Economic Analysis of Electric Power Industry : Competition versus Monopoly*

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Abstract

In this paper, we characterize and compare the competitive and monopolistic equilibrium paths in an intertemporal economy where the *electric power energy* can be supplied by using an exhaustible resource such as petroleum or coal, a renewable resource such as “water”, or a backstop technology such as nuclear fusion. It is shown that the size of the constant *rainfall* regenerated by nature significantly effects on these equilibrium paths and that the backstop technology is not always used forever. We also find that in all cases but for an exceptional case, monopolist is excessively conservative with respect to the renewable resource as well as the exhaustible, and that during the “final” interval in the exceptional case, the energy prices in competition and monopoly are both *equal* to the “demand” price at the constant rainfall.

1. Introduction

The economic theories of exhaustible resources and of renewable resources have been “separately” developed so far [see, Clark(1976) and Dasgupta and Heal(1979)]. At first sight, this gives us a misleading impression that these resources are entirely unrelated. However, in fact, many energies have been produced by using not only exhaustible resources but also renewable resources. For example, electric power energies can be supplied from both the “thermal” power generation using an exhaustible resource such as petroleum or coal and the “hydroelectric” power generation using a renewable resource such as *water*. Moreover, they are perfect substitutes as intermediate inputs and as consumption goods. Thus, in analyzing

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the problem of electric power energy and similar problems, we should no longer treat the exhaustible resource and the renewable resource separately but interdependently. And this will lead to a unified theory of these types of resources.

The purpose of this paper is to characterize and compare the competitive and monopolistic equilibrium paths of the electric energy production in an intertemporal economy where the electric power energy can be produced by using an exhaustible resource, a renewable resource, and a backstop technology such as nuclear fusion. Our electric energy model is based on a "partial" equilibrium model without the renewable resource developed by Stiglitz and Dasgupta(1982). They showed the following results :

- (a) The energy production patterns in both competition and pure monopoly are that the exhaustible resource with the "lowest" constant unit cost of energy production is to be used first, and that the backstop technology is always used after the resource depletion.
- (b) If the elasticity of energy demand is constant, decreasing with the output, or increasing for the small initial stock of the exhaustible resource, then the depletion date of exhaustible resource in monopoly is later than that in competition.

In this paper, we are interested particularly in (i) how to modify assertion (a) and (b) of Stiglitz & Dasgupta (1982) with the introduction of the renewable resource and (ii) the optimal use of the renewable resource. Let c_i ($i=1, 2, 3$) be the constant unit cost of electric power production from the renewable resource, the exhaustible resource and the backstop technology respectively, and assume that $c_2 < c_1 < c_3$. Then, our main conclusions are as follows ;

First, the characteristics of the competitive equilibrium path are remarkable different, according as the constant *rainfall* R of the renewable resource (water) regenerated by nature, exceeds the demand $D(c_1)$ for the electric energy at the price c_1 [formally, $R > D(c_1)$; Case 1], is between $D(c_1)$ and $D(c_3)$ [more specifically, $D(c_1) \geq R > D(c_3)$; Case 2], or is less than $D(c_3)$ [$D(c_3) \geq R$; Case 3]. Throughout these three cases, the exhaustible resource is to be used at first in producing the electric power energy. Its stock is entirely depleted at a finite date, and instead the energy production from the renewable resource starts. In Case 1, the renewable resource is *not* depleted and the backstop technology is *not* used forever. In Case 2, both resources are *depleted*. After that, the electric energy continues to be produced by using the constant rainfall alone. The backstop technology is *not* used in this case too. In Case 3, the backstop technology is *used simultaneously* with the constant rainfall during the final interval after the depletions of both resources.

Second, from the formal point of view, it should be noted that the energy production patterns in monopoly is determined in precisely the same manner as in competition by a mere replacement of the market demand function D with the inverse function of the marginal revenue function W , though the resultant paths of the energy price and output are different from those in competition.

Third, if the elasticity of energy demand is constant or decreasing with the output, then in all cases but an "exceptional" case : $D(c_3) < R \leq W(c_1)$, the depletion date of the *exhaustible resource* in monopoly is later than that in competition. Similarly, if the *renewable resource* is depleted in both competition and monopoly, then its depletion date in monopoly is later than that in competition in almost all cases. If it is not depleted at all, then the long-run stationary stock of the renewable resource in monopoly is larger than that in competition. Thus, in monopoly, the utilization of resources, renewable or exhaustible, may well be said conservative.

Finally, if the derivative of the elasticity of energy demand is non-positive or a sufficiently large positive value, then the monopolistic price is *higher* than the competitive one during the initial and final intervals in almost all cases. However, the competitive and monopolistic prices in the exceptional case are both *equal* to $D^{-1}(R)$ during the final interval, irrespective of the sign of the derivative of the elasticity of energy demand. The reason is that after the depletions of both resources, the electric power energies in both competition and monopoly can be produced only from the constant rainfall, R .

The rest of the paper is organized as follows ; In Section 2 and 3, the intertemporal competitive and monopolistic equilibrium paths of the electric energy production are stated respectively. Section 4 compares these equilibrium paths. Finally, some concluding remarks are stated in Section 5.

2. The Competitive Market

We begin with depicting the basic structure of our model. Let $x_{1t} \geq 0$, $x_{2t} \geq 0$, and $x_{3t} \geq 0$ respectively denote the electric power energy produced by using a renewable resource, an exhaustible resource, and a backstop technology at date t (≥ 0). For the sake of simplicity, we assume in addition that the electric power energies produced by different inputs are perfect substitutes in the final consumption and that every energy output can be produced at a *constant* unit cost, c_i ($i=1, 2, 3$) satisfying the condition that $c_2 < c_1 < c_3$. By y_t and p_t represent the total demand and price of the electric energy at date t , respectively. Then the market demand function for the electric energy can be written as $y_t = D(p_t)$. Assuming that $D' = dD/dt < 0$ everywhere, we have the inverse demand function $p_t = F(y_t)$ with the property that $F'(y) < 0$. In what follows, we further suppose that $\lim_{y \rightarrow 0} F(y) = \infty$. Since we proceed by the partial equilibrium analysis, social benefits, with the usual provisos, is thought to be measured by consumer surplus from the market demand function. Hence, as a preliminary, we define "gross" social utility by

$$U(y) = \int_0^y F(h) dh, \quad (1)$$

which is strictly concave since $d^2U/dy^2 = F'(y) < 0$.

Defining one unit of each resource as the quantity of the resource required to produce a unit of electric energy, the dynamic behaviour of resources are described by ;

$$\dot{S}_{1t} = -x_{1t} + G(S_{1t}), \quad (2)$$

$$\dot{S}_{2t} = -x_{2t}, \quad (3)$$

where $\dot{S}_{1t}(S_{2t})$ is the stock at the renewable (exhaustible) resource at date t , the dot above a variable represents the time derivative of the variable, and $G(S_{1t})$ gives rise to the rate of natural growth of the renewable resource. Specifying the energy output as the *electric power*, it will be reasonable to formulate $G(S_{1t})$ by ;

$$G(S_{1t}) = R - kS_{1t}, \quad (4)$$

where $R > 0$ and $k \geq 0$. From now on, we regard S_{1t} , R , and k as the stock of *water* accumulated in reservoirs or dams as a whole, the constant *rainfall* regenerated by nature, and the constant rate of evaporation and/or absorption, respectively. If $k=0$ as a special case, equation (2) reduces to $dS_{1t}/dt = R - x_{1t}$, which is exactly the same dynamic equation as Dasgupta and Heal (1974) analyzed in a generalized "cake-eating" problem. Since it is natural to consider the evaporative-absorbable effects do work in the case of water resource, we henceforth assume that k is positive. We also assume that the initial stock of the renewable resource is initially at the stationary level in "natural" state ; $S_{10} = R/k$.

In this section, we suppose that the stocks of two resources and the backstop technology are competitively owned and characterize the intertemporal competitive equilibrium path. We take it granted that the capital market is *perfect* and the market rate r of interest on "numeraire" asset is given. Then, as is well known, the *fundamental theorem* of welfare economics¹ asserts that the competitive equilibrium path coincides with the solution of a planner's social optimization problem. The planner's problem is to choose x_{it} ($i=1, 2, 3$) so as to maximize net social surplus.

$$\int_0^\infty [U(y_t) - c_1x_{1t} - c_2x_{2t} - c_3x_{3t}]e^{-rt} dt$$

subject to

$$y_t = x_{1t} + x_{2t} + x_{3t}, \quad (5-1)$$

$$\dot{S}_{1t} = R - kS_{1t} - x_{1t}, \quad (5-2)$$

1. We shall now sum up briefly that the intertemporal competitive equilibrium path is equivalent to the solution of the planner's optimization problem. Under the perfect foresight to future prices and the perfect capital market, three firms and the representative consumer must respectively solve the dynamic optimization problems as follows :

(a) *Firms* ($i=1, 2$ and 3)

$$\text{Max}_{(x_{it})} \int_0^\infty (p_t x_{it} - c_i x_{it}) \exp(-rt) dt$$

subject to $\dot{S}_{it} = R - kS_{it} - x_{it}$, $x_{it} \geq 0$, and $S_{it} \geq 0$, where for the firm 2, $R=k=0$, and for the firm 3, all the constraints are unnecessary.

$$\dot{S}_{2t} = -x_{2t}, \quad (5-3)$$

$$x_{1t} \geq 0, \quad x_{2t} \geq 0, \quad x_{3t} \geq 0, \quad (5-4)$$

$$S_{1t} \geq 0, \quad S_{2t} \geq 0. \quad (5-5)$$

Equation (5-1) represents the market equilibrium condition for the electric energy. Equations (5-2) and (5-3) express the dynamic equations concerning the stocks of the renewable and exhaustible resources, respectively. Finally, equations (5-4) and (5-5) denote the non-negativity conditions for the energy outputs and the stocks of two resources, respectively.

This dynamic optimization problem can be solved by using Pontryagin's maximum principle [see, for example, Arrow and Kurz(1970), Chapter II, Propositions 7-10]. Letting the current-value Lagrangian be

$$L = U(x_{1t} + x_{2t} + x_{3t}) - c_1 x_{1t} - c_2 x_{2t} - c_3 x_{3t} + q_{1t}(R - kS_{1t} - x_{1t}) - q_{2t}x_{2t} + \lambda_{1t}S_{1t} + \lambda_{2t}S_{2t},$$

the necessary conditions for optimality are that there exist the auxiliary variables (i.e., the *imputed* prices of two resources), q_{1t} and q_{2t} , and the Lagrange multipliers, λ_{1t} and λ_{2t} , such that they satisfy the following set of equations (6) to (12) for each t :

$$\partial L / \partial x_1 = U'(y) - c_1 - q_1 \leq 0, \quad x_1(\partial L / \partial x_1) = 0, \quad x_1 \geq 0, \quad (6)$$

$$\partial L / \partial x_2 = U'(y) - c_2 - q_2 \leq 0, \quad x_2(\partial L / \partial x_2) = 0, \quad x_2 \geq 0, \quad (7)$$

$$\partial L / \partial x_3 = U'(y) - c_3 \leq 0, \quad x_3(\partial L / \partial x_3) = 0, \quad x_3 \geq 0, \quad (8)$$

$$\dot{q}_1 / q_1 = r + k - \lambda_1 / q_1, \quad (9)$$

$$\dot{q}_2 / q_2 = r - \lambda_2 / q_2, \quad (10)$$

$$\lambda_1 \geq 0, \quad \lambda_1 S_1 = 0, \quad \lambda_1 \dot{S}_1 = 0, \quad (11)$$

$$\lambda_2 \geq 0, \quad \lambda_2 S_2 = 0, \quad \lambda_2 \dot{S}_2 = 0, \quad (12)$$

Equations (6)–(8) are the maximum principle for the control variables. Equations (9) and (10) are the so-called “asset market” equilibrium conditions. Since we find from (5-3) that x_2 must be zero when $S_2=0$, it holds that $S_{2t} \geq 0$ for all $t \geq 0$. Therefore, it follows from (12) that $\lambda_{2t}=0$ for $t \geq 0$. Thus, the dynamic equation (10) have the solution, $q_{2t} = q_{20} \exp(rt)$, which shows that the price of *exhaustible* resource increases exponentially with the market rate of interest. This is the so-called *Hotelling rule* (1931). Finally, equations (11) and (12)

(b) The Representative Consumer

$$\text{Max}_{(y_t, z_t)} \int_0^{\infty} [U(y_t) + z_t] \exp(-\delta t) dt$$

Subject to $\dot{Q}_t = rQ_t - p_t y_t - z_t$, $y_t \geq 0$, $z_t \geq 0$, and $Q_t \geq 0$, where z =consumption of *numeraire* goods, Q =numeraire asset accumulated, δ =private rate of time preference, and finally $U(y) + z$ =additively separable utility function, which is required to define the so-called consumer surplus in the partial equilibrium analysis.

Now, considering the market equilibrium condition for the electric energy, $y_t = x_{1t} + x_{2t} + x_{3t}$, it can be easily shown that the necessary and sufficient conditions of the above-mentioned dynamic optimization problems for both economic agents perfectly coincide with those of the planner's dynamic optimization problem in the text.

are the so-called complementarity slackness conditions for the non-negativity of the stocks of both resources.

There can be seven policies for the electric energy production in this model, because it has been assumed that $\lim_{y \rightarrow 0} U'(y) = \infty$. Let us now define a function $H(q_1)$ and the constant values α and β as follows :

$$H(q_1) \equiv q_1 + c_1 - c_2, \quad \alpha \equiv c_3 - c_1 < 0, \quad \text{and} \quad \beta \equiv c_3 - c_2 > 0.$$

Then, their policies are given by :

- Policy A* : $x_1 > 0, x_2 > 0, \text{ and } x_3 > 0$; $q_1 = \alpha, q_2 = \beta$, and
 $\beta = H(\alpha)$; $p = U'(x_1 + x_2 + x_3) = c_1 + q_1 = c_2 + q_2 = c_3$.
- Policy B* : $x_1 > 0, x_2 > 0, \text{ and } x_3 = 0$; $q_1 \leq \alpha, q_2 \leq \beta$, and
 $\beta = H(\alpha)$; $p = U'(x_1 + x_2) = c_1 + q_1 = c_2 + q_2 \leq c_3$.
- Policy C* : $x_1 > 0, x_2 = 0, \text{ and } x_3 > 0$; $q_1 = \alpha, q_2 \geq \beta$, and
 $\beta \geq H(\alpha)$; $p = U'(x_1 + x_3) = c_1 + q_1 = c_3 \leq c_2 + q_2$.
- Policy D* : $x_1 = 0, x_2 > 0, \text{ and } x_3 > 0$; $q_1 \geq \alpha, q_2 = \beta$, and
 $\beta \geq H(\alpha)$; $p = U'(x_2 + x_3) = c_2 + q_2 = c_3 \leq c_1 + q_1$.
- Policy E* : $x_1 > 0 \text{ and } x_2 = x_3 = 0$; $q_1 \leq \alpha \text{ and } q_2 \geq H(q_1)$;
 $p = U'(x_1) = c_1 + q_1 \leq c_2 + q_2 \leq c_3$.
- Policy F* : $x_2 > 0 \text{ and } x_1 = x_3 > 0$; $q_2 \leq \beta \text{ and } q_2 \leq H(q_1)$;
 $p = U'(x_2) = c_2 + q_2 \leq c_1 + q_1 \text{ and } c_2 + q_2 \leq c_3$.
- Policy G* : $x_3 > 0 \text{ and } x_1 = x_2 = 0$; $q_1 \geq \alpha \text{ and } q_2 \geq \beta$;
 $p = U'(x_3) = c_3 \leq c_1 + q_1 \text{ and } c_3 \leq c_2 + q_2$.

It should be now noted that the *demand* price of electric energy is p_t , but the *supply* prices on the basis of the renewable resource, the exhaustible resource, and the backstop technology are $c_1 + q_{1t}$, $c_2 + q_{2t}$, and c_3 , respectively.

These policies can be illustrated in the imputed price plane (q_1, q_2) , as is shown in the quadrant 2 of Figure 1. The regions (policies) *E*, *F* and *G* show the specialization in the electric energy production from the renewable resource, the exhaustible resource, and the backstop technology, respectively. The line (policy) *B* indicates a set of imputed prices which reduce the prices of energy output produced by using the renewable and exhaustible resources to be equal, i.e., $p_t = c_1 + q_{1t} = c_2 + q_{2t}$. Similarly, we can interpret the lines (policies) *C* and *D*. The intersection point *A* of three lines *B*, *C* and *D* corresponds to the policy *A*.

We can now establish Theorem 1. The proof is provided in the mathematical appendix.

Theorem 1 : Depending on whether the constant rainfall, R , of the renewable (water) resource regenerated by nature is $R > D(c_1)$, $D(c_1) \geq R > D(c_3)$, or $D(c_3) \geq R$, the characteristics of the competitive equilibrium path are remarkably different as follows :

Case 1 : $R > D(c_1)$, i.e., $F(R) < c_1$.

The optimal policy sequence is *FBE*, so that the competitive equilibrium path of electric

power energy consists of two phases. During the first interval $[0, T_1)$, $x_{2t} = D(p_t)$, $p_t = c_2 + q_{20} \exp(rt)$ where $q_{20} > 0$, and $x_{1t} = x_{3t} = 0$. At T_1 the exhaustible resource is depleted and instead the energy production from the renewable resource starts. During the second interval $[T_1, \infty)$, $x_{1t} = D(p_t) < R$ where $p_t = c_1$, and $x_{2t} = x_{3t} = 0$. The renewable resource is *not* depleted, that is, $\lim_{t \rightarrow \infty} S_{1t} = [R - D(c_1)]/k$. The backstop technology is *not* used forever.

Case 2 : $D(c_1) \geq R > D(c_3)$, i.e., $c_1 \leq F(R) < c_3$.

The optimal policy sequence is $FBEE^s$ where E^s is the stationary state in the policy E , so that the competitive equilibrium path consists of three phases (see Figure 1). The movements of energy output and price during the first interval $[0, T_1)$ are the same as those in Case 1. During the second interval $[T_1, T_2)$, $x_{1t} = D(p_t) > R$, $p_t = c_1 + q_{10} \exp(r+k)t$ where $q_{10} > 0$, and $x_{2t} = x_{3t} = 0$. At T_2 the stock of the renewable resource is *depleted*, but the electric energy production using the constant rainfall, R , starts. During the third interval $[T_2, \infty)$, $x_{1t} = D(p_t) = R$ where $p_t = F(R)$, and $x_{2t} = x_{3t} = 0$. The backstop technology is *not* used forever.

Case 3 : $D(c_3) \geq R$, i.e., $c_3 \leq F(R)$.

The optimal policy sequence is $FBEC$, so that the competitive equilibrium path consists of three phases. The movements of energy output and price during the first and second intervals are the same as those in Case 2. During the third interval $[T_2, \infty)$, $x_{1t} = R$, $x_{2t} = 0$, $x_{3t} = D(c_3) - R \geq 0$, and $p_t = c_3$. The backstop technology is used *simultaneously* with the rainfall R during the final interval.

3. Pure Monopoly

In this section, it is supposed that a single firm owns the renewable and exhaustible resources and has a patent on the backstop technology. Let $V(y_t) = y_t F(y_t)$ denotes the gross revenue for the monopolist. Then, the monopolist's problem is to choose three programs x_{1t} , x_{2t} , and x_{3t} so as to maximize

$$\int_0^\infty [V(y_t) - c_1 x_{1t} - c_2 x_{2t} - c_3 x_{3t}] e^{-rt} dt$$

subject to the same constraints (5) as the planner's social optimization problem. The necessary conditions for optimality are the same as those in the competitive market, except that we must replace $U'(y)$ in the conditions (6)–(8) with the marginal revenue, $V'(y) \equiv dV(y)/dy$ where it is assumed that $V''(y) < 0$. We shall define the inverse function of V' by W .

Following the similar proof to Theorem 1, we can now establish Theorem 2.

Theorem 2 : The energy production patterns in the intertemporal monopolistic equilibrium are formally the same as those in the competitive equilibrium, but the paths of energy price and output themselves are different as follows :

Case 1 : $R > W(c_1)$, i.e., $F(R) < F[W(c_1)]$,

During $[0, T_1)$, $x_{2t} = D(p_t)$, $p_t = [\epsilon/(\epsilon-1)][c_2 + q_{20}\exp(rt)]$ where $\epsilon \equiv -D'(p)p/y (>1)$ is the elasticity of energy demand and $q_{20} > 0$, and $x_{1t} = x_{3t} = 0$. During $[T_1, \infty)$, $x_{1t} = D(p_t)$ where $p_t = F[W(c_1)]$, and $x_{2t} = x_{3t} = 0$.

Case 2 : $W(c_1) \geq R > W(c_3)$, i.e., $F[W(c_1)] \leq F(R) < F[W(c_3)]$.

During $[0, T_1)$, the paths of energy output and price are the same as those in Case 1. During $[T_1, T_2)$, $x_{1t} = D(p_t)$, $x_{2t} = x_{3t} = 0$, and $p_t = [\epsilon/(\epsilon-1)][c_1 + q_{10}\exp(r+k)t]$ where $q_{10} > 0$. During $[T_2, \infty)$, $x_{1t} = D(p_t) = R$ where $p_t = F(R)$, and $x_{2t} = x_{3t} = 0$.

Case 3 : $W(c_3) \geq R$, i.e., $F[W(c_3)] \leq F(R)$.

During $[0, T_1)$ and $[T_1, T_2)$, the paths of energy output and price are the same as those in Case 2. During $[T_2, \infty)$, $x_{1t} = R$, $x_{2t} = 0$, $x_{3t} = W(c_3) - R \geq 0$, and $p_t = F[W(c_3)]$.

4. Comparison of the Competitive and Monopolistic Outcomes

We shall now compare the characteristics of the intertemporal competitive and monopolistic equilibrium paths given by Theorems 1 and 2, respectively. To do so, it is convenient for us to classify two cases : Case A : $D(c_3) \geq W(c_1)$ and Case B : $D(c_3) < W(c_1)$. When the stocks of the renewable and exhaustible resources are S_1 and S_2 , respectively, the competitive and monopolistic equilibrium paths of electric energy price can be represented by $p^c(S_1, S_2)$ and $p^m(S_1, S_2)$, where the superscripts "c and m" denote competition and pure monopoly, respectively. Then, we can prove the following lemma, which is proved in the mathematical appendix.

Lemma : If there exists a positive number $S_1 = \Phi$ such that $p^c(S_1, 0) = p^m(S_1, 0)$, then it holds that

$$\left| \frac{\partial p^c(\Phi, 0)}{\partial S_1} \right| \leq \left| \frac{\partial p^m(\Phi, 0)}{\partial S_1} \right| \text{ as } \epsilon'(\phi) \geq A(\phi), \quad (13)$$

where $\epsilon'(x_1) = d\epsilon/dx_1$, $A(x_1) = c_1/\{x_1[F(x_1) - c_1]\} > 0$, and $\phi = D[p^c(\phi, 0)] = D[p^m(\phi, 0)]$. Similarly, if there exists a positive number $S_2 = \Psi$ such that $p^c(S_{10}, S_2) = p^m(S_{10}, S_2)$, then it holds that

$$\left| \frac{\partial p^c(S_{10}, \Psi)}{\partial S_2} \right| \leq \left| \frac{\partial p^m(S_{10}, \Psi)}{\partial S_2} \right| \text{ as } \epsilon'(\psi) \geq B(\psi), \quad (14)$$

where $B(x_2) = c_2/\{x_2[F(x_2) - c_2]\} > 0$ and $\psi = D[p^c(S_{10}, \Psi)] = D[p^m(S_{10}, \Psi)]$.

Armed with this lemma, we can now establish a set of Theorems for each of Cases A and B. The proofs are given in the appendix.

4.1. Case A : $D(c_3) \geq W(c_1)$.

This case is classified into the following five subcases.

Case A-1 : $R \leq W(c_3)$. The intertemporal equilibrium paths in competition and monopoly are given by Case 3 in Theorem 1 and Case 3 in Theorem 2, respectively.

Case A-2 : $W(c_3) < R \leq W(c_1)$. They are given by Case 3 in Theorem 1 and Case 2 in Theorem 2.

Case A-3 : $W(c_1) < R \leq D(c_3)$. They are given by Case 3 in Theorem 1 and Case 1 in Theorem 2.

Case A-4 : $D(c_3) < R \leq D(c_1)$. They are given by Case 2 in Theorem 1 and Case 1 in Theorem 2.

Case A-5 : $D(c_1) < R$. They are given by Case 1 in Theorem 1 and Case 1 in Theorem 2.

Theorem 3 : If $\epsilon'(y) \leq 0$ for all $y > 0$, then we obtain the following two basic results, depending on either $R \leq W(c_1)$ or $R > W(c_1)$.

Result 1. For Cases A-1 and 2, i.e., $R \leq W(c_1)$, it holds that

$$p^m(S_1, 0) > p^c(S_1, 0) \text{ for all } S_1 \geq 0, \quad (15)$$

$$p^m(S_{10}, S_2) > p^c(S_{10}, S_2) \text{ for all } S_2 \geq 0, \quad (16)$$

$$T_1^m > T_1^c, \quad T_2^m > T_2^c, \text{ and } T_2^m - T_1^m > T_2^c - T_1^c. \quad (17)$$

Result 2. For Cases A-3, 4 and 5, i.e., $R > W(c_1)$, it holds that

$$p^m(S_1, 0) > p^c(S_1, 0) \text{ for } S_{10} \geq S_1 > \lim_{t \rightarrow \infty} S_t^m = [R - W(c_1)]/k, \quad (15)'$$

$$p^m(S_{10}, S_2) > p^c(S_{10}, S_2) \text{ for all } S_2 \geq 0, \quad (16)'$$

$$T_1^m > T_1^c. \quad (17)'$$

Theorem 4 : If $\epsilon'(y)$ is a sufficiently large positive value for all $y > 0$ such that $\epsilon'(x_1) > A(x_1) > 0$ and $\epsilon'(x_2) > B(x_2) > 0$ always hold, then Results 1 and 2 in Theorem 3 still continue to hold.

4.2. Case B : $D(c_3) < W(c_1)$.

This case is moreover classified into the following five subcases, Case B-1 : $R \leq W(c_3)$, Case B-2 : $W(c_3) < R \leq D(c_3)$, Case B-3 : $D(c_3) < R \leq W(c_1)$, Case B-4 : $W(c_1) < R \leq D(c_1)$, and Case B-5 : $D(c_1) < R$. Since the competitive and monopolistic equilibrium paths in all cases except for Case B-3 are the same as those in the corresponding subcases of Case A, it is clear that Theorem 3 and 4 continue to hold for these cases. Thus, we shall now concentrate on the analysis of Case B-3. The equilibrium paths in competition and pure monopoly in this case are given by Case 2 in Theorem 1 and Case 2 in Theorem 2, respectively.

We can now establish Theorem 5 and 6 for Case B-3, Which are proved in the appendix.

Theorem 5 : If $\epsilon'(y) \leq 0$ for all $y > 0$, then we obtain either of the following two results.

Result 1. It holds that

$$p^m(S_1, 0) \geq p^c(S_1, 0) \text{ for all } S_1 \geq 0, \text{ where } p^m = p^c \text{ is obtained only when } S_1 = 0, \quad (15)''$$

$$p^m(S_{10}, S_2) > p^c(S_{10}, S_2) \text{ for all } S_2 \geq 0, \quad (16)$$

$$T_1^m > T_1^c, \quad T_2^m > T_2^c, \text{ and } T_2^m - T_1^m > T_2^c - T_1^c. \quad (17)$$

Result 2. There exists a unique and positive number $\emptyset$ such that

$$p^m(S_1, 0) \leq p^c(S_1, 0) \text{ as } S_1 \leq \Phi, \quad (18)$$

and then there exists a critical value of the "initial" stock of the *renewable* resource, Ω where $\Omega > \Phi > 0$, such that

$$T_2^m - T_1^m \leq T_2^c - T_1^c \text{ as } S_{10} \leq \Omega. \quad (19)$$

Moreover, (i) if $S_{10} > \Phi$, then it holds that the inequality (16) and $T_1^m > T_1^c$, (ii) if $S_{10} = \Phi$, then either

$$p^m(\Phi, S_2) \geq p^c(\Phi, S_2) \text{ for all } S_2 \geq 0, \quad (20)$$

where $p^m = p^c$ only when $S_2 = 0$, or there exists some unique and positive number Ψ such that

$$p^m(\Phi, S_2) \leq p^c(\Phi, S_2) \text{ as } S_2 \leq \Psi, \quad (21)$$

and then there exists a critical value of the "initial" stock of the *exhaustible* resource, Θ where $\Theta > \Psi > 0$, such that

$$T_1^m \leq T_1^c \text{ as } S_{20} \leq \Theta, \quad (22)$$

and finally (iii) if $S_{10} < \Phi$, there exist Ψ and Θ such that satisfy the inequalities (21) and (22).

Theorem 6 : If $\epsilon'(y)$ is a sufficiently large positive value for all $y > 0$ such that $\epsilon'(x_1) > A(x_1) > 0$ and $\epsilon'(x_2) > B(x_2) > 0$ always hold, then Result 1 in Theorem 6 still continues to hold.

4.3. Economic Implications.

Theorems 3-6 have some important implications as follows :

(a) Depletion of the Exhaustible Resource.

If $\epsilon'(y) \leq 0$ for all $y > 0$, then in all cases exclusive of Case B-3 : $D(c_3) < R < W(c_1)$, the depletion date of the exhaustible resource in monopoly is later than that in competition, so that monopoly is excessively conservative. This outcome still continues to hold for all cases inclusive of Case B-3, only if $\epsilon'(y)$ takes a sufficiently large positive value for all $y > 0$. Such an intertemporal bias stemming from monopoly have been studied by Hotelling (1931), Stiglitz (1974), Stiglitz and Dasgupta (1982) and others in the economic theory of exhaustible resources. In our energy model also, we can make sure of the same bias in most cases.

(b) Depletion of the Renewable Resource.

If $\epsilon'(y) \leq 0$ for all $y > 0$ and if the renewable resource is depleted in both competition and monopoly, then its depletion date in monopoly is later than that in competition in all cases except for Case B-3. If it is not depleted at all, then it holds that

$$\lim_{t \rightarrow \infty} S_{it}^m = [R - W(c_1)]/k > [R - D(c_1)]/k = \lim_{t \rightarrow \infty} S_{it}^c.$$

These outcomes still continue to hold for all cases inclusive of Case B-3, only if $\epsilon'(y)$ takes a sufficiently large positive value for all $y > 0$. Hence, we can find that monopoly is excessively conservative with respect to the renewable resource also.

(c) Energy Price Movement.

If $\epsilon'(y) \leq 0$ for all $y > 0$ or if $\epsilon'(y)$ takes a sufficiently large positive value, then in all cases

except for Case B-3 the monopolistic price is higher than the competitive one during the initial and final intervals. This outcome is the same as that in the case of the energy being produced from *two* exhaustible resources and the backstop technology. But, the competitive and monopolistic prices in Case B-3 is equal to $F(R)$ during the final interval, irrespective of the sign of $\epsilon'(y)$:

$$p^c(T_1^c) = F(R) = p^m(T_1^m),$$

where

$$p^c(T_1^c) = c_1 + q^c(T_1^c) \text{ and } V'(R) = c_1 + q^m(T_1^m).$$

The reason is that after the exhaustible and renewable resources are both depleted, the electric power energies in both competition and monopoly can be produced only from the constant rainfall, R .

5. Concluding Remarks

In this paper, based on the partial equilibrium analysis developed by Stiglitz and Dasgupta (1982), we have characterized and compared the competitive and monopolistic equilibrium paths in the intertemporal economy where the electric power energy can be supplied by using an exhaustible resource, a renewable (*water*) resource, or the backstop technology. It has been clarified that the size of the constant *rainfall* of regenerated by nature significantly effects on (a) the intertemporal competitive and monopolistic paths of the electric energy production, (b) the availability and depletion of the renewable resource, (c) the availability of the backstock technology, (d) the upper limit of the energy price, and so on.

We have investigated an economic problem of the electric power energy in the intertemporal economy with the several simplified assumptions. It will be now useful to make particular remarks on some important assumptions in this paper.

First, we have assumed that the constant unit cost condition of the electric energy production is $c_2 < c_1 < c_3$. This assumption is *crucial* for our conclusions in this paper. But, in some countries the cost condition may be such that not the exhaustible resource but the *renewable* has the "lowest" constant unit cost : $c_1 < c_2 < c_3$. Further, the unit cost of the energy production may not be constant but depend on the energy output and the size of resource stock. Therefore, it will be really difficult that our assumption of the cost condition continues to hold for all future dates. However, it seems that to relax this assumption reduces our analysis for the electric power energy to be very complicated.

Second, we have also assumed that the renewable resource is the *water* resource. Instead, it could be specified as the *biomass* such as timber or an animal. Then, the natural growth function $G(S_1)$ can be generally formulated as follows [see Clark(1976) and Dasgupta and Heal(1979)] :

$$G''(S_1) < 0 \text{ for all } S_1 \geq 0. \quad (a)$$

$$G'(S_f) = 0, \quad (b)$$

$$G(S_f^s) = G(S_f^u) = 0, \text{ where } S_f^s > S_f^u \geq 0, \quad (c)$$

where S_f gives us the maximum sustainable yield, i.e., $G(S_f) > G(S_1)$ for all $S_1 \geq 0$, and S_f^s and S_f^u are the stable and unstable biomass in the natural state equilibria, respectively. In this case, the energy output in our model should be interpreted as not the electric power energy but the *thermal* energy or the *motive power* energy. We can do the same analysis for such an energy. Although it is somewhat complicated, it can be proved that the similar theorems to those in the previous sections continue to hold, only if we shall replace the constant rainfall R with the maximum sustainable yield $G(S_f)$ or the optimal long-run yield $G(S_f^s)$, where S_f^s is the biomass at which the own rate of return on it is equal to the market rate of interest, i.e., $G'(S_f^s) = r$.

Finally, although we have not set any constraint about the total demand for the electric power energy, it may be necessary for us to consider a *lower limit* on the energy demand due to its necessity property and the population or economic growth.

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Mathematical Appendix

A. Proof of Theorem 1 :

Let us first prove that $q_{20} > 0$. Suppose that it is not so. Then, since the energy price p_t at all date is equal to the "lowest" constant unit cost c_2 , the electric energy can be forever produced from the *exhaustible* resource alone. That is, $x_{2t} = D(c_2)$ for all $t \geq 0$. However, this is clearly not optimal, because the exhaustible resource is depleted at some finite date T , so that we can not entirely produce the energy output after that. Thus, the energy price must increase exponentially with the market rate of interest : $p_t = c_{20} + q_{20} \exp(rt)$ where $q_{20} > 0$. As it has been assumed that the unit cost condition is $c_2 < c_1 < c_3$, in any case it is optimal that during the first interval the policy F is selected. Next, consider the sign of q_{10} . If $q_{10} = 0$, then there exists some finite date $T_1 (> 0)$ such that the energy can be produced from the *renewable* resource alone after that. That is, $x_{1t} = D(c_1)$ for all $t \geq T_1$. At T_1 the optimal policy switches from the policy F to the policy E .

Now, if we shall consider that *Case 1* : $R > D(c_1)$, then it is possible to sustain $x_{1t} > 0$ and $S_{1t} > 0$ for all $t \geq T_1$. Because both $\lim_{t \rightarrow \infty} S_{1t} = [R - D(c_1)]/k > 0$ and it follows from (9) and (11) that $q_{1t} = 0$ for all $t \geq T_1$, the *transversality condition* :

$$\lim_{t \rightarrow \infty} e^{-rt} q_{1t} S_{1t} = 0, \quad (\text{A.1})$$

is satisfied. Here, letting (q_{20}, T_1) denote the solution of the following simultaneous equation with respect to (q_{20}, T) :

$$c_2 + q_{20} \exp(rT) = c_1 \text{ and } \int_0^T D[c_2 + q_{20} \exp(rt)] dt = S_{20},$$

the other transversality condition :

$$\lim_{t \rightarrow \infty} e^{-rt} q_{2t} S_{2t} = 0, \quad (\text{A.2})$$

is also satisfied. Considering the gross social surplus function $U(y)$ is strictly concave, it is clear that the optimal policy sequence is FBE . Hence, the intertemporal competitive equilibrium path given in Case 1 satisfies the necessary and sufficient conditions for optimality.

However, if $R \leq D(c_1)$, then $\lim_{t \rightarrow \infty} S_{1t} \leq 0$, so that it is impossible to sustain $x_{1t} > 0$ for all $t \geq T_1$. Therefore, q_{10} must be positive : $q_{10} > 0$. Since $q_{2t} = q_{20} \exp(rt)$ where $q_{20} > 0$, it is optimal that we select one of the policies C , E , and G as the final policy. However, the policy G is not optimal for the following reason. Since $x_{1t} = 0$ there, the dynamic equation (5-2) becomes $dS_{1t}/dt = R - kS_{1t}$, so that $\lim_{t \rightarrow \infty} S_{1t} = R/k$. This implies that because it follows from (9) and (11) that $q_{1t} = q_{10} \exp[(r+k)t]$ where $q_{10} > 0$, the transversality condition (A.1) is not satisfied. Next, suppose that the policy C is the optimal final one. Since $q_{1t} = a$ there, dq_{1t}/dt

$=0$. Therefore, we obtain $\lambda_{1t}=(r+k)\alpha>0$ from (9). Thus, $S_{1t}=0$ from (11) and $x_{1t}=R$ from (5-2). If $D(c_3)\geq R$, then $x_{3t}=D(c_3)-R\geq 0$, so that the policy C is feasible and satisfies the transversality condition (A.1), but otherwise it is not so. Finally, suppose that the policy E is the optimal final one. Since $x_{1t}=D(c_1+q_{1t})$ there, the dynamic equations (5-2) and (9) become respectively :

$$\dot{S}_{1t}=R-D(c_1+q_{1t})-kS_{1t} \text{ and } \dot{q}_{1t}=(r+k)q_{1t}-\lambda_{1t}.$$

A phase diagram for the path of solutions of these dynamic equations is shown in the quadrant 1 of Figure 1. In the stationary state E^s , it holds that $S_1^s=0$, $q_1^s=F(R)-c_1\geq 0$, $\lambda_1^s=(r+k)q_1^s>0$, and $x_1^s=R$. If $q_1^s<\alpha$, i.e., $R>D(c_3)$, then the path which converges to the stationary state E^s is feasible and satisfies the transversality condition (A.1), but otherwise it is not so.

Based on the above-mentioned analyses, let us now consider that *Case 2* : $D(c_1)\geq R>D(c_3)$. Then, the optimal final policy is E and the path which converges to the stationary state E^s is optimal. In order that the path which satisfies both necessary conditions and the transversality condition (A.1) satisfies also the other transversality condition (A.2), the exhaustible resource must have been depleted before we select the policy E . As we have already proven that in any case the optimal first policy is the policy F , the optimal policy sequence is $FBEE^s$. The exhaustible resource is depleted as soon as we choose the policy E , and the stock of the renewable resource is depleted as soon as the optimal path converges to the stationary state E^s . The competitive equilibrium path given in Case 2 corresponds to the optimal policy sequence $FBE(E^s)$ which satisfies the necessary and sufficient conditions for optimality.

Finally, let us consider that *Case 3* : $D(c_3)\geq R$. Then, the optimal final policy is C . Since $S_1=0$ there, we must have chosen the policy E before the policy C . Thus, the optimal policy sequence in this case is $FBEC$. This is the same as the competitive equilibrium path given in Case 3.

Q.E.D.

B. Proof of the Lemma :

We shall now prove only (13) since (14) can be shown in a similar way. During the *second* intervals in Cases 2 and 3 of Theorem 1, it follows from (6) and (9) that $p_t^c=c_1+q_{1t}^c$ and $dq_{1t}^c/dt=(r+k)q_{1t}^c$, so that $dp_t^c/dt=(r+k)(p_t^c-c_1)>0$. Thus, considering (5-2), i.e., $dS_{1t}^c/dt=R-x_{1t}^c-kS_{1t}^c<0$, in competition it holds that

$$\frac{\partial p^c(S_1^c, 0)}{\partial S_1^c} = \frac{dp_t^c/dt}{dS_{1t}^c/dt} = -\frac{(r+k)[p^c(S_1^c, 0)-c_1]}{x_1^c+kS_1^c-R} < 0. \quad (B.1)$$

Similarly, during the second intervals in Case 2 and 3 of Theorem 2, it follows that $V_t'=c_1+q_{1t}^m$ and $dq_{1t}^m/dt=(r+k)q_{1t}^m$, so that $dV_t'/dt=(r+k)(V_t'-c_1)>0$. Using $V_t'=p_t^m[1-(1/\epsilon)]$, $x_{1t}^m=D(p_t^m)$, and $\epsilon=-D'(p_t^m)p_t^m/x_{1t}^m$, we obtain :

$$\frac{\dot{V}_t'}{V_t'} = \frac{\dot{p}_t^m}{p_t^m} [1 - \epsilon' x_1^m / (\epsilon - 1)],$$

where we should note that $[1 - \epsilon' x_1^m / (\epsilon - 1)] > 0$, since $dV_t'/dt > 0$ and $dp_t^m/dt > 0$. The latter is derived from $V''(x_1^m) < 0$, $dV_t'/dt > 0$, $dx_{1t}^m/dt < 0$, and $dx_{1t}^m/dt = D'(dp_t^m/dt)$ where $D' < 0$. Eliminating now dV_t'/dt and V_t' from the above-mentioned equation, we obtain :

$$\dot{p}^m = \frac{(r+k)[p^m - \epsilon c_1 / (\epsilon - 1)]}{[1 - \epsilon' x_1^m / (\epsilon - 1)]} > 0.$$

Thus, in monopoly it holds that

$$\frac{\partial p^m(S_1^m, 0)}{\partial S_1^m} = - \frac{(r+k)[p^m(S_1^m, 0) - \epsilon c_1 / (\epsilon - 1)]}{(x_1^m + kS_1^m - R)[1 - \epsilon' x_1^m / (\epsilon - 1)]} < 0. \quad (B.2)$$

Now, setting $S_1^c = S_1^m = \emptyset$ in (B.1) and (B.2), and then considering $x_1^c = D[p^c(\emptyset, 0)] = D[p^m(\emptyset, 0)] = x_1^m$, we can prove (13). Q.E.D.

C. Proof of Theorem 3 :

We shall now prove only Case A-1 in Result 1 since the others can be shown in a similar way. By Theorems 1 and 2, when the renewable and exhaustible resources are depleted, it holds that

$$p^m(T_2^m) = p^m(S_1 = 0, S_2 = 0) = \underline{p}^m > c_3 = p^c(S_1 = 0, S_2 = 0) = p^c(T_2^c)$$

where $\underline{p}^m = [\epsilon / (\epsilon - 1)] c_3$. Therefore, if there exists some positive number $S_1 = \emptyset$ such that $p^c(S_1, 0) = p^m(S_1, 0)$, then because it follows from (B.1) and (B.2) that $\partial p^c(S_1, 0) / \partial S_1 < 0$ and $\partial p^m(S_1, 0) / \partial S_1 < 0$, we obtain $|\partial p^c(\emptyset, 0) / \partial S_1| \leq |\partial p^m(\emptyset, 0) / \partial S_1|$. However, since the assumption : $\epsilon' \leq 0$ and (13) in the lemma, it must hold that $|\partial p^c(\emptyset, 0) / \partial S_1| > |\partial p^m(\emptyset, 0) / \partial S_1|$, which is a contradiction. Therefore, the curve $p^m(S_1, 0)$ must lie entirely above the curve $p^c(S_1, 0)$ for all $S_1 \geq 0$. Hence, the inequality (15) is proved. Similarly, since the assumption, (14) in the lemma, and

$$p^m(T_1^m) = p^m(S_{10}, 0) > p^c(S_{10}, 0) = p^c(T_1^c), \quad (C.1)$$

the inequality (16) is proved.

Now, it follows from (5-2) and (5-3) that

$$\partial \dot{S}_1 / \partial p = - \partial x_1 / \partial p > 0, \quad (C.2)$$

$$\partial \dot{S}_2 / \partial p = - \partial x_2 / \partial p > 0. \quad (C.3)$$

Since (15) and (C.2), $T_2^m - T_1^m > T_2^c - T_1^c$, and since (16) and (C.3), $T_1^m > T_1^c$, we obtain $T_2^m > T_2^c$. Hence, the inequalities (17) are proved. Q.E.D.

D. Proof of Theorem 4

We shall now prove only Case A-1 in Result 1, as in Theorem 3. Since $\partial p^c(S_1, 0) / \partial S_1 < 0$, $\partial p^m(S_1, 0) / \partial S_1 < 0$, and

$$\lim_{S_1 \rightarrow \infty} p^c(S_1, 0) = c_1 < F[W(c_1)] = \lim_{S_1 \rightarrow \infty} p^m(S_1, 0), \quad (D.1)$$

if there exists some $S_1 = \emptyset > 0$ such that $p^c(S_1, 0) = p^m(S_1, 0)$, then we obtain $|\partial p^c(\emptyset, 0) / \partial S_1| \geq |\partial p^m(\emptyset, 0) / \partial S_1|$. However, by the assumption and (13) in the lemma, it must hold that $|\partial p^c(\emptyset, 0) / \partial S_1| > |\partial p^m(\emptyset, 0) / \partial S_1|$.

$0)/\partial S_1| < |\partial p^m(\Phi, 0)/\partial S_1|$, which is a contradiction. Hence, the inequality (15) is proved. Inequalities (16) and (17) can be shown in the similar way to the proofs of Theorem 3.

Q.E.D.

E. Proof of Theorem 5

Since the assumption, (13) in the lemma, (D.1), and $p^m(0, 0) = p^c(0, 0) = F(R)$, if there exists some $S_1 = \Phi > 0$ such that $p^c(\Phi, 0) = p^m(\Phi, 0)$ holds, then it must be unique. Thus, the relative position of the curves $p^c(S_1, 0)$ and $p^m(S_1, 0)$ is whether $p^m(S_1, 0)$ lies entirely above $p^c(S_1, 0)$ at $S_1 > 0$ or $p^c(S_1, 0)$ crosses $p^m(S_1, 0)$ precisely once from above at $S_1 = \Phi$. In the case of the former, we can prove Result 1 in the similar way to Theorem 3. On the contrary, in the case of the latter, there exists some unique and positive number Φ such that satisfies the inequality (18) and then it follows from (C.2) that there exists a positive critical value of the initial stock of renewable resource, $\Omega(>\Phi)$, such that satisfies the inequality (19). Now if $S_{10} > \Phi$, then the inequality (16) and $T_1^m > T_1^c$ hold for the same reason as Result 1. Next, if $S_{10} = \Phi$, then since the assumption, (14) in the lemma, $p^m(\Phi, 0) = p^c(\Phi, 0)$, and

$$\lim_{S_2 \rightarrow \infty} p^c(S_{10}, S_2) = c_2 < F[W(c_2)] = \lim_{S_2 \rightarrow \infty} p^m(S_{10}, S_2), \quad (E.1)$$

the relative position of the curves $p^c(S_{10}, S_2)$ and $p^m(S_{10}, S_2)$ is the same as that of the curves $p^c(S_1, 0)$ and $p^m(S_1, 0)$. Therefore, similarly either the inequality (20) holds or there exists $S_2 = \Psi$ and $S_{20} = \Theta$ such that satisfy the inequalities (21) and (22). Finally, if $S_{10} < \Phi$, then since the assumption, (14) in the lemma, $p^m(\Phi, 0) = p^c(\Phi, 0)$, and (E.1), there exists Ψ and θ .

Q.E.D.

F. Proof of Theorem 6 :

It has been already shown that if there exists some $S_1 = \Phi > 0$ such that $p^c(S_1, 0) = p^m(S_1, 0)$, it must be unique. Therefore, the inequality $|\partial p^c(\Phi, 0)/\partial S_1| \geq |\partial p^m(\Phi, 0)/\partial S_1|$ holds. However, by the assumption and (13) in the lemma, it must hold that $|\partial p^c(\Phi, 0)/\partial S_1| < |\partial p^m(\Phi, 0)/\partial S_1|$, which is a contradiction. Hence, the inequality (15)'' is proved. The inequalities (16) and (17) can be proved in the similar way to Theorem 3.

Q.E.D.