Commuting Frequency in the Information-oriented City : Equilibrium vs. Optimum[†]

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1. Introduction

After the industrial revolution, the industrial society was born. The work place moved out of the home and into the factory or office in the industrial society. With recent developments in new information technology, working at home is facilitated. With a growing information-oriented society, the work place may be about to move back in the home. On the traditional urban model in Urban Economics, workers must travel between their homes and office. Basic assumption of the monocentric urban model is that workers prefer less commuting frequency. Hence, the model predicts that workers will take advantage of any opportunity to reduce their commuting frequency. Since telecommuting becomes possible by the recent progress of information and telecommunication technology, workers need not travel to an office and can reduce their commuting frequency.

With a growing information-oriented society, clarification of the effects of the information technology on the spatial structure in the city is important. Higano and Orishimo[8] have developed a model of an information-oriented city with telecommuting. They analyzed the impacts of the progress of information technology on the spatial distribution of markets of home and office work. There are many previous studies about traffic congestion (e.g., [1] [10] [13] [15]). They compared an equilibrium of the market city with the optimum outcome and analyzed the spatial effects of congestion externalities. Henderson[5] constructed a flextime model of commuting and examined the phenomenon of peak-period travel, such the morning or evening rush hour for commuting. However, in these urban models, commuting frequency is exogenously given and not analyzed. We focus on commuting frequency in the information-oriented city.

In this paper, we construct a general equilibrium model of the land and labor markets in the closed circular city with telecommuting. Firms determine subjective optimal commuting frequency. Given the commuting frequency, households commute from their own houses to the office in the CBD. Therefore, there exists negative externalities of commuting frequency. This is different from the external diseconomies of traffic congestion. We derive equilibria by numerical computations using specific utility and production functions. We also derive the commuting tax which optimally control the externalities of commuting frequency. By numerical simulation, we analyze the impacts of the commuting tax on the spatial equilibrium of the city.

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2. The Model

We construct a general equilibrium model of the land and labor in the closed city. The main focus of this paper is to analyze the spatial structure of commuting frequency in the city. There exist three distinct economic sectors in the city: composite goods sector, real estate sector and household sector. The households living in the suburbs commute to the office in the CBD by the transportation networks and/or work at their own homes by the telecommunication networks. The number of households in the suburbs and the number of firms in the CBD are both fixed. Each household has one wage earner. The household maximizes its utility subject to the constraints on the time and budgets. The utility depends on housing space, leisure and composite goods. The household has a production function. The production function depends on not only home working time but also office working time. Monetary cost of commuting in the city is borne by the employers. On the other hand, time cost of commuting is borne by the commuters. Traffic congestion doesn't occur in the city.

The firms of the composite goods sector are located in the CBD and maximize their profits. The real estate sector leases land in the whole city from absentee landowners at an opportunity land rent of agricultural uses, and, in turn, leases it back to households and firms at the market land rent. The profits are shared equally by all the suburban households. Losses are, if any, covered through the assessment of an equal lump sum tax on each household. The firm of the composite goods sector determines the number of commuting trips so as to maximize its profit. There exists the negative externalities of commuting frequency. The externalities is caused because households commute from their own homes to firms by the number of commuting trips which is given by the firms. The public authority imposes the commuting tax on the firms in order to correct the inefficient resource allocation. The revenue of tax is evenly re-distributed in the lump sum to the suburban households in the city.

2.1 Behavior of Households

(1) Formulation

Household optimization problem is formulated as follows:

$$\max_{\{l_H, T_L, X\}} U(l_H, T_L, X), \quad (1)$$

s.t.

$$T - T_L - T_H - T_F - G(r)T_C(z, r) = 0, \quad (2)$$

$$w_s(z, r)S(T_H, T_F) + w_F(z, r)T_F + d_v + D_t - p_t(r)l_H - X \geq 0, \quad (3)$$

$$l_H, T_L, T_H, T_F, X \geq 0, \quad (4)$$

$$\eta \geq r \geq \varepsilon \geq z \geq 0. \quad (5)$$

Utility Function $U(\cdot)$ depends on the amount of land (l_H), leisure (T_L), and the amount of consumption of composite goods (X). We assume that the price of composite goods is numeraire. Variables $r(\varepsilon \leq r \leq \eta)$ and $z(0 \leq z \leq \varepsilon)$ are a residential and an office-working location variables, respectively. Both r and z are defined as the distance from the location to the CBD center. Parameter ε is the CBD boundary and parameter η is the city boundary. Parameter T is the total available time. Variable T_H is the time spent in home work.

Variable T_F is the time spent in office work. Parameter $T_c(z, r)$ is commuting time per one-way between residential location r and office-work location z and is defined as $T_c(z, r) = (r - z)/\bar{v}$. Parameter $\bar{v}$ is a given average commuting velocity. Parameter $G(r)$ is the number of commuting trips given by a firm. Total commuting time per period is calculated as $G(r)T_c(z, r)$. Production function $S(\cdot)$ depends on the time spent in home work (T_H) and office work (T_F). We will use the term "household production goods" to refer to the output. Parameter $w_s(z, r)$ is the price of household production goods. Parameter $w_F(z, r)$ is wage rate of office work. Parameter $p_l(r)$ is land rent at location r . Parameter d_v is an equal dividend of profits of firms. Parameter $D_t(>0)$ is an equal re-distributed of the surplus of the tax revenue. The solution for the above household's optimization problem of Eqs. (1)–(5) are represented by $l_H[*_H]$, $T_L[*_H]$, $T_H[*_H]$, $T_F[*_H]$, and $X[*_H]$ provided that residential and office-work location, r and z , are temporarily given. Symbol $[_H]$ means the vector of parameters, $[_H] = [w_s(z, r), w_F(z, r), T_c(z, r), G(r), p_l(r), d_v, D_t; z, r]$. Although parameters, $w_s(z, r)$, $w_F(z, r)$, $p_l(r)$, d_v , D_t , $G(r)$, ε , and η are taken as constants by its subjective optimization, these are endogenously determined in the urban system through the equilibrium of the market.

(2) Bid rent and Wage functions

Bid rent in the suburbs and wage offer functions are derived by the land arbitrage condition as follows: $U(l_H[*_H], T_L[*_H], X[*_H]) = \bar{U}(\varepsilon \leq r \leq \eta)$. Variable $\bar{U}$ is the equilibrium utility level of household. At the equilibrium, all the households attain the same utility level irrespective of their residential and office-work locations. The first derivative of the bid rent function with respect to r , provided that office-work location z is temporarily fixed, is given as follows:

$$\frac{dp_l(r)}{dr} = \frac{1}{l_H(r)} \left[\frac{dw_s(z, r)}{dr} S + \frac{dw_F(z, r)}{dr} T_F - \nu(r) \frac{d(G(r)T_c(z, r))}{dr} \right] (\varepsilon \leq r \leq \eta),$$

in which $\nu(r) = \mu(r)/\lambda(r)$. Variable $\nu(r)$ represents the monetary time cost of commuting. Variable $\mu(r)$ is the Lagrangean multiplier associated with Eq. (2) and refers to the marginal utility of time. Variable λ is the Lagrangean multiplier associated with Eq. (3) and refers to the marginal utility of income. On the other hand, the first derivative of the wage offer function for office work with respect to z , provided that residential location r is temporarily fixed, is given as follows:

$$\frac{dw_F(z, r)}{dz} = \frac{1}{T_F} \left[-\frac{dw_s(z, r)}{dz} S + \nu(r) \frac{d(G(r)T_c(z, r))}{dz} \right] (0 \leq z \leq \varepsilon).$$

2.2 Behavior of Firms

(1) Composite Goods Sector

Firm optimization behavior is formulated as follows:

$$\begin{aligned} \max_{(l_F, s, t_F, y, g)} \quad & \pi = (1 - p_c z) F(l_F, s, f(t_F, g), y) \\ & - p_l(z) l_F - w_s(z, r) s - w_F(z, r) t_F - y - g\{p_t(z, r) + T_x(z)\}. \end{aligned} \quad (6)$$

The firm maximizes profit (π). Production function, $F(\cdot)$, depends on the amount of land for office work (l_F), the input of household production goods (s), the labor input of office work in efficiency unit, $f(\cdot)$, and the intermediate inputs of composite goods (y). The labor input

of office work in efficiency unit depends on the office work in hours (t_F) and the number of commuting trips (g)¹. The monetary commuting cost of household is borne by the firm. Parameter $p_t(z, r)$ is the commuting cost per one-way between location z to location r and is defined as $p_t(z, r) = \bar{p}_t(r - z)$. Parameter $\bar{p}_t$ is a given price of the transportation service per unit distance. Parameter p_c is the transport cost per unit output of the composite goods per unit distance. The products are sold in the open market located at the city center at the c.i.f price $(1 - p_c z)$, namely the cost of the shipment of the products to the market is borne by the firm. The aggregate sales of the firm at location z is therefore $(1 - p_c z)F$. Parameter $T_x(z)$ is the tax charged on the commuting frequency. The solutions for the above firm's optimization problem of Eq. (6) are represented by $l_F[*_F]$, $s[*_F]$, $t_F[*_F]$, $y[*_F]$, and $g[*_F]$ provided that residential and office-working locations, r and z , are temporarily given, respectively. Symbol $[_F]$ means the vector of paramers, $[_F] = [w_s(z, r), w_F(z, r), p_t(z, r), p_c, T_x(z); r, z]$.

Bid rent function for the land in the CBD is derived by the land arbitrage condition as follows: $\pi(l_F[*_F], s[*_F], t_F[*_F], g[*_F], y[*_F]) = \bar{\Pi}$ ($0 \leq z \leq \varepsilon$). Variable $\bar{\Pi}$ is the amount of the equilibrium profit of the firm. At the equilibrium, all the firms acquire the same amount of profits irrespective of their workplace and employment locations. The first derivative of the bid rent function with respect to r , provided that residential location r is temporarily fixed, is given as follows:

$$\frac{dp_t(z)}{dz} = \frac{1}{l_F(r)} \left[-p_c F - s \frac{dw_s(z, r)}{dz} - t_F \frac{dw_F(z, r)}{dz} - g \frac{d(p_t(z, r) + T_x(z))}{dz} \right] (0 \leq z \leq \varepsilon). \quad (7)$$

(2) Real Estate Sector

The real estate sector intermediately leases all the land in the city from the absentee landowners at an opportunity agricultural rate, and leases it out to households at market rates. The leasing costs are negligibly small. The profit (Π_a) is calculated as: $\Pi_a = \int_0^{\varepsilon} \{p_t(r) - R_A\} 2\pi r dr$. Parameter R_A is an opportunity land rent of agricultural use.

3. Equilibrium Conditions

The laissez-faire equilibrium is simulated when $T_x(z)$ and D_t are equal to zero for all z and r . The conditions for the laissez-faire equilibrium are stated as follows:

(1) Arbitrage conditions in the land market

$$U(l_H[*_H], T_L[*_H], X[*_H]) = \bar{U} \quad (\varepsilon \leq r \leq \eta), \quad (8)$$

$$\pi(l_F[*_F], s[*_F], t_F[*_F], g[*_F], y[*_F]) = \bar{\Pi} \quad (0 \leq z \leq \varepsilon). \quad (9)$$

(2) Equilibrium conditions in the land leasing markets

$$p_t(r) = \max\{p_t(r), R_A\} \quad (0 \leq r \leq \eta), \quad (10)$$

$$p_t(\eta) = R_A, \quad (11)$$

$$2\pi z = l_F[*_F] e_F(z) \quad (0 \leq z \leq \varepsilon), \quad (12)$$

$$2\pi r = l_H[*_H] e_H(r) \quad (\varepsilon \leq r \leq \eta). \quad (13)$$

Eq. (11) is the boundary condition of the land rent function. Eqs. (12) and (13) mean that the total amount of the land for housing, or office space has to be equated with the amount of the

¹ We assume $\partial f / \partial t_F > 0$ and $\partial f / \partial g > 0$. The number of commuting trips means the frequency of meeting or contact with colleagues in the office. The labor input of office work in efficiency unit positively depends on the fact-to-fact contact at the office.

land available at location z or r , respectively. Variable $e_F(z)$ is the number of firms located at location z in the CBD. Variable $e_H(r)$ is the number of households residing at location r in the suburbs.

(3) Conditions of the closed city

$$\bar{P} = \int_{\epsilon}^{\eta} e_H(r) dr, \quad (14)$$

$$\bar{N} = \int_0^{\epsilon} e_F(r) dr. \quad (15)$$

Parameter $\bar{P}$ is the number of households in the suburbs and parameter $\bar{N}$ is the number of firms in the CBD.

(4) Equilibrium conditions in the household production goods and labor markets

$$S[*_H]e_H(r) = s[*_F]e_F(z(r))\dot{z}(r) \quad (0 \leq z \leq \epsilon \leq r \leq \eta), \quad (16)$$

$$T_F[*_H]e_H(r) = t_F[*_F]e_F(z(r))\dot{z}(r) \quad (0 \leq z \leq \epsilon \leq r \leq \eta), \quad (17)$$

in which $\dot{z}(r) = dz(r)/dr$. $z(r)$ is a function which defines residential location r associated with office-work location z . The households locating at r commute to the firms over location $z(r) + \dot{z}(r)$. Eq. (16) means that the supply of household production goods over location r is equal to the demand of household production goods over location $z(r) + \dot{z}(r)$. Eq. (17) means that the supply of office work over the households at location r is equal to the demand of office work over the firms over $z(r) + \dot{z}(r)$.

(5) Commuting conditions

Given the number of commuting trips, the household works at the office in the CBD. The number of commuting trips of household, $G(r)$, is calculated as follows:

$$G(r) = g(z(r))e_F(z(r))\dot{z}(r)/e_H(r) \quad (0 \leq z \leq \epsilon \leq r \leq \eta). \quad (18)$$

We assume that no cross commuting occurs in the city (ref. [4])². The office-work location $z(r)$ is defined so as to satisfy the following equation:

$$\int_0^{z(r)} t_f(\xi)e_F(\xi)d\xi = \int_{\epsilon}^r T_F(\xi)e_H(\xi)d\xi \quad (0 \leq z \leq \epsilon \leq r \leq \eta). \quad (19)$$

Integrating Eq. (6) from z to ϵ with respect to the office-work location variable, z , the market wage function of office work at the equilibrium is defined as follows:

$$w_F(z, r) = w_F(\epsilon, r) + [\nu(r)\{G(r)T_c(z, r) - G(r)T_c(\epsilon, r)\} - (w_s(z, r) - w_s(\epsilon, r))S]/T_F \quad (0 \leq z \leq \epsilon \leq r \leq \eta). \quad (20)$$

Variable $w_F(\epsilon, r)$ represents the wage rate of office work at CBD boundary which is paid to the household at location r . The second term on the right side represents the commuting cost from office-work location r to the CBD boundary ϵ . The third term represents the difference between the revenue from home work at location z and that of location ϵ .

(6) Dividend of profits

The dividend of profits is calculated as follows:

$$d_v = (\bar{I}\bar{N} + \Pi_a)/\bar{P}. \quad (21)$$

² The optimality of the no cross commuting is guaranteed when the commuting frequency and the cost of transportation per distance are fixed. In our model, because these variable depend on the location, the optimality of the no cross commuting is not guaranteed. This is a remaining problem.

Figure 1 Communing Frequency

Figure 2 Average Office Work Time

Figure 3 Office and Housing Space

Figure 4 Land Rent

Figure 5 Household Production Goods

Figure 6 Office Work

Figure 7 Wage Rate and Price of Home Work

Figure 8 Leisure, Home Work Time and Commuting Time

Figure 9 Disposable Income

Figure 10 Composite Goods

Figure 11 Marginal Utility of Income, Time and Marginal Leisure

Figure 12 Commuting Tax

4. Socially Efficient Resource Allocation

The laissez-faire equilibrium doesn't attain social optimum, because there exists the negative externalities of commuting frequency in the city. To obtain a criteria for the social efficient resource allocation in the city, we consider the optimization problem which maximizes the welfare of the city subject to the land arbitrage condition, and the constraints on the allocation of time, land labor, population, and goods (ref. Appendix 1). Comparing the conditions of market equilibrium and the first-order necessary conditions of household and firm with the first-order conditions of social optimum, we get the following system of taxation³:

$$T_x(z) = \nu(r) T_c(z, r), \quad (22)$$

$$D_t = \int_0^\varepsilon g(z) T_x(z) e_F(z) dz / \bar{P}. \quad (23)$$

Eq.(22) means that the commuting tax is the incremental commuting time cost which is caused by increase in the number of commuting trips. The lump sum transfer D_t is redistributed to the households in the city.

5. A Numerical Model

A set of spatially consecutive infinitesimal location rings is approximated by a discrete location ring. We specify that the utility, labor efficiency, and production functions are all the constant elasticity of substitution (CES) form.

Utility Function :

$$U(l_H, T_L, X) = A[\alpha_1 l_H^{-\rho_u} + \alpha_2 T_L^{-\rho_u} + \alpha_3 X^{-\rho_u}]^{-1/\rho_u} (A, \alpha_m > 0, \rho_u > -1; m=1, 2, 3),$$

Efficiency and Production Functions :

$$S(T_H, T_F) = B[\beta_1 T_H^{-\rho_h} + \beta_2 T_F^{-\rho_h}]^{-1/\rho_h} (B, \beta_1, \beta_2 > 0, \rho_h, \rho_h' > -1),$$

$$F(l_F, s, f, y) = C[\gamma_1 l_F^{-\rho_F} + \gamma_2 s^{-\rho_F} + \gamma_3 f^{-\rho_F} + \gamma_4 y^{-\rho_F}]^{-1/\rho_F} (C, \gamma_m > 0, \rho_F, \rho_{F'} > -1; m=1, 2, 3, 4),$$

$$f(t_F, q) = D[\delta_1 t_F^{-\rho_f} + \delta_2 q^{-\rho_f}]^{-1/\rho_f} (D, \delta_1, \delta_2 > 0, \rho_f, \rho_{f'} > -1).$$

6. Simulation

The simulation of the laissez-faire equilibrium is, in essence, to find out a set of prices, $W = \{w_s(z, r), w_F(z, r), p_l(r), p_l(z)\}$, which satisfy Eqs. (8)-(21), provided that $T_x(z)$ and D_t are equal to zero. On the other hand, the simulation of the social optimum is to find out a set of prices, W , and a set of policy variables, $I = \{T_x(z), D_t\}$, which satisfy Eqs. (22) and (23). The algorithm for the simulation is based on the non-linear optimum theory and the tatonnement process. The values of the parameters of the simulation are listed in the Appendix 2⁴.

7. Results of the Simulation

In order to analyze the effects of the commuting tax, we derive two equilibria by

³ We can consider various systems of taxes and subsidies by applying Coase's theorem. This commuting tax is one of them.

⁴ In order to get an equilibrium which we expect, the values of parameters are chosen through the calibration procedure. However, it is necessary to simulate of various values of parameters in order to examine robustness of simulation results.

numerical simulation. One is the laissez-faire equilibrium and another is the social optimum. Results of the simulation are summarized in Figures 1-12. Solid lines depict laissez-faire equilibria and dash lines depict optimum equilibria which are sustained by the commuting tax. It is observed that the number of commuting trips, land rent, demand of office work, wage rate of office work, price of household production goods, disposable income, and marginal utility of time decrease as the location are far from the city center (Figs. 1, 5, 6, 7, 9, 11). On the other hand, the average office working time, office space, housing space, supply of household production goods and office work, leisure, commuting time, and consumption of composite goods of household increase as the location are far from the city center (Figs. 2, 3, 5, 8, 10).

By the commuting tax (Fig. 12), the number of commuting trips decreases everywhere (Fig. 1). Due to the decrease in the commuting frequency, the demand and supply of office work decrease everywhere in the CBD and suburbs, respectively (Fig. 6). However, the average office working time per trips of household increases everywhere in the city (Fig. 2). It means that the decrease in the number of commuting trips is substituted by the increase in the average working time in the office by the commuting tax. The demand and supply of goods which is produced by household increase everywhere in the city though the office work decreases (Fig. 5). By the decrease in the commuting frequency, the commuting time per period decreases everywhere in the suburbs and leisure increases everywhere in the suburbs (Fig. 8). The disposable income decreases everywhere in the suburbs because both the wage rate of office work and the price of home work decrease everywhere (Figs. 7, 9). Therefore, the consumption of composite goods of household decreases everywhere in the suburbs (Fig. 10). The office space decreases at the inner rings in the CBD and increases at the outer rings in the CBD (Fig. 3). As a result, the CBD boundary expands. The city boundary also expands than the laissez-faire equilibrium because the housing space increases everywhere in the suburbs. The equilibrium level of profit decreases and the equilibrium utility level increases.

8. Conclusion

In this paper, we examined the commuting behavior in the city with telecommuting. It was shown that the commuting frequency decreases as the location are far from the city center. On the other hand, the average office work time per household increases as the location are far from the city center. We derived the commuting tax which attains the social optimum and analyzed the effects of the commuting tax on the spatial structure of the city. At the optimum, the commuting frequency decreases and the office working time decreases in the city. The home working time increases in the suburb and the housing space increases in the suburbs. As a result, the city expands than the laissez-faire equilibrium. Remaining task is to endogenously solve trip pattern of commuting between households and firms. A possible expansion is to introduce network externalities of telecommunication system and agglomeration economies of interaction of firms.

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Appendix 1

We consider the optimization problem which maximizes the welfare of the city, which is the sum of the utility, subject to the arbitrage condition for the land markets in the CBD and suburbs, and to the constraints on the allocation of time, land, population, labor, goods, and services. Given $z(t)$ and $\dot{z}(=dz(t)/dt)$ ($0 \leq z(t) \leq \varepsilon \leq t \leq \eta$; $z(\varepsilon)=0$, $z(\eta)=\varepsilon$), we formulate the following optimization problem :

$$\max_{\{l_h(t), T_L(t), T(t), X(t), l_F(z(t)), s(z(t)), l_F(z(t)), e_H(z(t)), e_F(z(t)), g(z(t)), G(t), \varepsilon, \eta; z(t), \dot{z}\}} \int_{\varepsilon}^{\eta} \bar{U} e_H(t) dt$$

subject to

$$\begin{aligned} \dot{M}_h(t) &= -G(t)e_H(t), \\ \dot{M}_f(t) &= g(z(t))e_F(z(t)), \\ G(t)e_H(t) - g(z(t))e_F(z(t)) &= 0, \\ \{U(l_h(t), T_L(t), X(t)) - \bar{U}\}e_H(t) &\geq 0, \\ \{T - T_L(t) - T_H(t) - T_F(t) - G(t)T_c(z(t), t)\}e_H(t) &\geq 0, \\ 2\pi z(t) - e_F(z(t))l_F(z(t)) &\geq 0, \\ 2\pi t - e_H(t)l_h(t) &\geq 0, \\ S(T_H(t), T_F(t))e_H(t) - s(z(t))e_F(z(t))\dot{z} &\geq 0, \end{aligned}$$

and

$T_F(t)e_H(t) - t_F(z(t))e_F(z(t))\dot{z} \geq 0$ for all t such that $\varepsilon \leq t \leq \eta$,

and also subject to

$$\begin{aligned} \bar{F}^t - \int_{\varepsilon}^{\eta} M_f(z(t)) + M_h(t) dt &\geq 0, \\ \bar{F} + \int_{\varepsilon}^{\eta} F(\cdot) e_F(z(t)) \dot{z} dt - \int_{\varepsilon}^{\eta} x(t) e_H(t) dt \\ - \int_{\varepsilon}^{\eta} y(z(t)) e_F(z(t)) \dot{z} dt - 2\pi R_A \int_{\varepsilon}^{\eta} t + z(t) \dot{z} dt &\geq 0, \\ \bar{F}^c - \int_{\varepsilon}^{\eta} z(t) F(\cdot) e_F(z(t)) \dot{z} dt &\geq 0, \\ \int_{\varepsilon}^{\eta} e_H(t) dt - \bar{P} &= 0, \\ \bar{N} - \int_{\varepsilon}^{\eta} e_F(z(t)) \dot{z} dt &= 0, \\ M_f(z(\eta)) &= M_h(\varepsilon), \\ M_f(z(\varepsilon)) &= M_h(\eta) = 0, \end{aligned}$$

in which $F(\cdot) = F(l_F(z(t)), s(z(t)), f[t_F(z(t)), g(z(t))], y(z(t)))$. $\bar{F}^t$, $\bar{F}$, and $\bar{F}^c$ are auxiliary variables for the formulation of the problem. $M_h(t)$ and $M_f(z(t))$ are the number of commuting trips through the ring at location t in the suburbs and at location $z(t)$ in the CBD, respectively. These are defined as follows :

$$M_h(t) = \int_t^{\eta} G(\xi) e_H(\xi) d\xi, \quad M_f(z(t)) = \int_{\varepsilon}^t g(z(\xi)) e_F(z(\xi)) \dot{z} d\xi.$$

Appendix 2

Values of the parameters are specified as follows :

Household

$$\begin{aligned} \alpha_1 &= 0.2; & \alpha_2 &= 0.6; & \alpha_3 &= 0.2; & \rho_u &= 0.8; & T &= 120 & R_A &= 2 & \bar{P} &= 200; \\ B &= 1; & \beta_1 &= 0.8; & \beta_2 &= 0.2; & \rho_u &= 0.1; & \rho_{h'} &= 0.11 & p_t &= 0.1; & \bar{v} &= 5. \end{aligned}$$

Firm

$$\begin{aligned} C &= 15; & \gamma_1 &= 0.25; & \gamma_2 &= 0.25; & \gamma_3 &= 0.25; & \gamma_4 &= 0.25; \\ \rho_F &= 0.1; & \rho_{F'} &= 0.12; & p_c &= 0.04; & \bar{N} &= 40; \\ D &= 1; & \delta_1 &= 0.8; & \delta_2 &= 0.2; & \rho_f &= 1.1; & \rho_{f'} &= 1.12. \end{aligned}$$