Longitude Origins on Moving Equator

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Abstract

Derived were the mathematical expressions of several longitude origins on a moving equator. Their motions in space were illustrated for a simple model of the motion of a pole. Noted was the global secular rotation of Guinot's non-rotating origin (NRO) (Guinot 1979) with respect to the inertial reference frame. Such secular rotation does not appear in the motions of some origins defined geometrically. Among them, recommended was the foot of the x-axis on the moving equator. This is partly because it is independent on the adopted value of obliquity at the epoch in the case of precessional motion of the Earth's pole. And this is partly because it is directly accessible through the angular observations referred to the moving equator such as the time measurement of prime meridian passage of quasars by VLBI facilities on the Earth.

1. Introduction

At its 24th General Assembly held at Manchester, U.K., in August 2000, the IAU passed a resolution to adopt Guinot’s non-rotating origin (Guinot, 1979; Capitaine et al., 1986) as a new longitude origin for celestial and terrestrial reference frames (IAU 2000). The new origins were named the Celestial Ephemeris Origin (CEO) for the celestial reference frame and the Terrestrial Ephemeris Origin (TEO) for the celestial reference frame, respectively. The CEO was created to replace the classical equinoxes. We note that the non-rotating origin (NRO) is equivalent with Newcomb’s departure point (Aoki and Kinoshita 1983).

The aim of the replacement of the equinox by using the concept of NRO is to exclude the effects of planetary precession as much as possible from the expression of right ascensions of stars and quasars. Namely, this is meant to separate the effects of Earth’s rotational and orbital motions, which are mixed in the current expression of precession and nutation. Of course, if for this purpose only, there are some other candidates such as the foot of the x-axis onto the equator (Kovalevsky and McCarthy, 1998). The most important property of the NRO which discriminates it from the others is its local irrotation with respect to the instantaneous pole. In other words, the NRO is defined as the point on the moving equator so as to have no component of instantaneous rotation around the pole whatever motion the pole takes (Capitaine et al., 2000).

Note that this property never assures that the NRO does not rotate globally with respect to the inertial frame. In fact, the pole is generally different from the instantaneous rotation axis as the Earth’s figure axis differs from its rotation axis. Also, the property defining the
NRO is of differential nature since it regulates only the instantaneous speed of rotation. Thus, it is not trivial whether this local non-rotation really leads to the global non-rotation with respect to the inertial reference frame, which we expect as the most important property of the longitude origins, especially that for the celestial reference frame. Unfortunately, the answer is no. As will be shown later, the NRO does have a secular component of rotation with respect to the inertial frame, which is a serious defect from the author’s viewpoint.

In Section 2, we will list some candidates of longitude origins on a moving equator. Then we will give their mathematical expressions explicitly. In Section 3, we will illustrate their global motions for a simple screw motion model of the motion of pole.

2. Mathematical Expressions

Consider the following five origins on the instantaneous moving equator (Kovalevsky and McCarthy); (1) the node, N, defined as the intersection with respect to the (fixed) x-y plane, (2) the prime meridian intersection, K, the intersection with respect to the (fixed) x-z plane, (3) the foot of the x-axis onto the moving equator, H, (4) the point, F, defined such that FN = XN, where X represents the x-axis, and (5) Guinot’s non-rotating origin (NRO), S. See Figure 1. Note that the NRO is usually designated as σ and the equinox is done as Y or γ. Sometimes F is also symbolized as Σ.

Let us denote a moving pole by P and the corresponding direction vector by P. The (moving) equator is defined as a great circle perpendicular to this vector. Then the direction vector pointing an arbitrary origin on the equator, say R, must satisfy the orthonormal relations as R · P = 0 and |R| = 1. Since R has three independent components, one more condition is necessary and sufficient to specify it. The last condition for the above origins are given except for the NRO as

\[ 0 = N \cdot Z = K \cdot Y = H \cdot (P \times X) = (F - X) \cdot N, \]  

where X, Y, and Z are the direction vectors pointing the x-, y-, and z-axis, respectively. The condition for the NRO will be described later in a different manner. The above conditions are explicitly solved and we obtained the following explicit expressions;

\[ N = \frac{Z \times P}{|Z \times P|}, \quad K = \frac{Y \times P}{|Y \times P|}, \quad H = \frac{(P \times X) \times P}{|(P \times X) \times P|}, \]

\[ F = \left[ Y - \left( \frac{P \cdot Y}{1 + P \cdot Z} \right) Z \right] \times P. \]  

The corresponding coordinate triads, i.e. the orthonormal basis vectors, are easily obtained from these by taking vector products such as (P, F, G) for F. Here G \( \equiv P \times F \). We note that thus obtained expression of the triad is no other than the precession-nutation matrix.

Next, consider the coordinate expressions. If we express P in the inertial frame as P = (X, Y, Z)\(^T\), then the above solutions are explicitly specified as

\[ N = \frac{1}{\sqrt{X^2 + Y^2}} \begin{pmatrix} -Y \\ X \\ 0 \end{pmatrix}, \quad K = \frac{1}{\sqrt{1 - Y^2}} \begin{pmatrix} Z \\ 0 \\ -X \end{pmatrix}, \]
Fig. 1. Longitude Origins on a Moving Equator. Shown are some longitude origins on a moving equator; N, H, F, K, and S. Here N is the node defined as the intersection with the x-y plane, H is the foot of the x-axis onto the moving equator, F is the point satisfying the condition FN = XN, K is the intersection with the x-z plane, and S is Guinot’s NRO. Note that P denotes a moving pole defining the equator.

\[
H = \frac{1}{\sqrt{1 - X^2}} \begin{pmatrix} 1 - X^2 \\ -XY \\ -XZ \end{pmatrix}, \quad F = \begin{pmatrix} 1 - X^2/(1 + Z) \\ -XY/(1 + Z) \\ -X \end{pmatrix} \tag{3}
\]

where we used the normalization condition, \(X^2 + Y^2 + Z^2 = 1\). Another type of specification is the relative location with respect to some specific origin, say F. One example of this relative location is the elongation upto F, i.e. the signed arc length measured from it upto F. We will denote the elongation by small italics as \(n = \overline{NF}, k = \overline{KF}, \) and so on. Then the expressions of this type are obtained as

\[
n = -\text{atan2}(X, -Y), \quad k = \tan^{-1}\left(\frac{XY}{1 + Z - Y^2}\right), \quad h = \tan^{-1}\left(\frac{-XY}{1 + Z - X^2}\right),
\]

\[
s = s_0 - \int_{t_0}^{t} \frac{1}{1 + Z} \left( X \frac{dY}{dt} - Y \frac{dX}{dt} \right) dt \tag{4}
\]

where \(s_0\) is a certain initial value. The last expression was first derived by Capitaine. et al.
Consider the time dependence of these origins. The origins N, K, H, and F depend on the pair of a certain fixed direction and a moving pole P. Thus their time dependences are only through that of P. In other words, once P is known, these origins are determined. While the equinox is a function of two moving directions, P and the ecliptic pole. Thus one has to know both motions precisely enough to specify the equinox with a precision comparable with that of VLBI and other state-of-the-art observational techniques. This is quite difficult (Kovalevsky and McCarthy, 1998). Therefore, we must replace the current standard origin, the equinox, by a more appropriate origin. On the other hand, we need the whole history of the motion of the pole P to specify the NRO. It is also a difficult task to do. As Capitaine et al. (2000) admitted, this is a drawback of NRO.

3. Global Motion

In order to illustrate the global motions of these longitude origins, we will draw their orbits on the unit sphere by assuming a simple model of the motion of the pole. As such a model, we consider a uniform screw motion around a fixed direction in the y-z plane, \( \mathbf{J} = (0, -\sin \epsilon, \cos \epsilon)^T \), where \( \epsilon \) is a constant satisfying \( 0 < \epsilon < \pi/2 \). Also we assume that the pole initially starts from the z-axis and moves in the clockwise sense. This is a simplified precessional motion of the Earth’s pole in the equatorial reference frame. If we denote the rotation angle by \( \psi \), then this screw motion is expressed as

\[
P = Z \cos \psi + (\mathbf{J} \cdot \mathbf{Z}) \mathbf{J}(1 - \cos \psi) + \mathbf{J} \times \mathbf{Z} \sin \psi = \begin{pmatrix} 
\sin \epsilon \sin \psi \\
-\sin \epsilon \cos \epsilon (1 - \cos \psi) \\
\cos^2 \epsilon + \sin^2 \epsilon \cos \psi 
\end{pmatrix}
\]

In this case, the node N moves along the fixed equator in the clockwise sense. Actually its longitude is expressed as an explicit function of \( \psi \) as

\[
n = -\frac{\pi}{2} + \frac{\psi}{2} - \tan^{-1} \left( \frac{\sin^2(\epsilon/2) \sin \psi}{\cos^2(\epsilon/2) + \sin^2(\epsilon/2) \cos \psi} \right)
\]

Note that its secular speed is the half of that of \( \psi \), say around 25″/yr for the precessional motion of the Earth’s pole. It is trivial that the point K oscillates along the zero meridian. Namely it has no secular motion with respect to the inertial reference frame. Also the points F and H move around K, or more strictly speaking around the x-axis, and therefore both of them have no secular rotation with respect to the inertial frame.

Consider the global motion of the NRO. Fortunately \( s \) is integrated explicitly as

\[
s = 2 \left( \frac{\sin^2 \epsilon}{2} \right) \psi - 2 \tan^{-1} \left( \frac{\sin^2(\epsilon/2) \sin \psi}{\cos^2(\epsilon/2) + \sin^2(\epsilon/2) \cos \psi} \right)
\]

where we chose the initial value of \( s \) such that \( s = 0 \) when \( \psi = 0 \). Note that the first term in the last expression represents the secular rotation of NRO with respect to the inertial
Fig. 2. Elongations of Longitude Origins. Shown are the elongations of the origins, K, H, and NRO measured up to F as functions of the precession angle, $\psi$. The angles are computed in the equatorial reference frame for the precessional motion of the Earth’s pole. The obliquity was set constant as $23.5^\circ$.

frame, since the point F has no secular rotation. Namely, in the case of precessional motion of the Earth’s pole, the NRO globally circulates with the speed of $2p \sin^2(\varepsilon/2)$, where $p$ is the general precession in longitude. The speed is as large as $\approx 415''/cy$. See Figure 2 for the elongation angles as functions of $\psi$ and Figure 3 for the resulting orbits of the origins in the inertial frame. As seen there, the orbits of F and H become oblique 8-figure like curves and that of K is a segment of a slanted great circle. While that of NRO is a partially curved zigzag pattern along the ecliptic.

4. Conclusion

As is clearly seen in Figures 2 and 3, NRO moves secularly with respect to the inertial reference frame. Therefore, we think that the adoption of NRO as a longitudinal origin is inappropriate since it introduces a secularly rotating coordinates. In this sense, we prefer one of geometrically defined origins like K, F, H, and their mixtures. Of course, when the precession angle $\psi$ is sufficiently small, the angle $\delta$ seems to contain no secular motion. However, this is temporally caused by the cancellation of the secular trend and the linear approximation of the periodic part.
Fig. 3. Orbits of Longitude Origins. Shown are the orbits of the origins, K, F, H, and NRO, in the equatorial reference frame for the precessional motion of the Earth's pole. Note that K oscillates on the meridian of 0 longitude, F and H wander on 8-figure like curves centered at the z-axis, while NRO makes a zigzag motion shifting secularly along the ecliptic.

The issue which of the geometrically defined origins is most appropriate depends on the type of observation mostly used. If the observation is mainly conducted by referring to the moving equator, as is usually done in the ground-based VLBI, GPS, LLR, optical and other existing observations, the foot, H, is the mostly suitable since it directly connects to the observables by way of the transit time difference. Also H has a property that it is insensitive on the initial value of obliquity.

References