On Viscoelastic Changes Appearing on Tiltmetric and Extensometric Records of the Ground*

Torao Tanaka
Disaster Prevention Research Institute, Kyoto University
(Received November 25, 1981)

Abstract
Secular ground tilts and strains observed with tiltmeters and extensometers show sometimes characteristic changes which can fully be represented by exponential functions with time constants of several years or shorter. We estimate the time constants of such exponential secular changes on the results obtained from various stations in Japan, excluding changes with time constants shorter than several months. It is found that these exponential secular changes are classified into two groups; one has rather long time constants of about 5 years or more and the other has shorter time constants of about 2 years. The origin of the former is considered to be instrumental drift due to stress relaxation within instrumental materials and that of the latter being real ground deformations due to stress relaxation in rocks around observation instruments. The latter might have some connection with post-seismic crustal movements usually observed in epicentral regions.

1. Introduction

Tanaka et al. [1] investigated secular ground tilt observed for about 29 years since 1952 with two horizontal pendulum tiltmeters (for short we call this type of tiltmeters HP tiltmeters hereafter) at Kishu Mine in the southeastern part of the Kii peninsula, and showed that the secular tilts are represented by exponential functions with a time constant

* 昭和 56 年 5 月 日本測地学会第 55 回講演会にて発表
of about 5 years. Furthermore, they attributed the exponential tilts to stress relaxation within rock materials around the tiltmeters, on the grounds that the time constants for both the two tiltmeters were equal to each other and their total amount of tilts for 29 years were considered too large to be wholly attributable to instrumental origins.

Table 1. Description of instruments used in the analysis and the time constants obtained.

<table>
<thead>
<tr>
<th>No</th>
<th>Station</th>
<th>Instrument</th>
<th>Symbol/Azimuth</th>
<th>Period analysed</th>
<th>Time const</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kishu</td>
<td>HP-T</td>
<td>A/N57°W</td>
<td>1953 - 78</td>
<td>4.8 yr</td>
<td>[1]</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>B/N33°E</td>
<td></td>
<td>5.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Kamioka</td>
<td></td>
<td>A/S45°E</td>
<td>1958 - 64</td>
<td>5.4</td>
<td>[2]</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>B/S45°W</td>
<td></td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Akatani</td>
<td></td>
<td>EW</td>
<td>1967 - 73</td>
<td>5.1</td>
<td>[3]</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>NS</td>
<td>1966 - 70</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Inuyama</td>
<td></td>
<td>N57°E</td>
<td>1968 - 80</td>
<td>4.8</td>
<td>[4]</td>
</tr>
<tr>
<td>8</td>
<td>Izunagaoka</td>
<td></td>
<td>N37°E</td>
<td>1960 - 63</td>
<td>1.2</td>
<td>[5]</td>
</tr>
<tr>
<td>9</td>
<td>Donzurubo</td>
<td></td>
<td>2A/N45°E</td>
<td>1969 - 78</td>
<td>2.2</td>
<td>[6]</td>
</tr>
<tr>
<td>10</td>
<td>Akibasan</td>
<td>Bimetal thermometer</td>
<td>Ta</td>
<td>1961 - 64</td>
<td>0.62</td>
<td>[7]</td>
</tr>
<tr>
<td>11</td>
<td>Tochigi</td>
<td>WT-T</td>
<td>EW</td>
<td>1950 - 78</td>
<td>9.7</td>
<td>[8]</td>
</tr>
<tr>
<td>12</td>
<td>Matsuyama</td>
<td></td>
<td>NS</td>
<td>1961 - 70</td>
<td>4.3</td>
<td>[9]</td>
</tr>
<tr>
<td>13</td>
<td>Inuyama</td>
<td></td>
<td>N57°E</td>
<td>1968 - 74</td>
<td>5.1</td>
<td>[4]</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td>EW</td>
<td>1968 - 80</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Osakayama</td>
<td></td>
<td>WT11/N38°E</td>
<td>1963 - 73</td>
<td>8.7</td>
<td>[10]</td>
</tr>
<tr>
<td>17</td>
<td>Nibetu</td>
<td></td>
<td>N25°E</td>
<td>1969 - 76</td>
<td>1.7</td>
<td>[12]</td>
</tr>
<tr>
<td>18</td>
<td>Yahiko</td>
<td></td>
<td>N02°W</td>
<td>1969 - 80</td>
<td>4.5</td>
<td>[13]</td>
</tr>
<tr>
<td>19</td>
<td>Ohmine</td>
<td></td>
<td>B'B/N57°W</td>
<td>1969 - 77</td>
<td>1.8</td>
<td>[14]</td>
</tr>
</tbody>
</table>

T: Tiltmeter, HP: Horizontal pendulum, WT: Water tube
S: Extensometer, FQ: Fused quartz tube, SI: Super-invar bar, BH: Borehole
Fig. 1. Secular changes observed with horizontal pendulum tilmeters. Exponential drifts should be linear on this semi-logarithmic plot after subtracting constant terms. Numbers correspond to those in Table 1.

Similar exponential changes with time constants of a few to several years are sometimes seen on tiltmetric and extensometric records, and they are familiar to us who are engaged in observations of crustal movements and earth tides with these kinds of instruments. With the purpose of clarifying their nature we have collected such exponential changes as many as possible from Japanese reports and papers, and estimated their time constants.

Based on the time constants obtained they are classified into two groups and interpreted as the changes due to viscoelastic stress relaxations within rocks around observation instruments and within instruments themselves.

2. Data

The data used in this analysis are summarized in Table 1. Yearly or half yearly values have been read from figures given on the literature referred to in the last column of the table. Although digitization errors are, therefore, not sufficiently small to be neglected, the conclusions obtained here would not be altered drastically by such errors. We have excluded from the present analysis short-term exponential changes usually seen just after
Fig. 2. Secular changes observed with water tube tiltmeters.

the commencement of observations or reset of instruments with time constants shorter than several months.

3. Determination of time constants

We assume that a secular change observed can be expressed as an exponential time function

\[ y = a \exp\left(-t/t_\tau\right) + b \]  \hspace{1cm} (1)

where \( t_\tau \) is the time constant, and \( a \) and \( b \) are constants to be estimated together with \( t_\tau \). The procedure we adopted here is as follows: First we estimate only \( t_\tau \) from the rate of the secular change plotted on semi-logarithmic coordinates and then calculate the two constants \( a \) and \( b \) by the method of least squares with an already obtained \( t_\tau \). Accordingly, if our assumption that the secular change \( y \) is expressed as an exponential function is correct, then \( y - b \) plotted on a semi-logarithmic graph paper will be a straight line.

4. Results

Analysed periods and obtained time constants are shown in Table 1. The exponential parts \( y - b \) of the secular changes observed with HP tiltmeters, water-tube tiltmeters (WT) and extensometers are shown in Figs. 1, 2 and 3, respectively. Numerals on each curve
correspond to those in Table 1. For convenience of comparison between time constants, Figs. 4, 5 and 6 have been drawn by taking the starting points of observations given in Figs. 1, 2 and 3 as the time origin. It is found that the secular changes are evidently represented by straight lines on semi-logarithmic coordinates, and moreover they are divided into two groups with larger time constants of about 5 years (hereafter we refer this group to S -slow- group) and with small time constants of about 2 years (we refer this to R -rapid- group); they are shown by solid and dashed lines in these figures, respectively.

Curve 10 in Fig. 1 shows a long-term change of room temperature in an underground vault recorded by a bimetal thermometer [7]. Its origin is considered to be stress relaxation within the bimetal, though a possibility of an actual increase of the room temperature cannot be completely excluded, because the vault had been shut by constructing wooden double doors at the entrance at the commencement of observation and electric power was used for lamp and recording devices. Its time constant obtained is 0.6 year and rather short compared with those of tiltmetric and extensometric changes analysed here. Therefore this exponential change of the room temperature is considered to be an initial drift of the same kind to those appearing on tiltmetric and extensometric records due to initial complex disturbances in observation instruments and their circumstances which we have
Fig. 4. Secular changes observed with horizontal pendulum tiltmeters. They are the same to those in Fig. 1, but the starting point (left end) of each curve is taken as the time origin and the tilt angle is so normalized as to be 1.0 at t=0 and 0.0 at t=∞. The numbers and slant lines at the right side and the bottom give the time constant by connecting them with the starting point.

In Fig. 1, R group from HP tiltmeters show fairly small amounts of tilts of about 50 µrad since the commencement of each observation compared with those of S group which reach 1 mrad or more (note that the ordinate is the logarithmic scale). On the contrary S group in WT tiltmeters in Fig. 2 show rather small tilts less than 10 µrad compared with R group which amount to 20-30 µrad. It is to be noted that the amount of tilts of R group in WT tiltmeters in Fig. 2 are nearly equal to those of R group in HP tiltmeters in Fig. 1.

Curves 5, 6 and 7 in Fig. 3 are secular volumetric strains observed in deep boreholes with Sacks Evertson strainmeters [21]. Curves 6 and 7 apparently belong to R group. Their secular strains, however, are large compared with those of other components (for example, curves 11, 13 and 14) of R group. If we assume that a horizontal linear strain is approximated by $e_{xx} = e_{yy} = (dV/V)/2(1-\sigma)$, where $\sigma$ is the Poisson's ratio, then $e_{tt}(=e_{yy})$ may be obtained for the borehole observations by dividing the volumetric strains by 1.5 assuming $\sigma=0.25$. The values thus estimated are still large, and this is probably due to
On Viscoelastic Changes Appearing on Tiltmetric Records

231

Fig. 5. Secular changes observed with water tube tiltmeters. These are the same to those in Fig. 2.

the difference of observing situations. For this reason it is reasonable to exclude the borehole observations from R group, and we may conclude that R group in the extensometric observations show smaller exponential secular strains than S group; this is the same to the result in Fig. 1 for HP tilometers.

The estimated time constants and mean squared errors are summarized in Fig. 7. The frequency centered around 2 and 5 years is high as expected from previous figures.

5. Viscoelastic Changes in Instrumental Drifts and Ground Deformations

The exponential secular changes analysed in the previous section may be interpreted as phenomena relating to viscoelastic stress relaxation within some materials. The viscoelastic stress relaxation in a Maxwell body is given by \( \sigma = \mu \epsilon_0 \exp(-\mu t/\eta) \), where \( \mu \) and \( \eta \) are elasticity and viscosity coefficients, respectively, and the ratio \( \eta/\mu \) is equal to the time constant \( t_r \) [22].

It has often been reported that secular changes recorded with HP tilometers are heavily contaminated with instrumental drifts because of their inherent instability and small size of the instruments. WT tiltmeters, on the other hand, are generally considered to be free from such instrumental drifts. However, even in the case of WT tiltmeters, instrumental drifts might reach one-hundredth or more of those for HP tiltmeters under
some unfavorable conditions, because the ratio of the effective dimension of both the types of tiltmeters is order of 100:1. In the case of extensometers with solid length standards the situation of instrumental drifts will be in the midpoint between the two types of tiltmeters, HP and WT, because of a possibility of the creep in length standards.

Since R groups in Figs. 1, 2 and 3 show nearly equal secular exponential changes of order of $10^{-6}$ (namely, they have nearly equal values of $a$ in (1) when we take $t=0$ at the time of starting point of published results), they are considered to originate from some common source. On the other hand the changes of S group are larger than those of R group in the cases of HP tiltmeters and extensometers, and vice versa in the case of WT tiltmeters. In addition, the amount of changes of S group of WT tiltmeters is order of one-hundredth of those of HP tiltmeters, and this is consistent with the above consideration as to instrumental drifts. Therefore we assert here that the exponential changes of R group are relating to viscoelastic stress relaxations in rocks around instruments and those of S group relating to instrumental drifts.

It is well known that similar exponential deformations of the earth have been observed following large earthquakes in their epicentral regions. The origin of these post-seismic deformations are considered to be due to slow creep on the fault zones [23] or viscoelastic after-effect relating to the asthenosphere [24]. Time constants of several post-seismic
deformations were calculated by Yamauchi [25], who assumed the exponential form of $a \exp(-bt) + ct + d$. In his results all of the estimated time constants are smaller than 3.5 years except the 1894 Off Nemuro earthquake, for which the time constant is given as 8 years. We have calculated the time constants $t$, and the constants $a$ and $b$ of post-seismic deformations of the Mitaka rhombus baseline after the 1923 Kwanto earthquake and of the level changes at Muroto and Kushimoto after the 1946 Nankai earthquake by the same procedure mentioned in §3. In these estimates we used figures and numerical data in papers by Fujita [26], Tsuboi [27], Okada and Nagata [28] and Okada [29]. The exponential parts $y - b$ of these post-seismic changes are shown in Fig. 8. Okada and Nagata [28] calculated the time constant of the level change at Muroto point and obtained 0.97 year, which is in good agreement with our result. Most of the time constants of these post-seismic deformations are smaller than 3 years. This suggests that exponential post-seismic crustal deformations and the exponential secular changes recorded by tiltmeters and extensometers with time constants of about 2 years might originate from some similar kind of viscoelastic relaxations of stresses which are set at the time of occurrence of earthquakes or artificial disturbances such as construction of vaults.

6. Discussion

It is to be noted that the exponential secular changes discussed here are rather unusual phenomena. In these cases there might have been some favorable circumstances under
Fig. 8. Post-seismic ground deformations. Slant lines and numbers at the right side and the bottom give the time constants. 1: Maximum shear of the Mitaka rhombus [26], 2: Angular change [26], 3: Areal change dNES [27] and 4: Areal change dSWN [27] of the Mitaka rhombus after the 1923 Kanto earthquake. 5: Level change at Muroto [28], and 6: Level change at Kushimoto [29], after the 1946 Nankaido earthquake.

which such exponential changes could be observed. For example, a newly dug underground vault will show large exponential deformations due to relaxation of the stress given artificially around the vault: Inuyama, Ohmine, Yahiko, Nibetu and Erimo seem to correspond to this case.

To estimate the constants \( a \) and \( b \) we have chosen the time origin of the exponential changes arbitrarily. If we could know the exact time when the stress was applied and adopt it as the time origin, discussions about the physical meaning of the value of \( a \) would become possible.

Shichi and Iida [30] pointed out that the tilt rate of the NS component of the HP tiltmeter at Inuyama (curve 7 in Fig. 1) was expressed by \( \delta = 6.0/t \) (secarc/day), namely as the form \( y = a \log t + b \). Scholz [31] plotted post-seismic crustal movements on the logarithmic time scale assuming a creep formula similar to that used by Shichi and Iida. We cannot decide at the present stage whether the logarithmic curve fits better to the observed secular change of the HP tiltmeter at Inuyama than the exponential curve we have obtained here. The secular changes recorded with HP tiltmeters at Kishu Mine, on the other hand, can apparently be expressed by exponential forms better than logarithmic curves such as \( a \log t + b \) or \( a \log (t+1) + b \) which are known as the creep formulae for rocks [32].

Yamauchi [25] concluded that the time constants of post-seismic deformations are dependent on earthquake magnitudes. His analysis includes exponential deformations observed by extensometers with time constants of a few days and smaller, which we have
excluded in the present analysis. It is probable that post-seismic deformations are not generated from only one source through some simple mechanism but generated from several different origins through complex mechanisms. Therefore, our conclusion that some part of post-seismic exponential deformations may be caused by viscoelastic stress relaxations within rocks in the crust, especially in superficial layers, does not conflict with the result by YAMAUCHI.

If the elasticity of rocks in the crust, which is relevant to such viscoelastic deformations, is assumed to be $3 \times 10^{11}$, then the viscosity is estimated as $2 \times 10^{19}$ for the time constant of 2 years. This value is extremely small compared with the rock viscosity generally accepted. If such low values of viscosity are universally valid, rocks would not support the masses of mountains, and it seems to be impossible for the earth's crust to store tectonic stress sufficient to generate large earthquakes. Therefore this type of stress relaxations is considered to occur under some restricted circumstances, probably due to stress concentrations associated with microcracks or small cavities within rocks, so that total deformations, namely the value of $a$ in (1), may not exceed some limiting values.

Curve 2 in Fig. 2, WT tiltmeter at Matsuyama, deviates remarkably from a linear decrease. In this case, as shown in Fig. 9, it might be better to divide the entire term
into two parts. This suggests that some event might have happened around 1960 to apply a stress within the tiltmeter. Similar divisions and/or coexistence of two or more exponential changes with different time constants must also be taken into considerations in closer investigations in the future.

We could not find any apparent relations between the time constants obtained and the depth of vaults, type of rocks or the period since the construction of vaults.

7. Conclusions

Exponential secular changes observed with tiltmeters and extensometers are classified into two groups by their time constants which they follow. We presume that exponential changes with time constants of 3 years or less are originated from stress relaxations within the ground and that post-seismic exponential deformations of the ground might be caused partly, to say the least of it, by stress relaxations in rocks of comparatively shallow parts of the crust. The other group of exponential secular changes with time constants of about 5 years is considered to be due to instrumental origins. From this point of view the exponential changes reported by TANAKA et al. [1] are concluded to be due to instrumental stress relaxation.

It is suggested that tiltmetric and extensometric records should be investigated taking the fact into considerations that exponential responses are sometimes observed with time constants of about 2 or 5 years after such events as applying stresses to the ground or observation instruments themselves, respectively.

Acknowledgements

We wish to thank Prof. T. MIKUMO of the Disaster Prevention Research Institute, Kyoto University for critical reading the manuscript and to thank the other members in Earthquake Prediction Research Section of the Institute for their helpful discussions.

References (*—in Japanese)

On Viscoelastic Changes Appearing on Tiltmetric Records 237


