according to the GUT relation \(m^D_\nu = m_u\) and the fact that \(m^R_\nu\) comes from the \(\overline{126}\)-Higgs fields, namely, \(m^R_\nu \propto m_d (m_e)\). The above neutrino texture indicates that; (i) One extra (sterile) neutrino in the 4 generation is left to be almost massless and may couple strongly to the second generation (muon) neutrino. (ii) The third generation right-handed Majorana mass is a little smaller than the others. This yields a heavier left-handed tau neutrino which can be the hot dark matter component. In the above texture we have assumed that the up-type quarks as well as neutrinos couple to 10-Higgs and especially, the 4-4 elements do not come from 126-Higgs (not \(\overline{126}\)). This may be easily realized when one introduces relevant Higgs multiplets with a flavor \(U(1)\) (gauge) symmetry (see the appendix). However, it is interesting that almost all parts of the above texture can be fixed from the characteristic IR property of this model without any symmetry arguments.

Now we are dealing with \(10 \times 10\) matrix totally because of the existence of majorana masses, we here analyse the neutrino masses and mixings. Most of the readers would think that this becomes too complicated to handle. However it is not indeed true! (1) First the texture indeed implies that the 1st as stated already. (2) Also the
3rd generation almost decouple from the extra generation, as seen from the textures (11)–(12). So we can be neglect them in the following analyses. (3) In the remaining part, two of six neutrinos (the second and fourth right-handed neutrinos) have their masses of the order of the intermediate scale $M_R$, and are decoupled leaving sea-saw masses to the reminders.

In this way the neutrino texture is immediately reduced to $4 \times 4$ matrix with light elements. Let us see whether the mixing angle between light neutrinos can become very large. After integrating out the heavy right-handed neutrinos of the second and fourth generations, we get the following mass matrix in the basis of $(\nu_2, \nu_4, \nu_2', \nu_4')$ (the second subscripts represent the transformation properties under the $SU(2)_L$),

$$
\begin{pmatrix}
2am & am' & m \\
am' & m & w \\
m & m' & -m & M \\
w & M & \\
\end{pmatrix}
$$

(13)

where $m$ and $m'$ are masses induced by seesaw mechanism ($m \sim \frac{w^2}{M_R}$, $m' \sim \frac{w M}{M_R}$) and are much smaller than $w$ and $M$. Therefore we are left with two very light neutrinos with masses $\sim O(m, m')$ which mainly come from $\nu_2$ and $\nu_4$. In the above matrix, $M$ is an invariant mass of the extra lepton doublets. Its range is estimated as $M \gtrsim 200$ GeV if one takes account of the constraints for the extra vector-like quark masses ($\gtrsim 1$ TeV) from the FCNC and $S, T$ and $U$ parameters, and the relative QCD enhancement factor ($\sim 5$) between quarks and leptons in this model. There also appear non-zero matrix elements with a factor $\alpha$ which come from the induced neutrino Dirac mass elements via one-loop renormalization group. This $\alpha$, representing the ratio of induced to tree-level Dirac masses, is almost independent of the input parameters ($\tan \beta, \alpha_{GUT}$, etc.) and its typical value is $|\alpha| \sim 0.1$. By taking the typical values of $\alpha$ and $w$, the mixing angle becomes,

$$
\sin^2 2\theta \sim \frac{1}{1 + \left(\frac{350}{M \text{ (GeV)}}\right)^2 \cos^2 \beta},
$$

(14)

with $\tan \beta$, a ratio of the vacuum expectation values of two doublet Higgses. From this, for $\tan \beta \gtrsim 3$, we can naturally get the large mixing angle for suitable parameter...
range \( (M \gtrsim 200 \text{ GeV}) \). This is naturally understood because \( m' \) is larger than \( m \), indicating the nondiagonal elements dominate, which yields quite naturally to a large mixing angle.

Numerical estimations are done: the third generation neutrino which is to be identified to the hot dark matter component, is almost decoupled from the other generations, we fix \( M_R \) as \( 10^{12} \text{ GeV} \sim M_R \sim 10^{13} \text{ GeV} \) from the eigenvalue \( m_3 \). We display acceptable solutions as an example,

\[
\Delta m_{12}^2 \simeq 1.0 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{e\mu} \simeq 0.012, \quad (15)
\]
\[
\Delta m_{24}^2 \simeq 1.1 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\mu\tau} \simeq 0.82, \quad (16)
\]
\[
m_3 \simeq \text{ a few eV}, \quad (17)
\]

for \( M_R \sim 4 \times 10^{12} \text{ GeV} \), \( \tan \beta \sim 30 \) and \( M \sim 250 \text{ GeV} \), which are in good agreement with the experimental observations of the solar, atmospheric and hot dark matter neutrinos.

Comments are in order concerning the other experimental results. In this model, the sterile neutrino has a large mixing with the muon neutrino to solve the atmospheric neutrino anomaly. The discrimination between two oscillation scenarios, \( \nu_\mu \leftrightarrow \nu_\tau \) and \( \nu_\mu \leftrightarrow \nu_s \), for the solution to the atmospheric neutrino anomaly will be made by the ongoing and forthcoming experiments observing various quantities. The recent Superkamiokande reports indicate that the observed suppression of the \( NC \) induced \( \pi^0 \) events is consistent with \( \nu_\mu \leftrightarrow \nu_\tau \) oscillation but they have not excluded \( \nu_\mu \leftrightarrow \nu_s \) oscillation as yet. The cosmological and astrophysical implications in the existence of the fourth light neutrino should also be addressed: the big-bang nucleosynthesis scenario which severely constrains the effective number of light neutrino species, or equivalently the mixing between the active and sterile neutrinos. However, according to the recent estimations more than four light neutrinos are acceptable and there is no constraint on the mixing angles.

In this way the extra neutrino is naturally supplied in the ESSM with an \( SO(10) \) framework. And the texture clearly requires that one extra (singlet sterile) neutrino which is left to be almost massless, couples strongly to the muon neutrino. This can explain the atmospheric neutrino deficit.
5 Bottom to Tau mass ratio

Now that the data suggests the existence of right handed neutrino with the $M_R \sim 10^{10-13}$ GeV, it affects the prediction of the value of $m_\beta/m_\tau$. Naive calculation of the right-handed neutrino mass $M_R$ is as follows; $m_\tau^{\text{exp}} = \sqrt{\Delta m_{21}^2 t m} = 10^{-1}$ eV, then it follows $M_R = \left(\frac{m_\nu_\tau}{m_\tau^{\text{exp}}}\right)^2$, which gives $10^{13}$ GeV by taking $m_\nu_\tau^{\text{exp}} = \eta_c m_\tau$ with a color RGE factor $\eta_c \sim 3$. If the hot dark matter candidate is ascribed as tau neutrino at the same time, we have to assume the almost degenerate masses for tau and muon neutrinos, i.e., $m_\nu_\tau \sim m_\nu_\mu \sim *$ eV, in which case we have $M_R = 10^{12}$ GeV. On the other hand, in our ESSM case in which the tau neutrino corresponds to a hot matter candidate, the predicted value is around $\epsilon \times 4 \times 10^{12}$ GeV. In any case, $M_R$ is on the lower boundary or even outside of the allowed region. So if the above effect is present it yields a very bad predicition of bottom tau mass ratio at least in the MSSM case.

This is because the existence of right handed neutrino Yukawa coupling compensates the enhanced effect of top Yukawa coupling in the following RGE equation,

$$R(\mu) = \frac{R}{16\pi^2}((Y_t^2 - Y_N^2) + 3(Y_\beta^2 - Y_\tau^2) - \frac{16}{3}g_3^2 - \frac{4}{3}g_1^2))$$

(18)

with $R(\mu) \equiv Y_\beta(\mu)/Y_\tau(\mu)$. Within $SO(10)$ framework, or, at least we start from some kind of left right symmetric models, we have the following Yukawa unification at GUT scale, $Y_t = Y_\nu$ and $Y_\beta = Y_\tau$. In MSSM it turns out to be difficult to reproduce the small ratio of the bottom to tau Yukawa couplings because up to $\mu = M_R$ they are cancelled out each other. This is inevitable in the case of MSSM so far as we take the boundary condition as above. It is our task, therefore, to investigate whether or not ESSM is in the same situation also. Fortunately enough, we can show that the presence of the right handed neutrino does not affect so seriously as in MSSM. The reason is as follows; As we noted before, in ESSM, due to the ANF character, the Yukawa couplings approach to their infrared fixed points very rapidly and the RGE effect of QCD leads the top Yukawa coupling to its infrared fixed point very rapidly irrespective of the initial condition compared with $Y_N$. This can be seen from the figure 1: you will find that $Y_t/g_3$ tends very rapidly to its infrared fixed point before the scale of $10^{12}$ GeV or so, while $Y_N/g_3$ becomes smaller and can be
neglected almost completely. This implies that the existence of $M_R$ from which the right-handed neutrino coupling is decoupled because of its mass, does not affect very much to the bottom tau ratio running. Thus the strong convergence of the relevant Yukawa couplings to IRFPs, because of the asymptotically non-free character, is particularly powerful to predict the low energy parameters.

6 Texture with $U(1)$ Assignments

It may be a little complicated to assign the a single family dependent $U(1)$ charge to reproduce our texture. However if one notes that we have two kinds of Higgs fields to get the Georgi Jarlskog type texture for the down quark and lepton mass ratio anyway, we need two kinds of Higgs fields having different $U(1)$ charges. Actually the following is just an example which reproduce the texture. Let us consider the following 2 $U(1)$ charges which ascribes to fermions as well as Higgs bosons. The first one ($F_u$) is responsible to determine the Higgs couplings of the up sector. In order to give zero for the (33,24,42)elements we assign the charges as $F_u(\psi_i(i = 1, 4)) = (3, 1, 0, -1))$ whose charge matrix is

\[
\begin{pmatrix}
6 & 4 & 3 & 2 \\
4 & 2 & 1 & 0 \\
3 & 1 & 0 & -1 \\
2 & 0 & -1 & -2
\end{pmatrix},
\]

with $F_u(H_u(10)) = 0, F_u(\theta) = -3$ with $\text{VEV}(\theta)/M = x = \lambda^4$ and with $F_u(H_d(10)) = -2$. For the down part it is more complicated to assign for the single Higgs, so we assume 2 126 Higgses. $F_d(\psi_i(i = 1, 4)) = (-1, 5, 0, -5))$ from which the charge matrix is

\[
\begin{pmatrix}
-4 & 3 & 1 & -7 \\
3 & 10 & 6 & 0 \\
1 & 6 & 2 & -4 \\
-7 & 0 & -4 & -10
\end{pmatrix}
\]

This can be made traceless by subtracting trace part (note that the 4th charge is already cancelled by its mirror fermions): $F_u'(\psi_i(i = 1, 4)) = (3, 1, 0, -1) - 4/3(1, 1, 1, 1) = (2, 0, -1, -2)$. However we neglect the anomaly problems for the moment.
with $F_d(H_d(126)) = 0$, $F_u(\theta') = -2$. With $\text{VEV}(\theta)/M = \lambda$ and $F_d(H_d(126)) = 10$, $F_u(\theta'') = -3$ with $\text{VEV}(\theta'')/M = x = \lambda^4$. This is not simple but if one reminds that we are considering $SO(10)$ unification, which have more stringent relations than $SU(5)$. Anyway this is just an example to show that it is in principle possible to describe such charge assignments to reproduce the texture.

References


