An Analysis of Economic Welfare under the Global Environment and International Trade†

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Abstract


In this paper, in a similar model to theirs, we consider the global environment which is affected by the pollution generated from the production activity of a manufacturing good in each country. Then, under the assumption that the productivity of an environmentally sensitive good (agricultural good) is influenced by the global environment differently between countries, we derive the following results. If the generating degree of pollution is low in a country where the agricultural production is more sensitive to the environment, both countries may lose from trade. On the other hand, if the generating degree of pollution is high in that country, there is a possibility that both countries gain from trade.

1. Introduction


In this paper, on the basis of these analyses, we generalize the existing studies further by considering the global environment which is common to both countries. Moreover, we assume that the productivity of an environmentally sensitive good (agricultural good) is

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affected by the global environment differently between countries. Then we analyze the
pattern of trade and the welfare effect of trade. The pattern of trade is determined by the
comparative production costs between countries. Therefore, in this paper, the difference in
the productivity of the environmentally sensitive good between countries plays a key role in
determining the pattern of trade as well as the pattern of production. On the other hand,
together with the pattern of production, the generating degree of pollution in each country
has an influence on the gains from trade. A simple intuition leads to the following. If trade
shifts production of the pollution emitting good to a country where the generating degree of
pollution is high, both countries may lose from trade through the deterioration of the global
environment. Conversely, if trade shifts it to a country where the generating degree of
pollution is low, both countries may gain from trade.

We will make a precise analysis and derive the following results in this paper. Independ-ently of the generating degree of pollution, a country with the high productivity of the
environmentally sensitive good has a comparative advantage in the production of that good.
Then the country exports the environmentally sensitive good and imports the pollution
emitting good. If the generating degree of pollution is low in a country with the high
productivity of the environmentally sensitive good, then both countries may lose from trade.
On the other hand, if the generating degree of pollution is high in a country with the high
productivity of the environmentally sensitive good, then both countries may gain from trade.
As a whole, it can be said that both countries would have a gain from trade only in the case
where the production of the pollution emitting good is shifted to a country with the
sufficiently low generating degree of pollution, but otherwise there is a possibility that trade
is harmful to at least one country.

The following section presents the model. In Section 3, we treat autarky. The trading
economy is considered in Section 4. In Subsection 4.1, we treat the case where the generat-
ing degree of pollution is the same between countries. In Subsection 4.2, we consider the
case where the generating degree of pollution is lower in a country where the agricultural
production is more sensitive to the global environment than the other country. Subsection
4.3 is devoted to the converse case. The last section is devoted to our conclusion.

2. The Model

We consider a world economy with two countries, Home and Foreign, and with two
goods labeled by $X$ and $Y$. Only labor is used for the production of those goods. The
productivity of good $Y$ is affected by the global environment\(^1\). So in the following, we call
good $Y$ as an agricultural good or an environmental good. The global environment is
supposed to be the type of ozone layer, for example. As a by-product of good $X$, pollution
is generated and it deteriorates the global environment. So we call good $X$ as a manufactur-
ing good or a polluting good. We now explain the Home economy. In the following
analysis we denote variables with and without "*" as those of Foreign and Home, respective-
ly. The state of the environment varies throughout time according to the following dynamic

and Suga [3] (2002) assumed that the productivity of the agricultural sector is affected by the
environment of their own countries.
process:

\[ \dot{K} = g(K - \bar{K}) - Z - Z^*, \]  

(1)

where \( K \) is the state of the global environment, \( \dot{K} \) is the differential of \( K \) by time, \( g(>0) \) is the recovering rate of \( K \), \( Z \) is the flow of pollution and \( \bar{K} \) is the maximum level of \( K \).

The production function of each sector is represented as

\[ X = L_X, \]  

(2)

\[ Y = K^e L_Y, \]  

(3)

where \( X(Y) \) is the output of the polluting (environmental) good and \( L_X(L_Y) \) is the labor input in the polluting (environmental) good. \( e \) is a parameter with \( 0 < e < 1 \), which affects the labor productivity of the environmental good.

We assume that \( \lambda \) unit of pollution is emitted per unit of output of the polluting good as follows:

\[ Z = \lambda X, \]  

(4)

where \( \lambda \) is a positive parameter.

When the labor endowment is \( L \), the full employment condition is imposed as

\[ L_X + L_Y = L \]  

(5)

Assuming that all consumers are identical, the aggregated utility function is given by

\[ U = D_X^\alpha D_Y^{1-\alpha} \]  

(6)

where \( \alpha \) is a parameter with \( \alpha \in (0, 1) \) and \( D_X(D_Y) \) is the demand for the polluting (environmental) good. Thus, we assume in this paper that pollution gives damage to production but not to utility.

3. Autarky

We begin with the analysis of autarky in Home. First we consider the supply side.

Substituting (2) and (3) for (5), the short run production possibility frontier under given \( K \) is obtained as

\[ X + Y/K^e = L. \]  

(7)

By (1) and (4), when \( \dot{K} = 0 \) the value of \( K \) becomes

\[ K = \bar{K} - (\lambda/g)X - (\lambda^*/g)X^*. \]  

(8)

Throughout the paper, we assume that the labor endowment is the same in both countries. We also assume that \( 1 \leq \bar{K} - (\lambda/g)L - (\lambda^*/g)L \) for the following analysis. Substituting (8) for (7), the long-run production possibility frontier in Home is given by

\[ Y = (L - X)(\bar{K} - (\lambda/g)X - (\lambda^*/g)X^*)^e. \]  

(9)

Supposing the environmental good to be a numeraire, we have

\[ \text{Throughout the paper, we treat the case where at least one country produces the environmental good. So, we have } \bar{K} - (\lambda/g)L - (\lambda^*/g)L < K = \bar{K} - (\lambda/g)X - (\lambda^*/g)X^*. \]  

(9)

Therefore, we assume that \( 1 < \epsilon^* < \epsilon \) for the following analysis.
from the profit maximization, where \( p \) and \( w \) are, respectively, the relative price of the polluting good and the wage rate. When both goods are produced, (10) and (11) yield

\[
p = K^e.
\]

By (8) and (12), if both goods are produced in the steady state, the relative price is expressed as

\[
p = (K - (\lambda/g)X - (\lambda^*/g)X^*)^e.
\]

On the demand side, maximizing the utility subject to the budget constraint that \( pD_x + D_y = wL \), the demand for each good is obtained as

\[
D_x = awL/p, \tag{14}
\]
\[
D_y = (1-a)wL. \tag{15}
\]

When both goods are produced, (14) and (15) become

\[
D_x = aL, \tag{14'}
\]
\[
D_y = (1-a)K^eL, \tag{15'}
\]

respectively, in view of (10) and (11).

When both countries are in the autarkic equilibrium, both goods must be produced to clear the good markets in each country. Throughout the paper, we assume that the utility function is the same in both countries. Then, by (8) and (14'), the steady state value of \( K \) is obtained as

\[
K = K_a \equiv K - (\lambda L/g)(\lambda + \lambda^*), \tag{16}
\]

where \( K = K_a \) is the steady state value of the global environment when both countries are in the autarkic equilibrium. By (16) and (12), the autarkic equilibrium price becomes

\[
p_a = (K_a)^e, \tag{17}
\]

where \( p_a \) is the relative price in the autarkic equilibrium. As shown in Copeland and Taylor [2](1999), when both countries are in the autarkic equilibrium, \( K = K_a \) is a globally stable and unique.\(^3\)

We have a similar argument on Foreign.

### 4. Trade between Two Countries

We now consider trade between two countries. Throughout the paper, the productivity of the environmental good is assumed to be higher in Home than in Foreign, that is \( \varepsilon > \varepsilon^* \). We also assume that the environment is global and both countries share the common environment.\(^4\) In this section, we compare the welfare in the autarkic equilibrium with that

\(^3\) Using (4), (14') and the fact that \( D_x = X \), we have \( Z = \lambda aL \) for Home. Similarly, we have \( Z^* = \lambda^* aL \) for Foreign. So, when both countries are in autarky, \( K \) evolves according to \( K = g(K - K) - \lambda aL - \lambda^* aL \). From this equation, it is shown that the autarkic equilibrium is globally stable.

\(^4\) For example, if we regard the global environment as the ozone layer, the degree in which it affects to the production is different between countries.
of the trading equilibrium.

Since $K$ is common to both countries, the relationship of the autarkic prices between countries is expressed as

\[ p_a = (K_a)^* > (K_a)^* = p_a^*, \]

from (17) and the assumption that $\varepsilon > \varepsilon^*$. Therefore, in trade, Home exports the environmental good and Foreign exports the polluting good. As seen in (18), only the difference between $\varepsilon$ and $\varepsilon^*$ determines the pattern of trade. That is, the amount of the labor endowment and the generating degree of pollution in any country do not affect the pattern of trade.

We will show that the pattern of trade depends on the degree of expenditure by the following proposition.

**Proposition 1**

Suppose that the productivity of the environmental good is more sensitive to the global environment in Home than in Foreign ($\varepsilon > \varepsilon^*$) and that these two counties are in trade. Then, the trading equilibrium is globally stable and unique, and in this equilibrium, the patterns of production are determined as follows: When the expenditure share on the polluting good, $a$, is $1/2 < a < 1$, Home produces both goods and Foreign specializes in the polluting good. When $a_K \leq a \leq 1/2$, Home specializes in the environmental good and Foreign specializes in the polluting good. When $0 < a < a_K$, Home specializes in the environmental good and Foreign produces both goods, where $a_K = [(K-(\lambda^*/g)L)^{\varepsilon^*}+1]^{-1}$.

**Proof.** See Appendix.

In the following analysis, we denote the relative international price of the polluting good as $p_l$ and express the demands for goods $X$ and $Y$ in the autarkic (trading) equilibrium as $D_X$, $D_Y$, $(D_X, D_Y)$ respectively. Then, from (14'), (15') and (16), we have $D_X = D_Y = aL$, $D_Y = (1-a)(K_a)^*L$ and $D_Y^* = (1-a)(K_a)^*L$. Furthermore, we define $K_X$, $K_S$ and $K_Y$ to be the states of the global environments in the trading equilibrium where both countries produce the polluting good, where both countries specialize and where both countries produce the environmental good, respectively. More precisely, from the proof of Proposition 1, we can show that $K_X = K-(\lambda^*/g)L(2-\varepsilon-\varepsilon^*)/(\lambda^*/g)L$, $K_S = K-(\lambda^*/g)L$ and $K_Y = K-(\lambda^*/g)L((K_Y)^{\varepsilon^*}+1)$.

### 4.1 The Case where the Generating Degree of Pollution is the Same between Countries

The purpose of this section is to see how the difference in the productivity of the environmental good between countries itself affects the welfare effect of trade. So we suppose that $\varepsilon > \varepsilon^*$ but countries are identical in the other aspects.

**Case 1: Both Countries Produce the Polluting Good** ($1/2 < a < 1$)

Comparing welfare in the autarkic equilibrium with that of the trading equilibrium, we can see that they are the same in Home while it improves by trade in Foreign. The reason is as follows: In the trading equilibrium, using (10) and (11), we obtain $w = w^* = p_l = (K_a)^*$. So, from (14) and (15), the demands for both goods in both countries can be expressed as $D_X$.
= aL, \( D^*_x = (1 - a)(K_x)^*L \), \( D^*_y = aL \) and \( D^*_y = (1 - a)(K_x)^*L \). Now we will show that \( K_a = K_x \).

From the proof of Proposition 1, we have \( K_x = \vec{K} - (\lambda^* / g)L(2a - 1) - (\lambda^* / g)L \). So using (16), \( K_a - K_x = (L / g)(a - 1)(\lambda - \lambda^*) \). Thus, we have \( K_a = K_x \) in view of the assumption that \( \lambda = \lambda^* \). So it holds that \( D^a_x = D^*_x \), \( D^a_y = D^*_y \), \( D^a_{y} = D^*_y \) and \( D^a_{y} < D^*_y \).

**Case 2: Both Countries Specialize \( (a_K \leq a \leq 1 / 2) \)**

The welfare effect of trade is ambiguous in both countries. The reason is as follows: From the proof of Proposition 1, in the trading equilibrium, the international price becomes \( p_i = (K_s)^* \). Also from (10) and (11), we have \( w = (K_s)^* \) and \( w^* = p_i \). Then using (14) and (15), we can verify that \( D^i_x = a(K_s)^*L / p_i = (1 - a)L \), \( D^i_y = (1 - a)(K_s)^*L \), \( D^i_{y} = aL \) and \( D^i_{y} = (1 - a)p_iL = a(K_s)^*L \). Now we will show that \( K_a > K_s \). From the proof of Proposition 1, we have \( K_s = \vec{K} - (\lambda^* / g)L \). So we can show that \( K_a - K_s = (L / g)(a(\lambda + \lambda^*) + \lambda^*) \) by using (16). Then, from the assumption that \( \lambda = \lambda^* \) and \( a \leq 1 / 2 \), we have \( K_a > K_s \). Therefore, we obtain \( D^a_x = D^*_x \), \( D^a_y > D^*_y \) and \( D^a_{y} = D^*_y \). The relationship between \( D^a_{y} \) and \( D^*_y \) is, however, ambiguous.

**Case 3: Both Countries Produce the Environmental Good \( (0 < a < a_K) \)**

The welfare effect of trade in Home is ambiguous. In Foreign, however, the level of welfare in the trading equilibrium is lower than that of the autarkic equilibrium. We can give the following reason. In the trading equilibrium, from (10) and (11), we have \( w = (K_y)^* \) and \( w^* = p_i \). Then the use of (14) and (15) yields \( D^i_x = a(K_y)^*L / p_i = (1 - a)L \), \( D^i_y = (1 - a)(K_y)^*L \), \( D^i_{y} = aL \) and \( D^i_{y} = (1 - a)p_iL = a(K_y)^*L \). Now we will show that \( K_a > K_y \). Using (16), we have \( K_a - K_y = \vec{K} - (aL / g)(\lambda + \lambda^*) - \{ \vec{K} - (\lambda^* / g)aL((K_y)^* + 1) \} = -(aL / g)(\lambda - \lambda^*(K_y)^* - c) \). Then, taking account of the assumption that \( \lambda < \lambda^* \) and \( c > c^* \), we obtain \( K_a > K_y \). So we have \( D^a_y < D^*_y \), \( D^a_y > D^*_y \), \( D^a_{y} = D^*_y \) and \( D^a_{y} > D^*_y \).

Summing up the above results, we can establish the following proposition.

**Proposition 2**

Suppose that the productivity of the environmental good is more sensitive to the global environment in Home than in Foreign \( (\varepsilon > \varepsilon^*) \). Also suppose that the generating degree of pollution is the same between two countries \( (\lambda = \lambda^*) \). Then, comparing the welfare in the autarkic equilibrium with that of the trading equilibrium, the following relationships hold depending on the expenditure share on the polluting good. If \( 1 / 2 < a < 1 \), Home welfare is the same between the situations in the trading equilibrium and in the autarkic equilibrium. On the other hand, the level of Foreign welfare is higher in the trading equilibrium than in the autarkic equilibrium. If \( 0 < a < a_K \), however, the level of Foreign welfare is lower in the trading equilibrium than in the autarkic equilibrium.

We can explain Proposition 2 intuitively. When \( 1 / 2 < a < 1 \), Home produces both goods and Foreign specializes in the polluting good. In this case, the demand for the polluting good is the same between before and after trade in both countries. The relative productivity of the polluting good is higher in Foreign than in Home. So by opening trade, the production
of the polluting good is shifted to Foreign. However, since the generating degree of pollution is the same between countries, the state of the global environment does not change by trade. So noting that the relative international price is equal to the Home productivity of the environmental good, that is $p_i = K_i$, the Home welfare stays the same after trade. On the other hand, since the Home productivity of the environmental good is higher than that of Foreign, the relative price of the polluting good in trade becomes higher than that of the Foreign autarkic relative price. Thus, Foreign, which is the polluting good exporting country, gains from trade.

When $0 < a < a_K$, Home specializes in the environmental good and Foreign diversifies. Then the demand for the polluting good in Foreign is the same between before and after trade. However, the demand for the polluting good in Home increases by trade. So, due to an increase in the total amount of pollution, the state of the global environment becomes worse. Because of the deterioration of the environment, the relative international price of the polluting good, which is the export good for Foreign, falls. So Foreign loses from trade.

4.2 The Case where the Generating Degree of Pollution is Lower in Home than in Foreign

Now we analyze the case where the generating degree of pollution is lower in Home than in Foreign. That is, we investigate the case where $\lambda < \lambda^*$. We maintain the assumption that $\varepsilon > \varepsilon^*$. Comparing the welfare in the autarkic equilibrium with that of the trading equilibrium, we have the following results.

Case 1: Both Countries Produce the Polluting Good ($1/2 < a < 1$)

The level of Home welfare in the trading equilibrium is lower than that of the autarkic equilibrium. In Foreign, however, the welfare effect of trade is ambiguous. We can give the following reason. The demands for both goods in both countries in the trading equilibrium are shown as in Case 1 of Subsection 4.1. Also we can see that $K_a > K_x$ under the assumption that $a > 1$ and that $\lambda < \lambda^*$, as shown in Case 1 of Subsection 4.1. Then we have $D^{*}_x = D_k$, $D^{*}_x > D_k$ and $D^{*}_y = D^{*}_l$. But whether $D^{*}_y$ is larger than $D^{*}_l$ or not is ambiguous.

When $a \leq 1/2$, the analysis is similar to that of Subsection 4.1. So the following proposition is obtained.

**Proposition 3**

Suppose that the productivity of the environmental good is more sensitive to the global environment in Home than in Foreign ($\varepsilon > \varepsilon^*$). Also suppose that the generating degree of pollution is lower in Home than in Foreign ($\lambda < \lambda^*$). Then, if $1/2 < a < 1$ ($0 < a < a_K$), the level of Home (Foreign) welfare is lower in the trading equilibrium than in the autarkic equilibrium.

We can give the following explanation for Proposition 3. By opening trade, the production of the polluting good is shifted to Foreign where the generating degree of pollution is relatively high. So independently of the expenditure share, the global environment deterio-
rates by trade.
When $1/2 < a < 1$, for Home which exports the environmental good, an environmental deterioration brings forth a fall in the productivity and a rise in the relative price of the exporting good. As the former effect is larger than the latter, Home loses from trade.
When $0 < a < a_0$, Foreign diversifies and exports the polluting good. Because of the environmental deterioration, both the productivity of the environmental good and the relative international price of the polluting good fall. So Foreign loses from trade.

4.3 The Case where the Generating Degree of Pollution is Higher in Home than in Foreign
We now consider the case where the generating degree of pollution is higher in Home than in Foreign. That is, we analyze the case where $\varepsilon > \varepsilon ^*$ and $\lambda > \lambda ^*$. The welfare effect of trade depends on the difference in the generating degree of pollution between countries. So, in the following analysis, we treat the case where the difference in the generating degree of pollution between countries is large and the case where it is small, separately.

4.3.1 The Case where the Difference in the Generating Degree of Pollution between Countries is Large
We assume that $\lambda > \lambda ^*$, $\varepsilon > \varepsilon ^*$ and $a \geq \lambda ^*/(\lambda + \lambda ^*)$. The last assumption is rewritten as $(\lambda - \lambda ^*)/\lambda ^* \geq (1 - 2a)/a$, which means that the difference in the generating degree of pollution between countries is relatively large. Using Proposition 1, we compare the welfare in the autarkic equilibrium with that of the trading equilibrium.

Case 1: Both Countries Produce the Polluting Good ($1/2 < a < 1$)
In Both countries, the level of welfare in the trading equilibrium is higher than that of the autarkic equilibrium. We can see that $K_a < K_X$ under the assumption that $a < 1$ and $\lambda > \lambda ^*$. So, we have $D_\tilde{g} = D_k$, $D_{\tilde{g}} < D_{\tilde{t}}$, $D_{\tilde{g}} ^* = D_{\tilde{t}} ^*$ and $D_{\tilde{g}} ^* < D_{\tilde{t}} ^*$, as shown in Case 1 of Subsection 4.1.

Case 2: Both Countries Specialize ($a_0 \leq a \leq 1/2$)
The level of Home welfare in the trading equilibrium is higher than that of the autarkic equilibrium. In Foreign, the level of welfare in the trading equilibrium is the same to or higher than that of the autarkic equilibrium. We can see that $K_a < K_S$ under the assumption that $a \geq \lambda ^*/(\lambda + \lambda ^*)$. So we have $D_\tilde{g} \leq D_k$, $D_\tilde{g} ^* \leq D_{\tilde{t}} ^*$, $D_{\tilde{g}} ^* = D_{\tilde{t}} ^*$ and $D_{\tilde{g}} ^* \leq D_{\tilde{t}} ^*$. The fact that $D_{\tilde{g}} ^* \leq D_{\tilde{t}} ^*$ is shown from the following equations and inequalities: $D_{\tilde{g}} ^* = (1 - a)(K_a)^* L$, $D_{\tilde{t}} ^* = (1 - a)p_1 L$, $(K_a)^* \leq p_1 \leq (K_S)^*$ and $K_a \leq K_S$.

Case 3: Both Countries Produce the Environmental Good ($0 < a < a_0$)
In Both countries, the level of welfare in the trading equilibrium is higher than that of the autarkic equilibrium. We can see that $K_a < K_Y$. It is because the total output of the polluting good is smaller when Foreign is diversified than when Foreign specializes in the polluting good. We also have the inequality, $K_a \leq K_S$, from the inequality, $a \geq \lambda ^*/(\lambda + \lambda ^*)$. Then it is shown that $K_a < K_Y$. So we obtain $D_\tilde{g} < D_k$, $D_{\tilde{g}}^* < D_{\tilde{t}}^*$, $D_{\tilde{g}}^* = D_{\tilde{t}}^*$ and $D_{\tilde{g}}^* < D_{\tilde{t}}^*$. 
Hence, we have the following proposition.

**Proposition 4**

Suppose that the productivity of the environmental good is more sensitive to the global environment in Home than in Foreign (\( \varepsilon > \varepsilon^* \)) and the generating degree of pollution is higher in Home than in Foreign (\( \lambda > \lambda^* \)). Also suppose that the difference in the generating degree of pollution between countries is relatively large (\( \alpha \geq \lambda^*/(\lambda+\lambda^*) \)). Then, comparing the welfare in the autarkic equilibrium with that of the trading equilibrium, the followings hold.

The level of Home welfare in the trading equilibrium is higher than that of the autarkic equilibrium. On the other hand, the level of Foreign welfare in the trading equilibrium is the same to or higher than that of the autarkic equilibrium.

An interpretation of Proposition 4 is as follows: By opening trade, the production of the polluting good is shifted to Foreign, since the productivity of the polluting good is relatively high in this country. In Foreign, the generating degree of pollution is sufficiently low. So, trade improves the global environment, even though the total output of the polluting good in the world increases by trade. Due to an improvement in the environment, the productivity of the environmental good rises. Then, Home which exports the environmental good gains from trade. Foreign which exports the polluting good also gains from trade through the improvement in the terms of trade. When Foreign produces the environmental good, the productivity of this good also rises by trade.

4.3.2 The Case where the Difference in the Generating Degree of Pollution between Countries is Small

We assume that \( \varepsilon > \varepsilon^* \), \( \lambda > \lambda^* \) and \( \alpha < \lambda^*/(\lambda+\lambda^*) \). The last inequality means that the difference in the generating degree of pollution between countries is relatively small. We have \( 0 < \lambda^*/(\lambda+\lambda^*) < 1/2 \), by the use of the assumption that \( \lambda > \lambda^* > 0 \). So, in the following analysis, we consider the case where \( 0 < \alpha < 1/2 \).

**Case 1: Both Countries Specialize** (\( \alpha_e < \alpha < 1/2 \))

We have \( K_a > K_s \) under the assumption that \( \alpha < \lambda^*/(\lambda+\lambda^*) \). So we can derive similar results to those of Case 2 of Subsection 4.1.

**Case 2: Both Countries Produce the Environmental Good** (\( 0 < \alpha < \alpha_e \))

The welfare effect of trade is ambiguous in Home. In Foreign, however, the level of welfare in the trading equilibrium is the same to or lower than that of the autarkic equilibrium. In the trading equilibrium, the demands for both goods in the both countries are shown as Case 3 of Subsection 4.1. So assuming that \( K_a \geq K_s \), we have \( D_a^x < D_b^x \), \( D_a^y \geq D_b^y \), \( D_a^x = D_b^x \) and \( D_a^y \geq D_b^y \).

5) Although we do not treat it in this paper, if we take the dynamic process into account, we need to assume that \( K_a \geq K_y \) for the one to one correspondence between the range of \( \alpha \) and production pattern.
So, the following proposition is derived.

**Proposition 5**

Suppose that the productivity of the environmental good is more sensitive to the global environment in Home than in Foreign ($\varepsilon > \varepsilon^*$) and the generating degree of pollution is higher in Home than in Foreign ($\lambda > \lambda^*$). Also suppose that the difference in the generating degree of pollution between countries is relatively small ($\alpha < \lambda^*/(\lambda + \lambda^*)$). Then, if $0 < \alpha < \alpha_K$, the level of Foreign welfare in the trading equilibrium is the same to or lower than that of the autarkic equilibrium.

When $\alpha$ is sufficiently small ($0 < \alpha < \alpha_K$), the Home demand for the polluting good increases by trade. On the other hand, the Foreign demand for the polluting good is the same between before and after trade. So, the world demand for the polluting good increases by trade. The difference in the generating degree of pollution between countries is small. So, although the production of the polluting good is shifted to Foreign with the low generating degree of pollution, the global environment becomes worse by trade. Then, by a similar reason to that of Case 3 of Subsection 4.1, Foreign loses from trade.

5. **Conclusion**

On the general equilibrium framework presented by Copeland and Taylor [2] (1999), we analyzed the welfare effect of trade. In our analysis, we assumed that both countries share the common environment and derived the following results. If the generating degree of pollution is low in a country with the high productivity of the environmental good, then both countries may lose from trade. On the other hand, if the generating degree of pollution is high in a country with the high productivity of the environmental good, then both countries may gain from trade.

The above results are explained by the following reason. The source of trade is the difference in the productivity of the environmental good between countries. So, a country with the high productivity of the environmental good has a comparative advantage in this good and thus exports it. Hence, if the generating degree of pollution is high in a country with the high productivity of the environmental good, then the production of the polluting good is shifted to a country with the low generating degree of pollution by trade. So, there is a possibility that the total amount of pollution in the world falls and the global environment improves. When the environment improves, the productivity of the environmental good rises. So, the environmental good exporting country gains from trade. In the polluting good exporting country, terms of trade become favorable by an improvement in the environment. When the polluting good exporting country produces the environmental good, the productivity of the environmental good also rises by an improvement in the environment. So both countries gain from trade. Conversely, if the generating degree of pollution is low in a country with the high productivity of the environmental good, then the production of the polluting good is shifted to the country with the high generating degree of pollution by trade. So, the total amount of pollution in the world increases and it results in the deterioration of
the global environment. Then, terms of trade improve but the productivity of the environmental good falls in the environmental good exporting country. When the latter effect is larger, this country loses from trade. On the other hand, in the polluting good exporting country, terms of trade become worse by the deterioration of the environment. When this country produces the environmental good, the productivity of this good also falls. So, the polluting good exporting country loses from trade.

As a whole, it can be said that it is likely that at least one country loses from trade. This is because both countries gain from trade only when trade promotes a production shift of the polluting good to a country with the sufficiently low generating degree of pollution.

It is also important to see the dynamic process from the autarkic equilibrium to the trading equilibrium. Transition of the welfare in the dynamic process is basically monotonic. However in some cases, the welfare leaps up after opening trade, then gradually falls with the deterioration of the environment. So, the final level of welfare depends on the net effect of these two movements.

Appendix

Proof of Proposition 1 Suppose that in the trading equilibrium, Home diversifies and Foreign specializes in the polluting good. Then, from (14) and (2), we have \( D_x - D_x^* = aL \) and \( X^* = L \). Putting these equations into the equation, \( D_x + D_x^* = X + X^* \), we obtain \( X = L(2a - 1) \). Since Home diversifies, it holds that \( 0 < X < L \). So, from the fact that \( X = L(2a - 1) \), we have \( 1/2 < a < 1 \). We now denote \( K_x \) to be the state of the global environment in the trading equilibrium where Home diversifies and Foreign specializes in the polluting good. From (1) and (4), \( K = g(K - K) - \lambda L(2a - 1) - \lambda^* L \). Putting \( K = 0 \), we have \( K = K_x = K - (\lambda^*/g)L(2a - 1) - (\lambda^*/g)L \). Since \( dK/dK = -g < 0 \), the equilibrium is unique and globally stable.

Suppose that in the trading equilibrium, Home specializes in the environmental good and Foreign specializes in the polluting good. From (2), (14), (11) and (14'), we have \( X = 0 \), \( X^* = L \), \( D_x = aK^* L / p_i \) and \( D_x^* = aL \), where \( p_i \) is the relative international price of the polluting good. Then using the equation, \( D_x + D_x^* = X + X^* \), we obtain \( a = ((K^*/p_i) + 1)^{-1} \). Noting that, if both countries specialize, the range of international price is \( (K)_{**}^* \leq p_i \leq (K)^* \), we have \( (K_{**}^*)^{-1} \leq a \leq 1/2 \). We now denote \( K_x \) to be the state of the global environment in the trading equilibrium where Home specializes in the environmental good and Foreign specializes in the polluting good. Since \( X = 0 \) and \( X^* = L \), we have \( K = g(K - K) - \lambda^* L \) from (1) and (4). Putting \( K = 0 \), we have \( K = K_x = K - (\lambda^*/g)L \). So it holds that \( [(K - (\lambda^*/g)L)^{-1} + 1]^{-1} \leq a \leq 1/2 \) in the trading equilibrium. Since we can see from the dynamic equation, \( K = g(K - K) - \lambda^* L \), that \( dK/dK = -g < 0 \), the equilibrium is unique and globally stable.

Suppose that Home specializes in the environmental good and Foreign diversifies in the trading equilibrium. Then, from (10) and (11), we have \( w = K^* \) and \( w^* = p_i = (K)^* \). So, from (14), the demand for the polluting good in each country becomes \( D_x = aLK^{**} \) and \( D_x^* = aL \). Since \( X = 0 \), the market clearing condition means that \( X^* = D_x + D_x^* \). Then we have \( X^* = aL(K^{**} + 1) \), which is rewritten as \( a = X^*/L(K^{**} + 1) \). In the trading equilibrium, using (8) and noticing that \( X = 0 \), we see that \( K = K - (\lambda^*/g)X^* \). Thus \( a \) is expressed as \( a = X^* / [L(K - (\lambda^*/g)X^*)^{**}] \). From this equation, we have \( da/dX^* > 0 \). This means that
in the equilibrium, the relationship between $a$ and $X^*$ is one to one. Since Foreign diversifies, it holds that $0 < X^* < L$. Then, the range of $a$ becomes $0 < a < [(K - (\lambda^* / g)L)^{e^*} + 1]^{-1}$. We now denote $K_Y$ to be the state of the global environment in the trading equilibrium where Home specializes in the environmental good and Foreign diversifies. Using (1), (4) and the fact that $X = 0$ and noticing that $X^* = aL(K^{e^*} + 1)$, we have $\dot{K} = g(K - K) - a\lambda^* L(K^{e^*} + 1)$. Putting $\dot{K} = 0$, we have $K = K_Y = K - (\lambda^* / g)aL((K_Y)^{e^*} + 1)$. Since $dK/dK = -g - a\lambda^* LK^{e^*} - 1(e - e^*) < 0$, the equilibrium is unique and globally stable.

It is shown that each production pattern determines the range of $a$, which is completely partitioned and each partitioned set of $a$ corresponds to each production pattern by one to one. Thus, conversely, each production pattern is determined by the corresponding partitioned set of $a$.

(Q.E.D.)

References


