Functionally Assigned Control for Flexible Structures Subject to Wind and Seismic Disturbances

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Abstract—In this paper, a method of active control is presented for suppressing random vibration of class of flexible structures subject to both wind and seismic disturbances. It should be noted that inherently each of them has mutually different characteristics. Therefore, it is reasonable from the practical point of view to provide two types of controllers, one of which excludes the vibration due to wind disturbance and another suppresses the vibration due to seismic input. Following to the idea, the state variable is split into two parts—the one is driven by wind disturbance and the other by the seismic one. Consequently the optimal controls are obtained in the form of feedback and feedforward.

1. Introduction and Problem Formulation

IN THIS PAPER, a new method of control system design is proposed for a class of mechanically flexible structures subject to wind and/or earthquake disturbances. Such disturbances are recognized as typical stochastic processes; that is, the wind disturbance may be viewed as a long-lasting, wide-band, stationary stochastic process, while the seismic input may be considered as a short-term, narrow-band, nonstationary process having strong excitation. So disturbances have mutually different characteristics. Hence it is reasonable from the engineering point of view to provide two types of controllers in such a way that each of them takes respective task against wind or seismic disturbance, by taking account of characteristics of disturbances. Thus, this idea can be expressed by the symbolic equation:

$$\begin{aligned} (\text{Controller}) &= (\text{Controller against Wind}) \\ &+ (\text{Controller against Earthquake}). \end{aligned} \quad (1)$$

That will increase the degree of freedom to design the control systems.

To fix the idea, consider a flexible cantilevered beam with length ℓ , mass density ρ , cross-sectional

area S and flexural rigidity EI , whose mathematical model is given by

$$\begin{aligned} \rho S \frac{\partial^2 u(t, x)}{\partial t^2} + c_D I \frac{\partial^5 u(t, x)}{\partial t \partial x^4} + EI \frac{\partial^4 u(t, x)}{\partial x^4} \\ = \sum_{i=1}^{n_{F1}} b_{1i} f_{1i}(t) \delta(x - x_{1i}) - \sum_{i=1}^{n_{M1}} e_{1i} f_{M1i}(t) \frac{\partial \delta(x - \eta_{1i})}{\partial x} \\ + \sum_{i=1}^{n_{F2}} b_{2i} f_{2i}(t) \delta(x - x_{2i}) - \sum_{i=1}^{n_{M2}} e_{2i} f_{M2i}(t) \frac{\partial \delta(x - \eta_{2i})}{\partial x} \\ - \rho S \ddot{e}(t) + g \gamma(t, x), \quad (t, x) \in (0, T) \times (0, \ell) \end{aligned} \quad (2)$$

with

$$\text{I.C.: } u(0, x) = u_0(x), \quad \frac{\partial u(0, x)}{\partial t} = \dot{u}_0(x) \quad (3)$$

$$\left. \begin{aligned} \text{B.C.: } u(t, 0) = \frac{\partial u(t, 0)}{\partial x} = 0 \\ \frac{\partial^2 u(t, \ell)}{\partial x^2} = \frac{\partial^3 u(t, \ell)}{\partial x^3} = 0. \end{aligned} \right\} \quad (4)$$

In (2) $\ddot{e}(t)$ is the earthquake-induced ground acceleration and $\gamma(t, x)$ is the (distributed) white Gaussian noise process; and these reflect the seismic and wind disturbances, respectively. The terms $f_{1i}(t)$ and $f_{M1i}(t)$ are pointwise shearing force and bending moment controls against the wind disturbance, and $f_{2i}(t)$, $f_{M2i}(t)$ are also the controls against seismic disturbance.

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The transverse displacement $u(t, \cdot)$ and velocity $\dot{u}(t, \cdot)$ are supposed to be measured in the following manner,

$$y_j(t) = c_j u(t, \xi_j) + d_j \dot{u}(t, \xi_j), \quad (5)$$

where ξ_j ($0 < \xi_j < \ell; j = 1, \dots, P$) is the location where the j -th sensor is installed and c_j, d_j are constants. The ground acceleration $\ddot{e}(t)$ is assumed to be measured directly by the seismometer set at $x = 0$.

In compliance with (1) our control problem is to construct the two types of control vectors, $f_1(t)$ and $f_2(t)$, such that they minimize respective cost functionals reflecting random vibration due to wind or seismic disturbance. In this sense we call such a couple of controls the “functionally assigned control (FAC).”

2. Reduced-Order Model and Constituent States

A. Reduced-Order Model

For the differential operator $\mathcal{A} := (EI/\rho S)d^4/dx^4$ let $\{\lambda_k\}$ and $\{\phi_k(x)\}$ ($k=1, 2, \dots$) be the eigenvalues and corresponding orthonormal eigenfunctions, and assume that $0 < \lambda_1 \leq \lambda_2 \leq \dots$, $\lim_{k \rightarrow \infty} \lambda_k = \infty$. Then the solution to (2) has the modal representation

$$u(t, x) = \sum_{k=1}^L u_k(t) \phi_k(x), \quad (6)$$

where L ($= \infty$, in theory) is (sufficiently) large integer, and $\{u_k(t)\}$ are solutions to

$$\begin{aligned} \ddot{u}_k(t) + \frac{c_D}{E} \lambda_k \dot{u}_k(t) + \lambda_k u_k(t) &= \sum_{i=1}^{n_{F1}} \frac{b_1}{\rho S} \phi_k(x_{1i}) f_{1i}(t) + \sum_{i=1}^{n_{M1}} \frac{e_1}{\rho S} \frac{\partial \phi_k(\eta_{1i})}{\partial x} f_{M1i}(t) \\ &+ \sum_{i=1}^{n_{F2}} \frac{b_2}{\rho S} \phi_k(x_{2i}) f_{2i}(t) + \sum_{i=1}^{n_{M2}} \frac{e_2}{\rho S} \frac{\partial \phi_k(\eta_{2i})}{\partial x} f_{M2i}(t) \\ &- \sigma_k \ddot{e}(t) + \frac{g}{\rho S} \gamma_k(t) \quad (k = 1, 2, \dots, L), \end{aligned} \quad (7)$$

where $\sigma_k := \int_0^\ell \phi_k(x) dx$, $\gamma_k := \int_0^\ell \gamma(t, x) \phi_k(x) dx$ and $b_\nu = b_{\nu i}$, $e_\nu = e_{\nu i}$ ($\nu = 1, 2$).

Selecting N as the modes to control [1], and defining the vectors

$v_N(t) := [u_1(t), \dots, u_N(t) : \dot{u}_1(t), \dots, \dot{u}_N(t)]'$ and $v_R(t) := [u_{N+1}(t), \dots, u_L(t) : \dot{u}_{N+1}(t), \dots, \dot{u}_L(t)]'$ corresponding to the first N and the residual ($L - N$) modes (where the prime denotes transpose), we get from (7) the versions:

$$\left. \begin{aligned} dv_N(t) &= A_N v_N(t) dt + B_N f(t) dt \\ &\quad - \sigma_N \ddot{e}(t) dt + G_N dw_N(t), \quad v_N(0) = v_{N0} \\ dv_R(t) &= A_R v_R(t) dt + B_R f(t) dt \\ &\quad - \sigma_R \ddot{e}(t) dt + G_R dw_R(t), \quad v_R(0) = v_{R0}, \end{aligned} \right\} \quad (8)$$

where $w_N(t)$ and $w_R(t)$ are Wiener processes with covariance $\mathcal{E}\{dw_i(t)dw_j(t)\} = W_{ij}dt$ related to $\gamma_N(t) = [\gamma_1(t), \dots, \gamma_N(t)]'$ and $\gamma_R(t) = [\gamma_{N+1}(t), \dots, \gamma_L(t)]'$, respectively ($\mathcal{E}$ is the mathematical expectation), $f(t) := [f_1'(t), f_2'(t)]'$,

$f_1 := [f_{11}, \dots, f_{1n_{F1}} : f_{M11}, \dots, f_{M1n_{M1}}]'$,

$f_2 := [f_{21}, \dots, f_{2n_{F2}} : f_{M21}, \dots, f_{M2n_{M2}}]'$, and

$$A_* := \begin{bmatrix} 0 & I_* \\ -\Lambda_* & -\frac{c_D}{E} \Lambda_* \end{bmatrix}, \quad B_* := \begin{bmatrix} B_{*1} & B_{*2} \end{bmatrix}$$

$$\sigma_* := \begin{bmatrix} 0 \\ \tilde{\sigma}_* \end{bmatrix}, \quad G_* := \frac{g}{\rho S} \begin{bmatrix} 0 \\ I_* \end{bmatrix} \quad (* = N \text{ or } R)$$

with $\Lambda_N := \text{diag}\{\lambda_1, \dots, \lambda_N\}$, $\Lambda_R := \text{diag}\{\lambda_{N+1}, \dots, \lambda_L\}$, $\tilde{\sigma}_N := [\sigma_1, \dots, \sigma_N]'$, $\tilde{\sigma}_R := [\sigma_{N+1}, \dots, \sigma_L]'$. Matrices $B_{N1}, B_{N2}, B_{R1}, B_{R2}$ are those having the components $\phi_k(x_{\nu i}), \partial \phi_k(\eta_{\nu i})/\partial x$.

The observation vector $y(t) := [y_1(t), \dots, y_P(t)]'$ is described in the modal form

$$y(t) = C_N v_N(t) + C_R v_R(t). \quad (9)$$

B. Constituent State

In order to make each controller operate effectively let the modal state vector $v_N(t)$ be split further into two parts:

$$v_N(t) = v_{N1}(t) + v_{N2}(t), \quad (10)$$

where $v_{N1}(t)$ is the solution to the following process

$$\left. \begin{aligned} dv_{N1}(t) &= A_N v_{N1}(t) dt + B_{N1} f_1(t) dt \\ &\quad + G_N dw_N(t), \quad v_{N1}(0) = v_{N0}; \end{aligned} \right\} \quad (11)$$

and $v_{N2}(t)$ is determined by

$$\left. \begin{aligned} \dot{v}_{N2}(t) &= A_N v_{N2}(t) + B_{N2} f_2(t) - \sigma_N \ddot{e}(t) \\ v_{N2}(0) &= 0. \end{aligned} \right\} \quad (12)$$

At this stage it should be noted that $v_{N1}(t)$ and $v_{N2}(t)$ are constituent states of the controlled vector $v_N(t)$ excited by wind disturbance $w_N(t)$ and

seismic input $\ddot{e}(t)$, and that $f_1(t)$ and $f_2(t)$ are control vectors for the states $v_{N1}(t)$ and $v_{N2}(t)$, respectively. Since the sample process of $\ddot{e}(t)$ can be measured directly, the random process $v_{N2}(t)$ is measurable so far as $f_2(t)$ is Y_t -measurable (where $Y_t := \{y(s), 0 \leq s \leq t\}$).

By subtracting the sample process $v_{N2}(t)$ from the observation data, define a new process $\tilde{y}(t) := y(t) - C_N v_{N2}(t)$ which satisfies

$$\tilde{y}(t) = C_N v_{N1}(t) + C_R v_R(t). \quad (13)$$

By regarding the observation spillover $C_R v_R(t)$ as colored observation noise to the state $v_{N1}(t)$, Kalman filter for $v_{N1}(t)$ can be constructed from the new observation data $\tilde{Y}_t := \{\tilde{y}(s), 0 \leq s \leq t\}$ (which is equivalent to Y_t because $v_{N2}(t)$ is Y_t -measurable): [2]

$$\left. \begin{aligned} d\hat{v}_{N1}(t) &= A_N \hat{v}_{N1}(t) dt + B_{N1} f_1(t) dt + K_0(t) d\nu(t) \\ \hat{v}_{N1}(0) &= 0, \end{aligned} \right\} \quad (14)$$

where

$$\left. \begin{aligned} d\nu(t) &:= dy_1(t) - H \hat{v}_{N1}(t) dt, \\ dy_1(t) &:= d\tilde{y}(t) - C_R A_R C_R^\# \tilde{y}(t) dt - C_N B_{N1} f_1(t) dt \\ &\quad - C_R B_R f(t) dt + C_R \sigma_R \ddot{e}(t) dt, \quad y_1(0) = y(0) \\ H &:= C_N A_N - C_R A_R C_R^\# C_N \\ C_R^\# &:= (C_R' C_R)^{-1} C_R' \\ K_0 &:= [P(t) H_N' + G_N (C_N G_N W_N + C_R G_R W_0)'] S^{-1} \\ &\quad \equiv L(t) S^{-1} \\ \dot{P}(t) &= A_N P(t) + P(t) A_N' + G_N W_N G_N' \\ &\quad + L(t) S^{-1} L'(t) \\ S &:= C_N G_N W_N G_N' C_N + C_N G_N W_0' G_R C_R \\ &\quad + C_R G_R W_0 C_N' G_N' + C_R G_R W_R G_R' C_R' \end{aligned} \right\} \quad (15)$$

3. Functionally Assigned Control

Our problem is to find the optimal controls $f_1^o(t)$ and $f_2^o(t)$ minimizing some cost functionals. To

make each controller fulfill a respective task we introduce two cost functionals.

A. Controller against Seismic Disturbance

The control $f_2(t)$ is to be designed to control random vibrations due to seismic disturbance only when the earthquake occurs. Invoking that $v_{N2}(t)$, which is a part of the controlled state $v_N(t)$, represents the state due to seismic disturbance, we propose a cost functional by

$$J_2(f_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \mathcal{E} \left\{ \int_0^T [v_{N2}'(t) Q_2 v_{N2}(t) + f_2'(t) R_2 f_2(t)] dt \right\}, \quad (16)$$

where Q_2 is a nonnegative-definite matrix and R_2 is positive-definite weighting matrix.

From the stochastic control theory, the optimal control $f_2(t)$ is obtained as

$$\begin{aligned} f_2^o(t) &= -R_2^{-1} B_{N2}' \Pi_N v_{N2}(t) \\ &\equiv -K_2^o v_{N2}(t), \end{aligned} \quad (17)$$

provided that $\mathcal{E}\{\ddot{e}(t)\} = 0$. The matrix Π_N is the nonnegative-definite solution to the following algebraic Riccati equation:

$$\Pi_N A_N + A_N' \Pi_N + Q_2 - \Pi_N B_{N2} R_2^{-1} B_{N2}' \Pi_N = 0. \quad (18)$$

B. Controller against Wind Disturbance

In determining the control $f_1(t)$, we would select such as to suppress the influence due to wind disturbance. However, we should be cautious for the two controllers $f_1(t)$ and $f_2(t)$ not to affect each other. For instance, if we design the control $f_1(t)$ so that it excludes only the vibration due to wind disturbance, and the state $v_{N1}(t)$ happens to be equal to $-v_{N2}(t)$, the control $f_1(t)$ would operate to negate $f_2(t)$ in spite of $v_N(t) \equiv 0$. After all that results in the loss of control energy. So we should avoid the conflict between two controls $f_1(t)$ and $f_2(t)$. Keeping this crucial point in mind, we select the cost functional for $f_1(t)$ as

$$J_1(f_1) = \lim_{T \rightarrow \infty} \frac{1}{T} \mathcal{E} \left\{ \int_0^T [v_N'(t) Q_1 v_N(t) + f_1'(t) R_1 f_1(t)] dt \right\}, \quad (19)$$

where Q_1 and R_1 are, as the same in the problem for $f_2(t)$, nonnegative- and positive-definite weighting matrices, respectively. Thus, the role of $f_1(t)$ is to suppress the vibration due to wind disturbance and to cover the rest task of $f_2(t)$.

Recalling that $\mathcal{E}\{\cdot\} = \mathcal{E}\{\mathcal{E}\{\cdot|Y_t\}\}$ and $f_1(t)$, $v_{N2}(t)$ are Y_t -measurable, the cost functional (19) is rewritten as

$$\begin{aligned}
J_1(f_1) &= \lim_{T \rightarrow \infty} \frac{1}{T} \mathcal{E} \left\{ \int_0^T \mathcal{E} \{ v'_N(t) Q_1 v_N(t) \right. \\
&\quad \left. + f'_1(t) R_1 f_1(t) | Y_t \} dt \right\} \\
&= \lim_{T \rightarrow \infty} \frac{1}{T} \mathcal{E} \left\{ \int_0^T [\hat{v}'_N(t) Q_1 \hat{v}_N(t) \right. \\
&\quad \left. + f'_1(t) R_1 f_1(t) + \text{tr}\{Q_1 P(t)\}] dt \right\} \\
&= \lim_{T \rightarrow \infty} \frac{1}{T} \mathcal{E} \left\{ \int_0^T [\hat{v}'_{N1}(t) + v_{N2}(t)]' Q_1 [\hat{v}_{N1}(t) \right. \\
&\quad \left. + v_{N2}(t)] + f'_1(t) R_1 f_1(t) + \text{tr}\{Q_1 P(t)\}] dt \right\}, \quad (20)
\end{aligned}$$

where $P(t) := \text{cov}\{v_N(t)|Y_t\}$, and this matrix is the same as that given in (15). This form of cost functional appears sometimes in the tracking problem. So one may look at the relevant control problem as follows: the problem is to find the optimal control $f_1(t)$ for the (estimation) process (14) such that the process $\hat{v}_{N1}(t)$ tracks the deterministic process with negative sign, $-v_{N2}(t)$, minimizing the cost functional (20).

It is a custom to construct the augmented system for solving that problem. Let us introduce the augmented vector $\tilde{v}_N(t) := [\hat{v}'_{N1}(t), v'_{N2}(t)]'$ which satisfies

$$\begin{aligned}
d\tilde{v}_N(t) &= \tilde{A}_N \tilde{v}_N(t) dt + \tilde{B}_N f_1(t) dt \\
&\quad - \tilde{\sigma}_N \tilde{e}(t) dt + \tilde{K}_0(t) d\nu(t), \quad \tilde{v}_N(0) = \tilde{v}_{N0}, \quad (21)
\end{aligned}$$

where

$$\begin{aligned}
\tilde{A}_N &:= \begin{bmatrix} A_N & 0 \\ 0 & A_N - B_{N2} K_2^o \end{bmatrix}, \quad \tilde{B}_N := \begin{bmatrix} B_{N1} \\ 0 \end{bmatrix} \\
\tilde{\sigma}_N &:= \begin{bmatrix} 0 \\ \sigma_N \end{bmatrix}, \quad \tilde{K}_0(t) := \begin{bmatrix} K_0(t) \\ 0 \end{bmatrix}.
\end{aligned}$$

In terms of $\tilde{v}_N(t)$, the cost functional (20) becomes

$$\begin{aligned}
J_1(f_1) &= \lim_{T \rightarrow \infty} \frac{1}{T} \mathcal{E} \left\{ \int_0^T [\tilde{v}'_N(t) \tilde{Q} \tilde{v}_N(t) \right. \\
&\quad \left. + f'_1(t) R_1 f_1(t) + \text{tr}\{Q_1 P(t)\}] dt \right\}, \quad (22)
\end{aligned}$$

where $\tilde{Q}$ is the block matrix with all entries equal to Q_1 . Therefore the tracking problem has become the standard control problem. The optimal control $f_1^o(t)$ is obtained in the augmented form of

$$f_1^o(t) = -R_1^{-1} \tilde{B}'_N \tilde{\Pi}_N \tilde{v}_N(t), \quad (23)$$

where $\tilde{\Pi}_N$ satisfies

$$\tilde{\Pi}_N \tilde{A}_N + \tilde{A}'_N \tilde{\Pi}_N + \tilde{Q} - \tilde{\Pi}_N \tilde{B}_N R_1^{-1} \tilde{B}'_N \tilde{\Pi}_N = 0. \quad (24)$$

Describe the matrix $\tilde{\Pi}_N$ in the form of block matrix

$$\tilde{\Pi}_N = \begin{bmatrix} \Pi_1 & \Pi_2 \\ \Pi_2' & \Pi_3 \end{bmatrix},$$

then the optimal control (23) is decomposed as

$$\begin{aligned}
f_1^o(t) &= -R_1^{-1} B'_{N1} \Pi_1 \hat{v}_{N1}(t) - R_1^{-1} B'_{N1} \Pi_2 v_{N2}(t) \\
&\equiv -K_{11}^o \hat{v}_{N1}(t) - K_{12}^o v_{N2}(t). \quad (25)
\end{aligned}$$

Once obtained a sample value of $\tilde{e}(t)$, $v_{N2}(t)$ can be regarded already as the deterministic process, thus the control $f_1^o(t)$ is obtained consisting of two parts of feedback and feedforward.

4. Simulation Studies

In simulation studies, a steel beam with physical parameters $\ell = 10$ [m], $S = 0.1 \times 0.02$ [m²], $E = 2.058 \times 10^{11}$ [N·m⁻²], and $\rho = 7.860 \times 10^3$ [kg·m⁻³] is assumed. The number of controlled mode is $N = 3$ and residual one is $R = 2$ (i.e., $L = 5$), and constant parameters are $b = e = 1$, $g = 0.01$, $c_j \equiv c = 10$, $d_j \equiv d = 1$. The location of controller f_1 against the wind disturbance is $x_{11} = 0.99\ell$, and that of controller $f_2(t)$ against seismic disturbance is also set at $x_{21} = 0.99\ell$ (The moment control is omitted for simplification). The each value of diagonal components of weighting matrices are $Q_{2ii} = 15000$, $R_{2ii} = 1$; $Q_{1ii} = 1000$, $R_{1ii} = 1$. First, to confirm if the controller $f_1(t)$ operates aptly, the case is considered where only the wind disturbance affects to the beam. Figure 1(a) illustrates the vibration of the beam without any control input; while Fig.1(b) shows the dynamic behavior suppressed by the applied control $f_1(t)$. Figure 2 depicts each motion of the beam at the top ($x=\ell$). From these figures we can observe that the random vibration due to wind disturbance is successfully regulated. The control actions are depicted in Fig.3.

Next, a real seismic data of Northwest California Earthquake in 1951 depicted in Fig.4 was used as a seismic disturbance $\tilde{e}(t)$. It occurs as well as the wind disturbance from $t = 5$ [sec].

The simulation results when both wind and seismic disturbance is added to the beam are shown in Figs. 5-7. The random vibration with large amplitude lasts in the control-free case (Figs.5(a) and 6(a)), however, it is suppressed in Figs.5(b) and 6(b) by the control $f_2(t)$ which started powerfully at $t = 5$ [sec].

5. Concluding Remarks

In this paper, a method has been proposed for designing control of flexible structures. Partitioning the controlled states $v_N(t)$ into $v_{N1}(t)$ and $v_{N2}(t)$, that has been accomplished in the form of functionally assigned control. It was found out that each control has different form by setting two cost functionals which adapt the controllers to respective tasks. Moreover, it was verified this control method is effective to the vibration control for the flexible structures by simulation studies.

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Fig.1. The time evolution of the beam subject to wind disturbance only:
(a) Control-free; (b) Controlled.

Fig.2. Motions of the beam at the top subject to wind disturbance:
(a) Control-free; (b) Controlled.

Fig.3. Actions of controls $f_1(t)$ and $f_2(t)$.

Fig.4. Ground acceleration of Northwest California Earthquake in 1951.

Fig.5. The time evolution of the beam subject to wind and seismic disturbances: (a) Control-free; (b) Controlled.

Fig.6. The behaviors of the beam at the top subject to wind and seismic disturbances: (a) Control-free; (b) Controlled.

Fig.7. Actions of controls $f_1(t)$ and $f_2(t)$.