Effects of Perturbation on Formation Flights around an Eccentric Reference Orbit*

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Abstract

In this study, we propose a basic method for the construction of a formation flight around an eccentric reference orbit under the influence of gravitational disturbances (J_2). The relative motion between two spacecraft in the formation flight is affected by J_2 . The effects of this perturbing force are analyzed, and the initial conditions of the formation flight for the suppression of these effects are derived on the basis of these analytical results.

Key words : Spacecraft, Formation Flight, J_2 , Eccentric Orbit

Nomenclature

μ : gravity constant of the Earth

R : equatorial radius of the Earth

J_2 : second-order zonal harmonic coefficient of the gravitational potential of the Earth ($J_2 = 1.0826 \times 10^{-3}$)

$\mathbf{r}$: position vector of the spacecraft in the inertial system

$\mathbf{v}$: velocity vector of the spacecraft in the inertial system

a : semimajor axis of the (chief) reference orbit

e : eccentricity of the reference orbit

i : inclination of the reference orbit

ω : argument of the perigee of the ref-

erence orbit

Ω : right ascension of the reference orbit

M : mean anomaly of the chief spacecraft

E : eccentric anomaly of the chief spacecraft

ν : true anomaly of the chief spacecraft

θ : true latitude of the chief spacecraft ($\theta = \nu + \omega$)

q_1, q_2 : components of the eccentricity vector ($q_1 = e \cos \omega, q_2 = e \sin \omega$)

n : mean orbit rate

Subscripts c and d indicate the variables of chief spacecraft and deputy spacecraft, respectively.

1. Introduction

There are zonal harmonic terms in the Earth's gravity potential since the Earth's shape is not perfectly spherical. These terms are usually expressed by coefficients J_k ($k = 2, 3, \dots$), and the J_2 harmonic that originates from the oblateness (equator-pole radius difference) of the Earth is the dominant harmonic among them. It is well known that the J_2 harmonic has effects on the formation flight of multiple spacecraft⁽¹⁾. In the case of a circular reference orbit, initial conditions that are expressed in terms of the Hill coordinates have already been derived for canceling the effects of J_2 ⁽²⁾. Even in the case of an elliptic reference orbit, the initial conditions for canceling the effects of J_2 have been derived⁽³⁾; however, the initial velocities are not expressed in terms of the time derivatives of the Hill coordinates, and thus, they are not necessarily suitable as the initial conditions for the deputy spacecraft. In this study, the initial velocity of the deputy spacecraft for canceling the effects of J_2 are derived in terms of

the time derivative of the Hill coordinates in the case of an elliptic reference orbit.

2. Transition matrix in Hill coordinates

2.1. Hill coordinates

A spacecraft that moves on a reference orbit is called the chief spacecraft, and a spacecraft in formation flight with respect to the chief spacecraft is called the deputy spacecraft. The local orthogonal coordinate system with the origin at the center of mass of the chief spacecraft is selected so that the x -axis is aligned with an imaginary line drawn from the Earth's center of mass to the chief spacecraft and the z -axis aligns with the normal (rotation vector) to the orbital plane of the chief spacecraft. These local coordinates are called the Hill coordinates. Set the position vector of the chief spacecraft and its second derivative in terms of the inertial coordinates as $\mathbf{r}_c$ and $\ddot{\mathbf{r}}_c$, respectively. The equations of motion of the chief spacecraft are as follows:

$$\ddot{\mathbf{r}}_c = -\frac{\mu_e}{r_c^3} \mathbf{r}_c + \mathbf{f}_{J_2}, \quad (1)$$

where $\mathbf{f}_{J_2}$ is the force exerted by the J_2 term on the chief spacecraft. Let the components of $\mathbf{f}_{J_2}$ be expressed in terms of the Hill coordinates as $\begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^T$. These components are expressed as follows by using the orbit elements of the chief spacecraft⁽⁴⁾:

$$f_x = \frac{3\mu J_2 R^2}{2r_c^4} (3 \sin^2 i_c \sin^2 \theta_c - 1) \quad (2)$$

$$f_y = -\frac{3\mu J_2 R^2}{2r_c^4} \sin^2 i_c \sin 2\theta_c \quad (3)$$

$$f_z = -\frac{3\mu J_2 R^2}{2r_c^4} \sin 2i_c \sin \theta_c \quad (4)$$

2.2. Transformation of state vector

Set the position vector $\mathbf{r}$ expressed in the Hill coordinates as

$$\mathbf{r} = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T \quad (5)$$

and the state vector $\mathbf{p}$ of the spacecraft as

$$\mathbf{p} = \begin{bmatrix} r_x & r_y & r_z & \dot{r}_x & \dot{r}_y & \dot{r}_z \end{bmatrix}^T \quad (6)$$

The state vector of the deputy spacecraft with respect to the chief spacecraft, $\delta\mathbf{p}$, is given as follows:

$$\delta\mathbf{p} = \mathbf{p}_d - \mathbf{p}_c \quad (7)$$

Set the orbit elements of the spacecraft as

$$\mathbf{q} = \begin{bmatrix} a & \theta & i & q_1 & q_2 & \Omega \end{bmatrix}^T \quad (8)$$

$$\mathbf{s} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T \quad (9)$$

and the differences between the orbit elements of the deputy spacecraft and the chief spacecraft are denoted as $\delta\mathbf{q}$ and $\delta\mathbf{s}$:

$$\delta\mathbf{q} = \mathbf{q}_d - \mathbf{q}_c, \quad \delta\mathbf{s} = \mathbf{s}_d - \mathbf{s}_c \quad (10)$$

We first obtain the time evolution of $\delta\mathbf{s}$ and then obtain the relation between $\delta\mathbf{p}(t)$ and $\delta\mathbf{p}(t_0)$ (t_0 : initial time) as a state transition matrix. Hereafter, we derive the relations required for this calculation.

2.3. Transformation between $\delta p \leftrightarrow \delta q$

The transformation between δp and δq is expressed as follows where the partial derivatives are evaluated at the chief spacecraft state:

$$\delta p = A_{Hq} \delta q, \quad A_{Hq} = \left. \frac{\partial p}{\partial q} \right|_c \quad (11)$$

$$\delta q = A_{qH} \delta p, \quad A_{qH} = A_{Hq}^{-1} = \left. \frac{\partial q}{\partial p} \right|_c \quad (12)$$

The matrices A_{Hq} and A_{qH} that are under the influence of J_2 have already been derived in the literature⁽⁵⁾.

2.4. Transformation between $\delta q \leftrightarrow \delta s$

As a , i , and Ω are the common components between the orbit elements q and s , the transformation between δq and δs is not a complex one. Let us set the following:

$$\delta q = A_{qs} \delta s, \quad A_{qs} = \left. \frac{\partial q}{\partial s} \right|_c \quad (13)$$

$$\delta s = A_{sq} \delta q, \quad A_{sq} = A_{qs}^{-1} = \left. \frac{\partial s}{\partial q} \right|_c \quad (14)$$

Since the relation between M and E is as follows:

$$M = E - e \sin E, \quad (15)$$

$\delta M = M_d - M_c$ around the chief spacecraft is obtained as⁽¹⁾:

$$\delta M = \frac{\sqrt{1-e_c^2}}{(1+e_c \cos \nu)^2} \left[(1-e_c^2) \delta \nu_c - \sin \nu_c (2+e_c \cos \nu_c) \delta e \right] \quad (16)$$

Thus, by using $\delta \nu = \delta \theta - \delta \omega$, we obtain

$$\delta \theta = \frac{(1+e_c \cos \nu_c)^2}{(1-e_c^2)^{\frac{3}{2}}} \delta M + \delta \omega + \frac{\sin \nu_c (2+e_c \cos \nu_c)}{1-e_c^2} \delta e_c \quad (17)$$

Using this relation, the non-zero components of A_{qs} are obtained as follows, where the (i, j) -component A_{qs} is denoted as $A_{qsi j}$:

$$A_{qs11} = A_{qs24} = A_{qs33} = A_{qs64} = 1$$

$$A_{qs22} = \frac{\sin \nu_c (2+e_c \cos \nu_c)}{1-e_c^2}$$

$$A_{qs25} = \frac{(1+e_c \cos \nu_c)^2}{(1-e_c^2)^{\frac{3}{2}}}$$

$$A_{qs42} = \cos \omega_c, \quad A_{qs45} = -e_c \sin \omega_c$$

$$A_{qs52} = \sin \omega_c, \quad A_{qs55} = -e_c \sin \omega_c$$

2.5. Time evolution of δs

Finally, we obtain the time evolution of δs as a state transition matrix. We can calculate the state transition matrix in terms of the Hill coordinates by multiplying the above coefficient matrices associated with the variable transformations. By substituting f_x , f_y , and f_z given by Eqs. (2)–(4) into the Gauss planetary equations and averaging them in one orbit, the variation in the mean orbit elements due to J_2 are obtained as follows⁽¹⁾, where the mean orbit elements corresponding to the osculating orbit elements a_c , e_c , i_c , ω_c , and M_c are expressed as $\bar{a}_c$, $\bar{e}_c$, $\bar{i}_c$, $\bar{\omega}_c$, and $\bar{M}_c$, respectively:

$$\frac{d\bar{a}_c}{dt} = 0 \quad (18)$$

$$\frac{d\bar{e}_c}{dt} = 0 \quad (19)$$

$$\frac{d\bar{i}_c}{dt} = 0 \quad (20)$$

$$\frac{d\bar{\Omega}_c}{dt} = -\frac{3}{2}J_2n_c\left(\frac{R}{p_c}\right)^2 \cos i_c \quad (21)$$

$$\frac{d\bar{\omega}_c}{dt} = \frac{3}{4}J_2n_c\left(\frac{R}{p_c}\right)^2 (5 \cos^2 i_c - 1) \quad (22)$$

$$\frac{d\bar{M}_c}{dt} = \bar{n}_c + \frac{3}{4}J_2n_c\left(\frac{R}{p_c}\right)^2 \eta_c (3 \cos^2 i_c - 1) \quad (23)$$

We can obtain the variations in the mean orbit elements using the above equation and approximate the variations in the osculating orbit elements. In Eqs. (21) and (22), the osculating orbit elements are used in their right hand sides. Because the differences between the mean orbit elements and the osculating orbit elements depend on J_2 and the second-order terms of J_2 can be neglected, both the orbit elements can be used in the right hand sides of Eqs. (21) and (22). However, $\bar{n}_c$, which is the mean angular motion based on $\bar{a}_c$, should be used in the right hand side of Eq. (23), because this term is not multiplied by J_2 . In this case, by expressing $\bar{a}_c$ as

$$\bar{a}_c = a_c (1 + \Delta_a), \quad (24)$$

$\bar{n}_c$ can be expressed as follows:

$$\bar{n}_c = \sqrt{\frac{\mu}{\bar{a}_c^3}} = \sqrt{\frac{\mu}{a_c^3}} \left(1 - \frac{3}{2}\Delta_a\right) \quad (25)$$

where Δ_a is calculated as⁽¹⁾

$$\Delta_a = -\frac{J_2R^2}{2a_c^2(1-e_c^2)^3} \left[(3 \cos^2 i_c - 1) \left((1 + e_c \cos \nu_{c0})^3 - (1 - e_c^2)^{\frac{3}{2}} \right) + 3 \sin^2 i_c (1 + e_c \cos \nu_{c0})^3 \cos(2\omega_c + 2\nu_{c0}) \right]$$

Let us summarize Eqs. (18)–(23) as follows:

$$\frac{d\bar{s}}{dt} = g(s) \quad (26)$$

In order to obtain the time evolution of δs , we obtain the following approximation on the basis of the above equation.

$$\frac{d\delta s}{dt} = A_s \delta s, \quad A_s = \left. \frac{\partial g}{\partial s} \right|_c \quad (27)$$

By solving this equation, we obtain $\delta s(t)$ as follows:

$$\delta s(t) = \Phi_s(t, t_0) \delta s(t_0), \quad \Phi_s(t, t_0) = \exp(A_s(t - t_0)) \quad (28)$$

In general, it is not easy to obtain Φ_s ; however, Φ_s can be easily obtained in this case because the upper triangle part of A_s are zeros except for the (6,6) component. Let the (6,6) component of A_s be a_{s66} , and it is given as follows:

$$a_{s66} = -\frac{9J_2R^2n_c \sqrt{1+e_c} \sin \omega_c \cos \omega_c \sin^2 i_c}{a_c^2(1-e_c)^{\frac{9}{2}}} \quad (29)$$

The time exponential function in $\Phi_s(t, t_0)$ is $\exp(a_{s66}(t - t_0))$ only. If we consider the approximation

$$\exp(a_{s66}(t - t_0)) \approx 1 + a_{s66}(t - t_0),$$

the nonzero components of $\Phi_s(t, t_0)$ can be expressed as follows, where Φ_{sij} and α represent the (i, j) components of Φ_s and $J_2R^2n_c(t - t_0)/a_c^2$, respectively:

$$\Phi_{s11} = \Phi_{s22} = \Phi_{s33} = \Phi_{s44} = \Phi_{s55} = 1$$

$$\begin{aligned}
 \Phi_{s41} &= \frac{21\alpha \cos i_c}{4a_c(1 - e_c^2)^2} \\
 \Phi_{s42} &= -\frac{6\alpha e_c \cos i_c}{(1 - e_c^2)^3} \\
 \Phi_{s43} &= \frac{3\alpha \sin i_c}{2(1 - e_c^2)^2} \\
 \Phi_{s51} &= -\frac{21\alpha(5 \cos^2 i_c - 1)}{8a_c(1 - e_c^2)^2} \\
 \Phi_{s52} &= \frac{3\alpha e_c(5 \cos^2 i_c - 1)}{(1 - e_c^2)^3} \\
 \Phi_{s53} &= -\frac{15\alpha \sin i_c \cos i_c}{2(1 - e_c^2)^2} \\
 \Phi_{s61} &= -\frac{3n_c(t - t_0)}{2a_c} + \frac{21\alpha(3 \sin^2 \omega_c \sin^2 i_c - 1)}{4a_c(1 - e_c^2)^3} \\
 \Phi_{s62} &= -\frac{9\alpha(3 \sin^2 \omega_c \sin^2 i_c - 1)}{2(1 - e_c^2)^4} \\
 \Phi_{s63} &= -\frac{9\alpha \sin^2 \omega_c \sin i_c \cos i_c}{(1 - e_c^2)^3} \\
 \Phi_{s65} &= -\frac{9\alpha \sin \omega_c \cos \omega_c \sin^2 i_c}{(1 - e_c^2)^3} \\
 \Phi_{s66} &= 1 - \frac{9\alpha \sqrt{1 + e_c} \sin \omega_c \cos \omega_c \sin^2 i_c}{(1 - e_c^2)^{\frac{9}{2}}}
 \end{aligned}$$

2.6. Time evolution of δp

Using the abovementioned procedure, the state transition matrix between $\delta p(t)$ and $\delta p(t_0)$ can be obtained. Let the state transition matrix be

$$\delta p(t) = \Phi_p(t, t_0) \delta p(t_0), \quad (30)$$

where $\Phi_p(t, t_0)$ is expressed as follows:

$$\Phi_p(t, t_0) = A_{Hq}(t) A_{qs}(t) \Phi_s(t, t_0) A_{sq}(t_0) A_{qH}(t_0) \quad (31)$$

It should be noted that the components of A_{Hq} , A_{qs} , Φ_s , A_{sq} , and A_{qH} are the functions of the orbit elements of the chief spacecraft: a_c , e_c , i_c , ω_c , and v_c , respectively. Therefore, the orbit elements at time t_0 are used to obtain $\Phi_s(t, t_0)$, $A_{sq}(t_0)$ and $A_{qH}(t_0)$ and those at time t are used to obtain $A_{Hq}(t)$ and $A_{qs}(t)$. However, as the time variations in the mean orbit elements a_c , e_c , and i_c can be neglected under the influence of J_2 , the values of these elements can be considered to be the same at t_0 and t . The true anomaly v_c is used to designate time as well as t . The mean orbit element of the argument of the perigee ω_c varies as shown in Eq. (22) and it is approximated as

$$\omega_c(t) = \omega_c(t_0) + \beta, \quad \beta = \frac{3\alpha(5 \cos^2 i_c - 1)}{4(1 - e_c^2)^2} \quad (32)$$

Then, $\sin(\omega_c(t))$ and $\cos(\omega_c(t))$ are approximated as follows:

$$\sin(\omega_c(t)) \approx \sin(\omega_c(t_0)) + \cos(\omega_c(t_0))\beta \quad (33)$$

$$\cos(\omega_c(t)) \approx \cos(\omega_c(t_0)) - \sin(\omega_c(t_0))\beta \quad (34)$$

We set the time at the perigee as the initial time t_0 and the true anomaly of the chief spacecraft at time t as v_c . We also set each component of δp at time t in terms of the Hill coordinates as

$$\delta p = \begin{bmatrix} p_x & p_y & p_z & v_x & v_y & v_z \end{bmatrix}^T \quad (35)$$

where $v_x = \dot{p}_x$, $v_y = \dot{p}_y$, and $v_z = \dot{p}_z$. By using the subscript 0 to represent the variables at time t_0 , p_x , p_y , and p_z are calculated by using $\Phi_p(t, t_0)$ from p_{x0} , p_{y0} and p_{z0} , respectively. When the effects of J_2 are ignored, the following equations should be satisfied in order to cancel the time-dependent terms of p_x and p_y :

$$v_{y0} = \frac{(2 + e_c)n_c}{\sqrt{(1 + e_c)(1 - e_c)^3}} p_{x0} \quad (36)$$

This equation is known as the formation condition for an elliptic reference orbit⁽¹⁾. Let us express v_{y0} as follows by using this equation:

$$v_{y0} = \frac{(2 + e_c)n_c}{\sqrt{(1 + e_c)(1 - e_c)^3}} p_{x0} + \Delta v_{y0} \quad (37)$$

Then, p_x and p_y are expressed as follows:

$$p_x = \cos v_c p_{x0} + \frac{(1 - e_c)^{\frac{3}{2}} \sin v_c}{n_c \sqrt{1 + e_c}} v_{x0} + \left[\frac{2\sqrt{1 - e_c^2}(1 - \cos v_c)(1 + e_c + e_c \cos v_c)}{n_c(1 + e_c \cos v_c)} - \frac{3e_c \sin v_c(t - t_0)}{1 - e_c} \right] \Delta v_{y0} \quad (38)$$

$$p_y = \frac{(2 + e_c \cos v_c) \sin v_c}{1 + e_c \cos v_c} p_{x0} + \frac{1 + e_c}{1 + e_c \cos v_c} p_{y0} - \frac{(1 - e_c)^{\frac{3}{2}}(2 + e_c + e_c \cos v_c)(1 - \cos v_c)}{n_c \sqrt{1 + e_c}(1 + e_c \cos v_c)} v_{x0} + \left[\frac{2\sqrt{1 - e_c^2}(2 + e_c \cos v_c) \sin v_c}{n_c(1 + e_c \cos v_c)} - \frac{3(1 + e_c \cos v_c)(t - t_0)}{1 - e_c} \right] \Delta v_{y0} \quad (39)$$

$$p_z = \frac{(1 + e_c) \cos v_c}{1 + e_c \cos v_c} p_{z0} + \frac{\sqrt{1 + e_c}(1 - e_c)^{\frac{3}{2}} \sin v_c}{n_c(1 + e_c \cos v_c)} v_{z0} \quad (40)$$

As described later, there are time-dependent terms in p_x , p_y , and p_z due to the influence of J_2 . Some of these time-dependent terms can be cancelled by using the time-dependent terms obtained using Δv_{y0} . This procedure is described in the next section.

3. Initial conditions considering the effects of J_2

3.1. Formation condition

When the chief spacecraft and deputy spacecraft are in formation flight, there is a gradual change in the formation flight due to the effects of J_2 . In this study, we consider the initial conditions to minimize the changes in formation due to the effects of J_2 . Hereafter, by considering $\delta \mathbf{p}$ as a function of v_c , we set $\delta \mathbf{p}$ at the perigee as follows:

$$\delta \mathbf{p}(v_c = 0) = \begin{bmatrix} p_{x0} & p_{y0} & p_{z0} & v_{x0} & v_{y0} & v_{z0} \end{bmatrix}^T \quad (41)$$

For this initial condition, we set $\delta \mathbf{p}$ at $v_c = v_{cf}$ as follows:

$$\delta \mathbf{p}(v_c = v_{cf}) = \begin{bmatrix} p_{xf} & p_{yf} & p_{zf} & v_{xf} & v_{yf} & v_{zf} \end{bmatrix}^T \quad (42)$$

In the following two cases, the effects of J_2 on the formation flight are analyzed:

- Case 1 : $p_{yf} = p_{y0}$ at $v_{cf} = 2N\pi$ (N : integer)
- Case 2 : $\frac{d}{dt}(\delta M + \delta \omega) = 0$

In Case 1, the deputy spacecraft returns to the initial position in the Hill coordinates after orbiting N times, where N is assumed to be in the range of $\beta \ll 1$. In Case 2, the deviations in the orbit elements $M + \omega$ of the deputy spacecraft from those of the chief spacecraft do not change with time.

3.2. Case 1

Set the time at $\nu_c = 2N\pi$ as t_f . When v_{y0} is set as shown in Eq. (37), p_{xf} , p_{yf} , and p_{zf} are obtained in the following form with the inclusion of the effects of J_2 :

$$p_{xf} = p_{x0} \quad (43)$$

$$p_{yf} = p_{y0} + p_{yJ2} - \frac{3(1+e_c)(t_f - t_0)}{1 - e_c} \Delta v_{y0} \quad (44)$$

$$p_{zf} = p_{z0} + p_{zJ2} \quad (45)$$

where p_{yJ2} and p_{zJ2} are the terms that are introduced due to the effects of J_2 . As shown in Eq. (43), the effects of J_2 do not appear in p_{xf} in the first order. Therefore, Δv_{y0} is determined from the relation $p_{yf} = p_{y0}$:

$$\Delta v_{y0} = \frac{1 - e_c}{3(1 + e_c)(t_f - t_0)} p_{yJ2} \quad (46)$$

In this case, p_{zJ2} is obtained as follows:

$$p_{zJ2} = -\frac{3J_2 R^2 (t_f - t_0) \cos \omega_c \sin i_c}{a_c^2 (1 + e_c)^3 (1 - e_c)^2} \left(2n_c e_c \cos i_c p_{x0} + n_c (1 + e_c) \sin \omega_c \cos i_c p_{z0} \right. \\ \left. + \frac{1}{4} \sqrt{1 + e_c} (1 - e_c)^{\frac{3}{2}} (1 + 2 \cos^2 \omega_c \sin^2 i_c - 5 \cos^2 i_c) v_{z0} \right) \quad (47)$$

Although p_{zJ2} cannot be cancelled by Δv_{y0} , the effects of p_{zJ2} is relatively small as compared with those of p_{yJ2} . This is because the initial condition of p_{y0} (in the direction of motion) does not affect p_{zJ2} and the effects of the initial condition of v_{z0} (in the out-of-plane direction) are cancelled by a small phase difference in the vicinity of $\nu_c = 2N\pi$, as shown in Eq. (40). Δv_{y0} is obtained using Eq. (46) as follows, where the orbit elements of the chief spacecraft are the initial values:

$$v_{y0} = \left[-\frac{n_c(2+e_c)}{\sqrt{(1+e_c)(1-e_c)^3}} + \frac{3J_2 R^2 n_c (3 \sin^2 \omega_c \sin^2 i_c - 1)}{2a_c^2 \sqrt{1+e_c}(1-e_c)^{\frac{7}{2}}} - \frac{J_2 R^2 n_c e_c (3 \cos^2 i_c - 1)}{a_c^2 (1+e_c)^4 (1-e_c)} \right] p_{x0} \\ - \frac{3J_2 R^2 n_c \sin 2\omega_c \sin^2 i_c}{2a_c^2 \sqrt{1+e_c}(1-e_c)^{\frac{7}{2}}} p_{y0} - \left[\frac{3J_2 R^2 n_c \sin \omega_c \sin 2i_c}{a_c^2 \sqrt{1+e_c}(1-e_c)^{\frac{7}{2}}} + \frac{J_2 R^2 n_c \sin \omega_c \sin 2i_c}{a_c^2 (1+e_c)^3 (1-e_c)} \right] p_{z0} \\ - \frac{J_2 R^2 \sqrt{1-e_c} \cos \omega_c \sin 2i_c}{a_c^2 (1+e_c)^{\frac{7}{2}}} v_{z0} \quad (48)$$

3.3. Case 2

Since the condition of Case 2 is obtained from the deviation of the orbit elements, it is evaluated as follows from the time evolution of δs in Eq. (27):

$$\frac{d\delta s}{dt} = \mathbf{A}_s(t_0) \mathbf{A}_{sq}(t_0) \mathbf{A}_{qH}(t_0) \delta \mathbf{p}(t_0) \quad (49)$$

When v_{y0} is given in Eq. (37) in the initial condition of $\delta \mathbf{p}(t_0)$, the term

$$\frac{d}{dt} (\delta \omega + \delta M)$$

can be evaluated by adding the fifth row and the sixth row of Eq. (49). When the value of the above term is 0, the effects of J_2 are decreased in the orbital plane. v_{y0} is derived from this condition and the result is obtained as follows:

$$v_{y0} = \left[-\frac{n_c(2+e_c)}{\sqrt{(1+e_c)(1-e_c)^3}} + \frac{3J_2 R^2 n_c (3 \sin^2 \omega_c \sin^2 i_c - 1)}{2a_c^2 \sqrt{1+e_c}(1-e_c)^{\frac{7}{2}}} - \frac{J_2 R^2 n_c e_c (5 \cos^2 i_c - 1)}{a_c^2 (1+e_c)^{\frac{5}{2}} (1-e_c)^{\frac{5}{2}}} \right] p_{x0} \\ - \frac{3J_2 R^2 n_c \sin 2\omega_c \sin^2 i_c}{2a_c^2 \sqrt{1+e_c}(1-e_c)^{\frac{7}{2}}} p_{y0} - \left[\frac{3J_2 R^2 n_c \sin \omega_c \sin 2i_c}{a_c^2 \sqrt{1+e_c}(1-e_c)^{\frac{7}{2}}} + \frac{5J_2 R^2 n_c \sin \omega_c \sin 2i_c}{4a_c^2 (1+e_c)^{\frac{5}{2}} (1-e_c)^{\frac{5}{2}}} \right] p_{z0} \\ - \frac{5J_2 R^2 \cos \omega_c \sin 2i_c}{4a_c^2 (1+e_c)^3 (1-e_c)} v_{z0} \quad (50)$$

This result is slightly different from Eq. (48).

4. Numerical simulation results

4.1. Initial conditions of an elliptic orbit

By focusing on the case where the chief spacecraft moves on an elliptic orbit, we set the initial conditions of the deputy spacecraft in terms of the Hill coordinates as follows:

$$\begin{aligned} p_{x0} &= 0, & p_{y0} &= l, & p_{z0} &= 0 \\ v_{x0} &= \frac{n_c \sqrt{1+e_c} l}{2(1-e_c)^{3/2}}, & v_{y0} &= 0, & v_{z0} &= \frac{n_c l}{1-e_c} \end{aligned}$$

These initial conditions are set so that a nearly circular formation is realized in the case of Keplerian motion without any disturbances⁽⁶⁾. Simulation results are shown in Figs. 1 and 2 when the orbit elements of the chief spacecraft at the initial time are set as

$$\begin{aligned} e_c &= 0.5, & l &= 1[\text{km}], & \Omega_c &= 0 \\ \omega_c &= 60^\circ, & i_c &= 30^\circ, & a_c &= \frac{R + 500[\text{km}]}{1-e_c} \end{aligned}$$

Figure 1 shows the case without the effects of J_2 , while Fig. 2 shows the case with the effects of J_2 . As shown in these figures, the formation trajectory of the deputy spacecraft is gradually disturbed by the effects of J_2 .

Fig. 1 Nominal deputy motion expressed in the Hill Coordinates (without the effects of J_2)

Fig. 2 Nominal deputy motion expressed in the Hill coordinates (with the effects of J_2)

4.2. Correction of initial conditions

Next, when the effects of J_2 are present, the simulation results with the correction of the initial conditions are shown. Figure 3 shows the simulation results with the correction of Eq. (48) for Case 1, and Fig. 4 shows the simulation results with the correction of Eq. (50) for Case 2. As is clear from these results, the drift in the y component can be suppressed by the correction of the initial conditions. In Case 1, the deputy spacecraft almost returns to the initial position in the Hill coordinates at the perigee. In Case 2, although there is a very small difference between the position of the deputy spacecraft at the perigee and its initial position in the Hill coordinates, the whole variation of the relative position is also suppressed.

Fig. 3 Deputy motion expressed in the Hill coordinates (with v_{y0} correction in Case 1)

Fig. 4 Deputy motion expressed in the Hill coordinates (with v_{y0} correction in Case 2)

5. Conclusions

In this study, we have analyzed the effects of J_2 on the formation flight in an elliptic reference orbit and have derived the initial velocity in the direction of motion of the deputy spacecraft for canceling the effects of J_2 . We have used two methods: one is to derive the state transition matrix in terms of the Hill coordinates and to obtain the initial velocity on basis of the transition matrix in order to cancel the changes in the position after orbiting N times. The other one is to derive the initial velocity which makes the time change of the difference of the orbit elements between two spacecraft zero. The effectiveness of the initial velocities in canceling the effects of J_2 is verified by numerical simulations.

The method of this paper is a kind of feedforward adjustment of the relative velocity of the deputy spacecraft in the along-track direction for canceling the effects of J_2 . The J_2 effects are actually observed in the variation of the relative distance between two spacecraft at every orbit period. Therefore, a kind of feedback compensation of the J_2 effects is possible when the relative velocity of the deputy spacecraft is set in the same way as above at every orbit period based on the observation of the relative distance.

References

- (1) Schaub, S. and Junkins, J.L., Analytical Mechanics of Space Systems, AIAA Educational Series, pp. 489–543, 593–628, 693–696, 2003
- (2) Shima, T., Yamada, K., Jikuya, I. and Yoshikawa, S., Trajectory Planning of a Spacecraft based on the Modified Hill's Equation, *Transactions of the Japan Society of Mechanical Engineers*, Vol. 72, No. 722, pp. 3364–3371, 2006 (in Japanese)
- (3) Sengupta, P., Sharma, R. and Vadali, S.R., Periodic Relative Motion Near a Keplerian Elliptic Orbit with Nonlinear Differential Gravity, *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 5, pp. 1110–1121, 2006
- (4) Wie, B., Space Vehicle Dynamics and Control, AIAA Educational Series, pp. 233–240, 1998
- (5) Gim, D. and Alfriend, K.T., State Transition Matrix of Relative Motion for the Perturbed Noncircular Reference Orbit, *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 6, pp. 956–971, 2003
- (6) Yamada, K., Shima, T. and Yoshikawa, S., Spacecraft Formation Flying in Eccentric Orbits, *Proceedings of 16th Workshop on JAXA Astrodynamics and Flight Mechanics*, pp. 22–27, 2006