Quantitative Parametric Approach to Estimating Snowflake Size Distributions
Using an Optical Sensing Disdrometer

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Abstract

We evaluated a method by which to fit gamma distributions with parameters of snowflake size distributions by measuring three physical quantities using an optical sensing disdrometer. The three physical quantities are the diameters of the snowflakes that have 50 and 99 percentiles of volumes ($D_{50}$ and $D_{99}$, respectively) and the sum of the sixth powers of the diameters in a unit volume ($Z$). Snowflake size distribution was well fitted to a gamma distribution between $D_{50}$ and $D_{99}$, inclusive. This method prevented the snowflake volume from being considerably underestimated. Although the mean absolute error based on snowflake volume for this method was large compared with that obtained using a moment method, good estimates of snowflake volume were obtained for some samples using this method, for which the snowflake size distribution was less influenced by snowflakes 1 mm or less in diameter. The correlation coefficient was 0.989, as determined by regression analysis based on the observed and estimated snowflake volumes using this method. The estimation of the snowflake volume using this method depends on the quality control of the optimum shape parameter and requires a continuous probability distribution of snowflakes for diameters above 1 mm.


1. Introduction

Snowflake size distribution parameters are more useful than nonparametric snowflake size distributions for expressing the fundamental physical quantities that are used for radar and cloud physics studies. This is because the form of snowflake size distributions can be expressed quantitatively using such parameters, and we can then integrate these distributions analytically to calculate the snowfall rate ($R$). Furthermore, parametric snowflake size distributions are statistically stable, i.e., are little affected by outliers.

In early studies on parametric snowflake size distributions, Gunn and Marshall (1958) used the filter paper method to analyze 20 snowflake size distributions. They reported that, above $D = 1$ mm (where $D$ is the diameter of the melted snowflakes), the average distributions analyzed by Gunn and Marshall (1958) were well fitted by $N(D) = N_0 \exp(-\mu D)$, which is the same function that is used for raindrop size distributions (Marshall and Palmer 1948). Here, $N_0$ and $\mu$ are the intercept and slope parameters, respectively, of the size distribution. Ulbrich (1983) proposed a gamma distribution, $N(D) = N_0 D^\gamma \exp(-\Lambda D)$, where $\gamma$ is a shape parameter. This is a general form of the equation of Marshall and Palmer (1948).

Increased understanding of snowflake size distributions has contributed to the understanding of cloud physics. Actual snowflake diameters as measured by aircraft observation were fitted to a gamma distribution. The parameters of $N_0$ and $\Lambda$ have contributed to a better understanding of the collision and aggregation processes in frontal clouds (Houze et al. 1979; Passarelli and Srivastava 1979) and individual clouds (Lo and Passarelli 1982).

Snowflake size distribution parameters are also used to estimate $R$ from radar data. It is especially important that $R$ be temporally and spatially accurate when predicting extreme snowfall events. The relationships between the sixth powers of the diameters in a unit volume ($Z$, or radar reflectivity factor) and $R$ have been used to estimate $R$ in experiments for decades (e.g., Gunn and Marshall 1958). However, accurate estimation of $R$ by radar is difficult without verification from observational data on the ground (Fujiyoshi et al. 1990), because the radar reflectivity factor ($Z$) contains uncertainties, which are due primarily to the shape, velocity, density, and mass of snowflakes. In subsequent studies, the $Z$-$R$ relationship was determined using a radar-based estimation method for determining $R$ from the snowflake size distribution parameters (Matrosov 1992, 1998). In this method (Matrosov 1992, 1998), the snowflake size distribution parameters were estimated by dual-frequency radar. In order to determine the $Z$-$R$ relationship, it is essential to compare the snowflake size distribution parameters estimated by radar with those measured on the ground. Several studies have applied gamma distribution parameters to observed snowflakes (e.g., Brandes et al. 2007; Newman et al. 2009). Vivekanand et al. (2004) also estimated the gamma distribution parameters for raindrops by using three moments of size distribution, such as the second, fourth, and sixth powers ($Z$). Note that $Z$ is strongly influenced by large particles, and so it is important for ground-based verification and correction of the $Z$-$R$ relationship to accurately estimate the size distribution parameters for large-diameter snowflakes ($D > 1$ mm) and to avoid underestimation of snowflake volume ($W$), especially when predicting extreme snowfall events.

The purpose of the present study is to estimate the snowflake size distribution parameters ($N_0$, $\Lambda$, $A$) using three physical quantities, in particular, the diameters for 50 and 99 percentiles of snowflake volumes ($D_{50}$ and $D_{99}$, respectively) and $Z$, with an optical sensing disdrometer (Sections 2 and 3). The snowflake size distribution was well fitted to a gamma distribution in the range between $D_{50}$ and $D_{99}$, inclusive. We discuss quality control of the data for size distribution in Section 4 and present our conclusions in Section 5.

2. Observations

An optical sensing disdrometer, the Scintec Parsivel (R) M300 (for simplicity, Parsivel) was used to measure snowflakes during 48 days, from 25 December 2009 to 27 March 2010, in the Minakami highlands, Gunma Prefecture, Japan. Snowflake size distributions were estimated every 10 minutes. Figure 1 shows the location of the observation site.

The Parsivel measures the particle size and falling velocity (Löffler-Mang and Joss 2000). The Parsivel consists of a pair of sheet laser devices: one that generates a horizontal light beam and another that receives the beam. The sheet laser is 180 mm long, 27 mm wide, and 1 mm thick. Any particles passing through the beam cause optical extinction, resulting in a reduction in the
output voltage. Particle size (volume-equivalent diameter, $D$) is calculated from the maximum signal amplitude, which provides the ratio between the largest obscured area and the cross-sectional area of the light beam. Particles with $D < 1$ mm are assumed to be spherical, and particles with $D > 1$ mm are regarded as oblate spheroids (flattened ellipsoids of revolution). Particle velocity is derived from the signal duration and the particle size, along with the shape assumption. Particles are divided into a total of 32 size classes (from 0 to 26 mm) and 32 velocity classes (from 0 to 22.4 m s$^{-1}$). The size and velocity of an individual class are determined by the mean values within that class.

In the present study, we estimated the size distribution parameters for “snow” as classified by the Parsivel. In order to determine the precipitation types, it is necessary to have a total cross-sectional area of at least 10 mm$^2$. The type of precipitation determined by the Parsivel is related to the number of particles recorded within a 60-second interval. The number of particles counted must be at least 5 for rain, mixed precipitation, or snow, 10 for graupel, and 20 for drizzle.

The size distribution refers to the number of particles per unit volume. From the Parsivel data, the size distributions $N(l)$ [mm$^{-3} \cdot$ m$^{-3}$] were calculated as follows:

$$N(l) = \sum_{i=1}^{32} \frac{n(i,l)}{s \cdot v(i,l) \cdot wid(l)},$$

where $l$ is the size class, $i$ is the velocity class, $n(i,l)$ is the number of particles, $v(i,l)$ is the velocity [ms$^{-1}$], $wid(l)$ is the width of the particle size class [mm], $t$ is the observation time [s], and $s$ is the surface area of the horizontal sheet laser (0.00486 m$^2$).

According to Battaglia et al. (2010), Parsivel data underestimate the size distributions of snowflakes due to inaccurate observations of their size and falling velocity. However, the present study focuses on the evaluation of a method by which to fit the gamma distributions with parameters of snowflake size distributions.

### 3. Methods

In the present paper, snowflake size distributions $N(D)$ [mm$^{-3} \cdot$ m$^{-3}$] are assumed to be gamma distributions (Ulbrich 1983). Note that $W$ and $Z$ can be expressed by formulas that include higher-order moments of the size distribution. The $n$th moments of the size distribution can be expressed using a gamma function (Doviak and Zrnić 1993):

$$m_n = \int_0^\infty N_i D^{n+1} \exp(-\Lambda D) = N_0 \frac{\Gamma(n+1)}{\Lambda^{n+1}},$$

where $\Gamma(n+1) = (n+1)!$. According to Sekhon and Srivastava (1971), any errors derived from the integral range between $D_{min} = 0$ and $D_{max} = \infty$ are negligible. Here, $W = (x/6)m_3$, and $Z = m_2$.

### 3.1 Shape and slope parameters

In order to calculate $\mu$ and $\Lambda$ [mm$^{-1}$], we introduce the equations for 50% and 99% of $W$:

$$\frac{\Lambda^{n+1}}{\Gamma(n+4)} \int_0^{D_{0,n}} D^{n+3} \exp(-\Lambda D) dD = 0.50,$$

$$\frac{\Lambda^{n+1}}{\Gamma(n+4)} \int_0^{D_{99,n}} D^{n+3} \exp(-\Lambda D) dD = 0.99.$$
decreases, the errors of snowflake volume became larger. We applied a gamma distribution to the Parsivel data using three fitting methods. Here, $RE$ calculated using the linear least-squares method ($RE$) and Vivekanandan et al.’s (2004) method ($RE_\nu$) varied from $-0.26$ to $8.11 \times 10^3$ (when the correlation coefficient was more than 0.5) and from $-0.38$ to 0.57, respectively. The proposed estimation method best prevented snowflake volume from being underestimated. In terms of the mean absolute values of $RE_\nu$, $RE$, and $RE_\nu$, were 0.554, 0.076, and 1.397 $\times 10^3$, respectively. Although $RE_\nu$ was the smallest value among the three methods, 21 samples had the relation $|RE_\nu| < |RE|$, and 19 out of 21 samples had the relation $|RE| < 0.1$. Figure 3a shows the snowflake size distributions estimated by the present study and Vivekanandan et al.’s (2004) method, and the observed histogram, where $RE = 0.007$ and $RE_\nu = 0.13$. The present study, in which the result showed less influence from observed snowflakes 1 mm or less in diameter, could accurately estimate the snowflake volume.

Of the 636 samples for which $W$ was estimated using snowflake size distribution parameters, only 17 samples (2.7%) were underestimated. The proposed estimation method prevented the snowflake volume from being considerably underestimated. Snowflake size distributions estimated using this method when $|RE| < 0.1$ can be used to evaluate size distributions estimated by radar. This method is useful for verification and correction of the Z-R relationship when the snowflake density is estimated through comparison with other instruments. The corrected Z-R relationship allows for safety decisions related to avalanche warnings and controlling entry to mountainous areas during extreme snowfall events.

The reason that $W$ was overestimated in 619 size distributions is related to the characteristics of the tails for large particle size in the observed snowflake size distributions. The tails of the distributions affect the estimation of $N_{w_0}$, because $N_{w_0}$ is proportional to $D^2$. However, in the tails of the distributions, the observed snowflake size distributions (i.e., nonparametric size distributions) do not decrease smoothly, as compared to the estimates (i.e., parametric size distributions), because the Parsivel measures individual snowflakes. This difference in the tails of the distributions results in overestimations of $W$ for almost all size distributions.

### 4.2 Optimum shape parameter

The optimum value for the shape parameter ($\mu$) is determined from the minimum difference ($\Delta \Lambda$) between the slope parameters $\Lambda_1$ and $\Lambda_2$, which are calculated from $D_{0_1}$ and $D_{0_2}$, respectively. Table 1 shows the minimum and maximum values of $\Delta \Lambda$ between the slopes $\Lambda_1$ and $\Lambda_2$, which are calculated from $D_{0_1}$ and $D_{0_2}$, respectively.

### Table 1. Minimum and maximum values of $\Delta \Lambda$ and $D_{0_1}/D_{0_2}$ when $\mu$ is an integer between $-3$ and 3, inclusive.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\Delta \Lambda$</th>
<th>$D_{0_1}/D_{0_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$1.230-1.420$</td>
<td>$1.15-1.54$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$0.010-0.177$</td>
<td>$3.57-4.72$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$0.004-0.069$</td>
<td>$2.93-3.30$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0.003-0.045$</td>
<td>$2.61-2.72$</td>
</tr>
<tr>
<td>$1$</td>
<td>$0.019-0.041$</td>
<td>$2.53-2.58$</td>
</tr>
<tr>
<td>$2$</td>
<td>$0.002-0.253$</td>
<td>$1.62-2.23$</td>
</tr>
<tr>
<td>$3$</td>
<td>$0.023-0.253$</td>
<td>$1.69-2.23$</td>
</tr>
</tbody>
</table>

A small difference between $D_{0_1}$ and $D_{0_2}$ gives large errors between two slope parameters, indicating that it is difficult to accurately determine $\Lambda$.

### 4.3 Comparison between the estimated size distribution function and the observational histogram

We investigated the relationship between the estimation accuracy of the snowflake size distributions and the characteristics of the size distributions around $D_{0_2}$. The number of zero classes ($NZC$) is defined to be the number of classes for which the diameter of the snowflakes is from 1 mm to $D_{0_2}$, but in which no snowflakes were measured. Figure 3c shows an example of an observed snowflake size distribution (red histogram) and an estimated snowflake size distribution (blue line) where $RE = 2.53$ and $NZC = 5$. Large snowflakes, with $D = 11.0$ mm, were observed. This was possibly due to the aggregation of snowflakes. When $NZC \geq 2$ (green dots in Fig. 2), the $RE$s were relatively large, and the maximum $RE$ was 2.53, whereas when $NZC \leq 1$ (red dots in Fig. 2), the $RE$ was smaller, and the maximum $RE$ was 1.47. The approach proposed in the present study is thus appropriate when snowflakes are continuously counted from $D = 1.1$ mm to $D_{0_2}$. However, for discrete probability distributions, $RE$ could not attain small values, even if we control $NZC = 0$ by making wider ranges of particle size bins in the Parsivel setting.

Figure 3d shows an example of an observed snowflake size distribution (red histogram) and an estimated snowflake size distribution (blue line) where $RE = 0.07$ and $NZC = 0$. The observed histogram was well fitted to a gamma distribution between $D_{0_1}$ and $D_{0_2}$, inclusive. For the case in which quality control was applied when $\mu \neq -3$ and $NZC \leq 1$, Fig. 4 shows the relationship between $W_{0_1}$ and $W_{0_2}$ at logarithmic scale. The correlation coefficient was 0.989.

### 5. Conclusions

We estimated three parameters of snowflake size distributions ($N_{w_0}$, $\mu$, $\Lambda$) using an optical sensing disdrometer to measure three physical quantities: the diameters of the snowflakes that have 50 and 99 percentiles of volumes ($D_{0_1}$ and $D_{0_2}$, respectively) and the sum of the sixth powers of the diameters in a unit volume ($Z$). The snowflake size distribution observed by the disdrometer was fitted to a gamma distribution between $D_{0_1}$ and $D_{0_2}$, inclusive. Of the 636 samples of snowflake size distribution parameters, 2.7% underestimated the volume. Although the mean absolute error based on snowflake volume for the proposed method was large compared to that obtained using the method of Vivekanandan et al. (2004), some samples showed good estimates of snowflake volume using the proposed method. As a result of regression analysis based on observed and estimated snowflake volumes using the proposed method, the correlation coefficient was 0.989. The estimation of the snowflake volume using the proposed method depends on the optimum parameter of $\mu$ and requires a continuous probability distribution of snowflakes for diameters above 1 mm.

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