Bid Values and their Role in Land Development Over Time and Under Uncertainty[†]

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Abstract

Recently, Batabyal (2000) has demonstrated that when the land development question is studied in an intertemporal and random setting, it is possible that a landowner will never develop (always preserve) his or her land. This result arises because the landowner in Batabyal (2000) maximizes the likelihood of accepting the best possible bid of development. What happens when a landowner employs the following decision rule: Develop land as long as the dollar value of a bid exceeds a stochastic reservation level of revenue? In this paper, we provide an interesting answer to this question. First, we establish that the likelihood of developing land with this decision rule is always positive. Even so, we point out that on average, a landowner who uses this decision rule will always end up preserving his or her land.

Keywords: Decision Rule, Dynamics, Land Development, Preservation, Reservation Revenue Level
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1. Introduction

The subject of land development over time and under uncertainty has interested natural resource and urban economists at least since the publication of Weisbrod (1964). In later work, Arrow and Fisher (1974) and Henry (1974) identified a concept known as option value. The so called Arrow-Fisher-Henry (AFH) concept of option value, or quasi-option value as it is sometimes called, tells us that when development is both indivisible and irreversible, a landowner who ignores the possibility of procuring new information about the consequences of such development will unfailingly undervalue the benefits of preservation and hence skew the binary choice develop or preserve decision in favor of development.

Does this AFH result hold when the land development question is studied in a multi-period setting with uncertainty? From a mathematical standpoint, it is salient to understand that the AFH analysis is conducted in a two period model. Consequently, the pertinent development question in this mathematical setting is "Do I develop today or tomorrow?" In contrast, in a multi-period mathematical setting, the appropriate question is "When do I develop?" This follows from the fact that a landowner's decision problem now is not over

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two periods but over $n > 2$ periods.

Markusen and Scheffman (1978), Arnott and Lewis (1979), and Capozza and Helsley (1989) have all analyzed models of land development. The mathematical models used to study land development in these papers are multi-period and deterministic (non-stochastic) in nature. However, when the appropriate development decision is irreversible, the use of a certainty or non-stochastic framework will bias conclusions about the timing of land development. In fact, as we have learned from the investment under uncertainty literature,¹ uncertainty will typically add an option value to undeveloped land and thereby postpone the development of land from, say, agricultural to urban use. Accordingly, if we are to understand when land ought to be developed in the presence of irreversibilities, it is crucial that we expressly account for uncertainty.

Recently, Titman (1985), Capozza and Helsley (1990), and Batabyal (1996, 1997, 2000) have studied the subject of land development under uncertainty. In Titman's (1985) model, he first assumes that the value of vacant land in the first period exceeds the wealth of a landowner who wishes to construct a building. He then shows that a wealth maximizing landowner is better off leaving his or her land vacant. However, the bulk of Titman's (1985) mathematical analysis is carried out in a two period model. Because of this feature, the Titman (1985) paper is essentially unable to address the multi-period nature of the land development question.

The mathematical structure of the Capozza and Helsley (1990) paper is quite different. They analyze a "first hitting time" model.² With the help of this stochastic model, these authors show that land ought to be converted from rural to urban use at the first moment in which the land rent exceeds the reservation rent. Batabyal (1996, 1997) uses a particular stochastic process, namely a Markov decision process, to provide, respectively, a discrete-time, multi-period and a continuous-time, multiperiod examination of the "When do I develop land" question. In the discrete-time analysis of Batabyal (1996), the one stage look ahead policy (OSLAP) is used to decide when a random "revenue from development" process ought to be halted. Analogously, in the continuous-time analysis of Batabyal (1997), the infinitesimal look ahead stopping rule (ILASR) is utilized to determine when a stochastic "revenue from development" process should be terminated.

The paper that is closest to our paper is Batabyal (2000). From a mathematical standpoint, this Batabyal (2000) paper is unusual because it uses a probabilistic objective function to study the decision problem of a landowner who wishes to answer the "When do I develop land" query. Specifically, the landowner being studied maximizes "the *probability* of accepting the best possible offer of development, given that these offers are received sequentially over time" (Batabyal, 2000, p. 150, italics ours). In this mathematical setting, Batabyal (2000) demonstrates that it may be optimal for the landowner to always preserve or never develop his or her land. We stress that this conclusion has nothing to do with the landowner's time and decision making horizon. Rather, it arises because the landowner's optimization problem involves the maximization of a probability.

In contemplating the question of when—if ever—to develop land, it seems sensible to

¹ For more on this literature, see Pindyck (1991), Dixit and Pindyck (1994), and Hubbard (1994).

² For additional details on this, see Dixit and Pindyck (1994, pp. 83-84) and Ross (1996, pp. 363-366).

suppose that a landowner would have a reservation level of revenue from development in mind. Put differently, this landowner will develop his or her land if and only if (s)he receives a bid to develop land that surpasses this reservation level of revenue. Consequently, in this paper, we wish to determine the outcome when a landowner uses the following decision rule: Develop land as long as the dollar value of a bid exceeds a *random* reservation level of revenue.

Let us now succinctly explain why the reservation level of revenue in this paper is random. Fixed or deterministic reservation levels have been used to analyze questions in many different branches of economics such as contract theory (see Tirole (1986)) and labor economics (see Groot and Oosterbeek (1994)). Even in natural resource economics, the question of land development over time and under uncertainty with a fixed reservation revenue level has been analyzed in Batabyal (2002). Nevertheless, to the best of our knowledge, the impact of a stochastic reservation revenue level on the land development decision has *not* been analyzed previously in the literature. In addition, it is our contention that it is helpful to think of a stochastic reservation revenue level when a landowner is considering the development of a plot of land that has not been on a land market for quite some time. This is the second reason for concentrating on a stochastic reservation revenue level in this paper.

Specifically then, the primary objective of this paper is to provide an answer to the hitherto unanalyzed issue discussed in the previous two paragraphs. The precise question of interest is this. What happens when a landowner uses the following decision rule: Develop land as long as the dollar value of a bid surpasses a stochastic reservation level of revenue? As we shall soon see, in an intertemporal and stochastic setting, an interesting conclusion arises and that conclusion is this: Although the likelihood of land development is always positive, in an expected waiting time sense, our landowner will wait for an infinite amount of time before developing his or her land. In other words, on average, a landowner who utilizes the above decision rule will always preserve his or her land. We will also point out that a key part of the development decision depends significantly on whether the reservation level is deterministic or stochastic.

The rest of this paper is organized as follows: Section 2 provides a detailed discussion of the theoretical framework and the results of this paper. Section 3 concludes and offers suggestions for future research on land development over time and under uncertainty.

2. The Theoretical Framework

2.1. Preliminaries

Consider an individual who owns a plot of land. The decision problem faced by this landowner concerns when, if ever, to develop his or her plot of land. Keeping with the AFH tradition, we assume that the development decision is both indivisible³ and irreversible. This means that the possibility of partial development of the plot is disallowed and a decision to develop, once made, cannot be changed. The landowner solves his or her problem in an intertemporal and stochastic setting. The setting is stochastic because the decision to

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³ For more on divisible land development, see Epstein (1980), Hanemann (1989), and Batabyal (1999).

develop depends primarily on the receipt of bids to develop land. These bids materialize sequentially over time, in accordance with some independent and identically distributed stochastic process. In other words, a bid of a particular dollar value to develop land is received in time period t with a certain likelihood, independent of preceding or subsequent bids. These bids are received over time, one bid per time period. Therefore, the decision making framework of our paper is dynamic in the sense that this framework requires the landowner to decide when land ought to be developed on the basis of his or her observation of the intertemporal and random bid generation process. The actual source of these bids is not critical to our analysis. It could be the consequence of activities undertaken by private land developers or it could be the outcome of tangible governmental policies.

Upon receipt of a bid, our landowner must determine whether to accept the bid (develop land) or to decline the bid (preserve land) and wait for additional bids. If the landowner accepts a bid, i.e., if (s)he develops his or her land, then the question of subsequent bids is superfluous. Consequently, the stochastic bid generation process ceases. We now need to stipulate a decision rule for our landowner. A straightforward decision rule, and the one that we analyze in this paper, is the following: Let $D_0, D_1, D_2, D_3, \dots$ be independent and identically distributed, non-negative random variables that denote the dollar values of the bids that are received by our landowner sequentially over time. In other words, D_0 is the dollar value of the first bid, D_1 is the dollar value of the second bid, and so on. We suppose that our landowner's stochastic reservation level of revenue is D_0 , the dollar value of the first bid.⁴ Then, our landowner's decision rule is to accept the first bid that exceeds D_0 in dollar terms.

From the perspective of our landowner, the state of affairs delineated in the previous two paragraphs involves acting in a sequential decision making framework. To this end, let us denote the common and the continuous distribution and the density functions of these random bids $D_0, D_1, D_2, D_3, \dots$ by $F(\cdot)$ and $f(\cdot)$, respectively. Now, given our landowner's decision rule (see the previous paragraph), let N be the first index k for which $D_k > D_0$. Put differently, $N=1$ if $D_1 > D_0$, $N=2$ if $D_1 \leq D_0$ and $D_2 > D_0$, and so on. In words, N is an index for the first bid whose dollar value exceeds the reservation level of revenue D_0 . To comprehend the meaning of the index N in concrete terms, consider the following example: If $N=50$, then looked at in terms of the waiting period, our landowner has to wait forty-nine time periods—because one bid is received every time period—before his or her land is developed. Looked at in terms of the number of rejected bids, our landowner turns down forty-nine bids and then accepts the fiftieth bid to develop his or her land.

The reader will note that the mathematical setting of our paper is different from the setting employed in the papers discussed in section 1 in three important ways: First, as far as the bids are concerned, we work with a general stochastic process and we do not make any distributional assumptions. Second, our analysis involves working with a stochastic—and not a deterministic—reservation level of revenue. Third, as far as the land development decision is concerned, we analyze the formal properties of a previously unstudied decision rule.

⁴ The reader will note that our landowner's reservation level of revenue is obtained from the stochastic bid generation process.

2.2. Analysis

To comprehend the nature of the land development decision in the setting of this paper, let us first compute the probability mass function $Prob\{N=n\}$ for the positive and integer valued random variable N . To this end, note that

$$Prob\{N>n\} = \int_0^\infty [1-F(d')]^n f(d') dd' = - \int_0^\infty [1-F(d')]^n d[1-F(d')]. \quad (1)$$

Now substitute $y=[1-F(d')]$ in the right most integral in equation (1). This gives

$$Prob\{N>n\} = \int_0^1 y^n dy = \frac{1}{n+1}. \quad (2)$$

Using equation (2) we can infer that

$$Prob\{N=n\} = Prob\{N>n-1\} - Prob\{N>n\} = \frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}. \quad (3)$$

Equation (3) gives us the probability mass function for the random variable N . Inspecting equation (3), we see four interesting features of the land development decision. First, for any finite N , the probability of land development is always positive. Second, the maximal likelihood of land development is 0.5 and this probability occurs when $N=1$. Third, as N increases, the probability of land development falls over time. Finally, in the limit as $N \rightarrow \infty$, the likelihood of land development is zero. Together, these four features tell us that the likelihood of land development is higher early in the landowner's time and decision making horizon. For instance, the probability that the tenth bid ($N=10$) will result in development is only $1/110$ which is approximately equal to 0.009.

Thus far we have seen that, for all practical purposes, i.e., for all finite N , the probability of land development is positive. However, one can certainly ask the following question: What is the expected wait before our landowner's land is developed? In particular, given the limiting result of the previous paragraph, is it possible that in an expected value sense our landowner will never develop his or her land? To answer these questions, we now compute the mathematical expectation of N , $E[N]$. Because N is a non-negative and integer valued random variable, we can use equation (2) and equation 5.2 in Taylor and Karlin (1998, p. 44) to compute its expectation. We get

$$E[N] = \sum_{n=1}^{\infty} Prob\{N \geq n\} = \sum_{n=0}^{\infty} Prob\{N > n\} = \sum_{n=0}^{\infty} \frac{1}{n+1} = \infty. \quad (4)$$

Equation (4) answers the two queries stated in the previous paragraph. Specifically, we see that even though the probability of land development for all finite N is positive, the expected wait until the land in question is developed is infinity. Consequently, in an expected waiting time sense, our landowner will never develop (always preserve) his or her land. Batabyal (2000) showed that when a landowner maximizes the probability of accepting the best bid of development, it is possible that (s)he will never develop (always preserve) his or her land. Our equation (4) result complements the Batabyal (2000) finding. Specifically, this result tells us that even when a landowner's focus is not on maximizing the probability of accepting the best bid of development, the use of the decision rule delineated in this paper can still lead, in an expected waiting time sense, to a preservation bias in the decision to

develop land. In this sense, our last result runs counter to the AFH finding that there will often be a development bias in the decision to develop land. Does this preservation bias exist when the reservation level of revenue is deterministic and not stochastic? Batabyal (2002) and our own work in progress tell us that the answer to this question is no. In other words, in an intertemporal and stochastic setting, the existence of a preservation or a development bias in the land development decision is very much a function of the properties of the underlying decision rule.

2.3. Practical Considerations

Generally speaking, many contemporary problems in regional science involve striking an appropriate balance between the development and the preservation of land. In this regard, our paper has shown that the decision rule one selects affects the balance between land development and preservation. Therefore, the type of analysis that we've conducted in this paper does shed light on a number of practical regional and urban problems. We now briefly discuss two such problems.

First, in determining the optimal ways of dealing with the problem of urban sprawl, it is important to comprehend the impacts that alternate land development policies have on the sprawl phenomenon. This has been noted, *inter alia*, by Dascher (2001) in his discussion of urban sprawl in Dublin, Ireland, and by Danielsen *et al.* (1999) in their discussion of urban sprawl in the US states of Maryland and Oregon. Our paper contributes to this literature by pointing out the effects that a particular (bid driven) policy rule has on the decision to develop land. Moreover, if we identify increased development with increased sprawl, then the analysis in this paper tells us that the sprawl phenomenon is more likely to occur relatively early in a landowner's time and decision making horizon.

Second, when analyzing the pros and cons of alternate land development policies, it is salient to understand what effects these alternate policies have on the desire expressed by many communities to preserve land. In this regard, Larson *et al.* (2001) have pointed out that although south-eastern Pennsylvania in the US has seen increasing urbanization, communities in this region are nonetheless very interested in preserving some agricultural land. Similarly, Fleming *et al.* (2001) have documented the interest shown by area residents in preserving land in a historic New Mexico community in the US, namely, the La Cienga Valley in Santa Fe County. Our paper contributes to this literature about land preservation by showing that the use of this paper's (bid driven) land development rule results in a distinct preservation bias. Indeed, as we showed in section 2.2, on average, a landowner who uses the land development rule studied in this paper will always end up preserving his or her land.

3. Conclusions

In this paper we studied the land development question in an intertemporal and stochastic framework. We used this framework to answer the following hitherto unexamined question: What happens when the focus of a landowner is not on maximizing a likelihood but instead on developing land as long as the dollar value of a bid exceeds a stochastic reservation level of revenue D_0 ? Our analysis revealed that for all finite N , the likelihood of land development is always positive. This notwithstanding, in an expected waiting time sense,

our landowner waits for an infinite amount of time before developing his or her land. What this means is that, on average, our landowner ends up never developing (always preserving) his or her land.

The analysis of this paper can be extended in a number of directions. In what follows, we suggest two potential extensions. First, instead of working with an iid stochastic process, one could analyze the multi-period land development question in a setting in which the random bids are independent but not identically distributed over time. Second, if the landowner learns about the statistical properties of the bid generation process, then it is possible that this landowner will eventually know the distribution from which the development bids are generated. One could study the effect of this knowledge on the "When to develop land" question. Papers that analyze these facets of the problem will provide additional insights into the criteria that govern the development of land over time and under uncertainty.

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