Profit Determination for a Guided Tour Provider Operating in a Stochastic Environment†

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Abstract
In this note, we study a guided tour providing firm that operates in a stochastic environment. The environment is stochastic because this firm’s costs are deterministic but its revenues are stochastic. Since the revenues are stochastic, so the profits of this firm are also stochastic. For such a firm, we show how to compute the expected profit function for two cases. In the first case, the revenues accrued by the firm over time are continuous random variables and in the second case these same revenues are discrete random variables.

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1. Introduction
Tourists visiting attractions such as fiords, game parks, museums, and cities routinely take guided tours of varying levels of both length and quality. The provider of these guided tours is typically a tour operator and, in this note, we shall think of such an operator as a firm. The activities of such firms have recently been analyzed in both case and theoretical studies in the tourism literature. Focusing on South Africa, Strydom and Nel (2006) note that Bloemfontein is frequently excluded from guided tours and other touristic activities because tour operators are largely unfamiliar with the tourist offerings of this city. Huang and Wang (2007) contend that if guided tours in Great Britain are to be quality experiences for tourists from China then the tour guides will need to acquire intercultural competence. Baez Montenegro et al. (2009) use the notion of a hypothetical guided walking tour to value historical sites in Valdivia, Chile.

Three recent theoretical papers have analyzed the provision of guided tours to tourists during the off-peak or slack season. Batabyal and Yoo (2010) have determined the long run fraction of demand that is lost to a firm providing guided tours because of a capacity constraint. Batabyal and Beladi (2011) have studied the case in which the wait plus tour or excursion times of tourists is exponentially distributed. Finally, Batabyal (2011) has used the so called “scheduling by numbers” and “scheduling by time” approaches to optimize the provision of guided tours to tourists.

The studies discussed in the preceding two paragraphs have certainly advanced our understanding of the provision of guided tours by specialized firms. This notwithstanding, to the best of our knowledge, there are no theoretical studies that examine the way in which a guided tour providing firm’s activities impact this firm’s profit function. Given this lacuna in the extant literature, the purpose of this note is to determine the profit for a guided tour providing firm that operates in a stochastic environment. Section 2.1 below explains why the

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operating environment for this firm is stochastic. Given this stochastic environment, the profit for the guided tour providing firm is a random variable. Therefore, section 2.2 computes the expected or mean profit function when the revenues accruing to this firm are continuous random variables. Section 2.3 undertakes a similar exercise for the case in which the revenues obtained by the firm are discrete random variables. Section 3 concludes and then discusses two ways in which the research described in this note might be extended.

2. The Theoretical Framework

2.1. Preliminaries

Consider a firm that is in the business of providing guided tours to arriving tourists. Like any firm, the objective of this firm is to maximize the profit from its business operations and profit, denoted by \( \Pi \). This is the difference between total revenues (\( R \)) and total costs (\( C \)). In symbols, we have \( \Pi = R - C \). We suppose that this firm either knows or that it has a very good idea about the cost of providing guided tours to arriving tourists. Formally, this means that at any time \( t \) the deterministic cost function \( C(t) \) describes the costs confronting our firm at this time \( t \). Since costs are deterministic, the expected or mean cost at time \( t \) is \( E[C(t)] = C(t) \), where \( E[\cdot] \) is the expectation operator. In contrast, the revenues from the provision of guided tours depend on the demand for such tours from tourists and this demand is much harder for our firm to either know or to estimate with any degree of precision. We model this revenue uncertainty by supposing that the revenues accruing to our firm over time are random variables.

Now, suppose that tourists seeking guided tours arrive at the office of our firm in accordance with a stationary Poisson process with parameter or rate \( \xi > 0 \). Each time a tourist arrives at the office, this tourist observes the prices for the various guided tours that are on offer. (S)he then pays for one or more tours and, as a result, a certain amount of revenue accrues to our firm. If this tourist leaves without paying for a guided tour then the revenue amount accruing to our firm is clearly zero. On the other hand, depending on the type of guided tour that is paid for (one-hour tour, half-day tour, full-day tour, etc.), the revenue amount will typically be positive but random.

Let \( r_i \) denote the non-negative revenue amount contributed by the \( i \)th arriving tourist. A tourist arriving at time \( s \) will, independent of the past, contribute a random amount of revenue with cumulative distribution function \( G_s(\cdot) \) where \( s \geq 0 \). In other words, \( G_s(\cdot) \) is the cumulative distribution function of the various revenue amounts, that is, the \( r_i/s \). The total number of tourists who arrive at our firm’s office by time \( t \) is denoted by \( M(t) \) and \( M(t) \) is itself a Poisson distributed random variable with parameter \( \lambda > 0 \). Finally, let \( R(t) \) denote the sum of all the revenue amounts contributed by the various arriving tourists to our firm by time \( t \). In symbols, we have \( R(t) = \sum_{i=1}^{M(t)} r_i \). The twin tasks before us now are to determine what kind of random variable \( R(t) \) is and then to provide closed-form expressions for our guided tour providing firm’s expected revenue and profit functions. We now proceed to these tasks in the case where the \( r_i \) revenue amounts are continuous random variables.

2.2. Continuous revenue amounts

Our first objective is to determine the distribution function of \( R(t) \). To do so, we proceed intuitively and in two steps. First, let us condition on \( M(t) \) or the total number of tourist arrivals at our firm’s office by time \( t \). Second, we use theorem 2.3.1 in Ross (1996, p. 67). This tells us that given the total number of tourist arrivals \( M(t) \), the unordered set of \( M(t) \) tourist arrival times are independent and uniformly distributed random variables on the interval \((0, t)\). Putting the results of these two steps together, we deduce that \( R(t) \) is a compound Poisson random variable with parameter \( \lambda = \xi t > 0 \).

\(^1\) Note that the Poisson process is a particular kind of renewal process in which the so called “inter-arrival times” are exponentially distributed. See chapters 2 and 3 in Ross (1996) for a textbook treatment of Poisson and renewal processes.

\(^2\) We are modeling the arrival of tourists with a stationary Poisson process. This means that we are ruling out the possibility that there may be more tourist arrivals during certain times and less such arrivals at other times.
Next, from Ross (1996, p. 83) it follows that the expected value of $R(t)$ or the mean amount of revenue accruing to our guided tour providing firm by time $t$ is

$$E[R(t)] = \zeta t E[r],$$

where $E[r] = E[r_i]$. Given this expression for the expected revenue, we infer that our firm’s expected profit by time $t$ is simply the difference between its expected revenues and expected costs. This tells us that

$$E[R(t)] - E[C(t)] = \zeta t E[r] - C(t).$$

(2)

This expression conforms well with our intuition about profit. Inspecting the right-hand-side (RHS) of equation (2) we see that our guided tour providing firm’s profit increases in the tourist arrival rate $\xi > 0$ and the expected individual revenue contribution $E[r_i]$ and decreases in the costs $C(t)$. Given that $R(t)$ is a compound Poisson random variable, our final task in this note is to determine a closed-form expression for the expected profit function for our firm when the $r_i$ revenue amounts are discrete random variables.

### 2.3. Discrete revenue amounts

When the individual tourist revenue amounts $r_i$ are discrete, following Ross (1996, p. 83), it is possible to represent the total revenue accruing to our firm by time $t$ or $R(t)$ as a linear combination of independent Poisson random variables. Specifically, we have

$$\text{Prob} \{ r = j \} = p_j, \quad j = 1, 2, \ldots, k, \quad \sum_{j=1}^{k} p_j = 1. \quad (3)$$

Now, if we let $M_j$ denote the number of the $r_i$s that are equal to $j$, $j = 1, 2, \ldots, k$, then we can express $R(t)$ as a weighted sum of the various $M_j$s. This gives us

$$R(t) = \sum_{j=1}^{k} jM_j. \quad (4)$$

From Ross (1996, pp. 32-33) it follows that the various $M_j$ are independent Poisson random variables with mean $\chi p_j$, $j = 1, 2, \cdots, k$. Now, the expected revenue accruing to our guided tour providing firm is

$$E[R(t)] = \sum_{j=1}^{k} jE[M_j] = \sum_{j=1}^{k} j \chi p_j = \chi E[r]. \quad (5)$$

where the expression for the expected revenue on the RHS of equation (5) matches the corresponding expression for the “continuous revenue amounts” case in equation (1) because $\chi = \zeta$. Finally, the expression for our firm’s expected profit from the provision of guided tours is, once again, given by equation (2) where $E[R(t)]$ is now given by equation (5).

### 3. Conclusions

In this note, we analyzed a guided tour providing firm that operated in a stochastic environment. The environment was stochastic because this firm’s costs were deterministic but its revenues were stochastic. For such a firm, we showed how to compute the expected profit function when the revenues accruing to this firm over time were, respectively, continuous and discrete random variables.

Here are two suggestions for extending the research described here. First, one could generalize the analysis by studying the case in which the costs incurred by the firm are also stochastic. In this case, the more interesting case would appear to be the one in which the fixed costs are deterministic but the variable costs are probabilistic. Second, following Beladi and Oladi (2006), it would be useful to study the firm specific impacts of advertising and the offer of volume discounts to tourists demonstrating high demand for guided tours. Studies of profit determination that incorporate these aspects of the problem into the analysis will provide further insights into the functioning of guided tour providing firms that are an important part of the tourism industry.

### References


