Recent Topics on Subband Adaptive Filtering

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Abstract In this paper, recent topics on subband adaptive filtering are reviewed. Especially, delayless structures are considered. In these structure adaptive weights in each subband are computed by the LMS algorithm and then transformed into those in fullband. The conventional subband adaptive filter has transmission delay and aliasing effects associated with the filter bank. Such defects are completely avoided in these schemes. Also, a characteristic equation is derived to discuss stability of the adaptation algorithm using the Hadamard transform.

1 INTRODUCTION

There have been many works concerning subband adaptive filters (ADF) [1][2]. Due to the improvement of the condition number of the covariance matrices of the input signals, the subband ADF with LMS adaptive algorithm gives better convergence property over the fullband ADF[2]. However, it is shown in [1][2] that the overall transfer function of the conventional subband ADF in Fig.1 is approximately a delayed version of $W_{opt}(z)$, i.e., $W_{opt}(z)z^{-m}$ where $W_{opt}(z)$ is the transfer function of the optimal Wiener filter for the fullband ADF and m is the delay of the filter bank. Also, the increased error variance due to the aliasing effect is produced.

Recently, a delayless subband ADF has been proposed by Morgan and Thi[3]. Their idea is that the tap coefficients of the subband ADF are transformed into those of the fullband ADF using the frequency sampling method in Fig.2. They have presented some *qualitative* analysis to show that the scheme is delayless and alias-free.

In [4] the closed loop scheme in Fig.2 has been analyzed more *quantitatively* and a new adaptation algorithm using the Hadamard transform has been proposed. In this review, we mainly describe the scheme in [4]. The general stability condition of the adaptation algorithm has been given in [4] and a more explicit condition for a special case is men-

tioned.

Figure 1: Conventional subband ADF

2 2-BAND DELAYLESS SUBBAND ADF

We first consider the 2-band delayless subband ADF with the closed loop structure in Fig. 2, where $H_i(z)$ and $G_i(z)$ ($i = 0, 1$) are analysis and subband adaptive filters, respectively, and $x(n)$ and $d(n)$ are stationary input and desired signals with zero mean, respectively. We assume that the signals are real. The input signal $x(n)$ and the error signal $e(n)$ are decomposed into sets of subband signals using lowpass and highpass filters, and in each subband the signals are decimated by a factor 2. Here we define the subband weight and input

Figure 2: 2-band delayless subband ADF

vectors of dimension $N/2$ as

$$\mathbf{g}_i(n) = [g_i(0) \ g_i(1) \ \dots \ g_i(\frac{N}{2} - 1)]^T \text{ at sub-time } n \quad (1)$$

$$\mathbf{x}_i(n) = [x_i(n) \ x_i(n-1) \ \dots \ x_i(n - \frac{N}{2} + 1)]^T, \quad (2)$$

respectively, where N is the number of the fullband taps. Then the vector $\mathbf{g}_i(n)$ is updated using the LMS type algorithm

$$\mathbf{g}_i(n+1) = \mathbf{g}_i(n) + \mu_i \mathbf{x}_i(n) e_i(n) \quad (3)$$

where μ_i ($0 < \mu_i \ll 1$) is the adaptive step size.

On the other hand, the equivalence between the subband filters $G_i(z)$ and the fullband one $W(z)$ is given by

$$W(z) = \begin{cases} G_0(z^2) & \text{(lowpass band)} \\ G_1(z^2) & \text{(highpass band)} \end{cases} \quad (4)$$

in the frequency domain. As a most natural transformation of the subband taps into the fullband ones $w(k)$ ($k = 0, 1, \dots, N-1$) Morgan and Thi [3] used the frequency sampling type transform

$$w(k) = \frac{1}{N} \sum_{m=0}^{N-1} W(e^{j\frac{2\pi}{N}m}) e^{j\frac{2\pi}{N}km}. \quad (5)$$

Using (4) in (5), after a lengthy calculation we have

$$w(k) = \begin{cases} \frac{1}{2} \{g_0(\frac{k}{2}) + g_1(\frac{k}{2})\} & (k : \text{even}) \\ \frac{1}{N} \sum_{l=0}^{\frac{N}{2}-1} \sum_{m=-\frac{N}{4}}^{\frac{N}{4}-1} \{g_0(l) - g_1(l)\} e^{-j\frac{2\pi}{N}(2l-k)m} & (k : \text{odd}) \end{cases} \quad (6)$$

In (6) the term corresponding to $m = -N/4$ is purely imaginary and the other terms are real in

total. This implies that for the real coefficient case we need to delete it. But this in turn causes the corresponding transformation matrix to be singular. That is, in some cases there are no $g_0(k)$, $g_1(k)$ giving $w_{opt}(k)$. Morgan and Thi [3] actually used noncritically decimated scheme with $M/2$ decimation for M band decomposition to avoid the above defect. But this scheme is not computationally efficient.

To avoid the above defect, in [4] we have proposed a nonsingular transformation

$$w(k) = \frac{1}{2} \{g_0(\lfloor \frac{k}{2} \rfloor) + (-1)^k g_1(\lfloor \frac{k}{2} \rfloor)\} \quad (7)$$

where $\lfloor x \rfloor$ denotes the largest integer not exceeding x . This is based on the above transformation and coincides with (6) for even k . The computational complexity is significantly reduced to a total of $N \log 2$ multiplications per subband input sample compared to a total of about $2N \log N$ for (6).

Next, we investigate the overall transfer function of the delayless subband ADF in stationary state. In the frequency domain, from (7) the fullband filter is related to the subband one as

$$W(z) = \frac{1}{2} [(1+z^{-1})G_0(z^2) + (1-z^{-1})G_1(z^2)]. \quad (8)$$

The cross spectral density between $e_i(n)$ and $x_i(n)$ $S_{e_i x_i}(z)$ is expressed as

$$S_{e_i x_i}(z) = \frac{1}{2} [|H_i(z^{\frac{1}{2}})|^2 S_{ex}(z^{\frac{1}{2}}) + |H_i(-z^{\frac{1}{2}})|^2 S_{ex}(-z^{\frac{1}{2}})] \quad (9)$$

where $S_{ex}(z)$ is the cross spectral density between $e(n)$ and $x(n)$ given by

$$S_{ex}(z) = S_{dx}(z) - W(z)S_{xx}(z). \quad (10)$$

where $S_{xx}(z)$ and $S_{dx}(z)$ are the spectral density of $x(n)$ and the cross spectral density between $d(n)$ and $x(n)$, respectively. Substituting (10) into (9) and solving the linear equations $S_{e_i x_i}(z) = 0$ for $i = 0, 1$, which is equivalent to the stationarity condition of (3), i.e., $E[x_i(n-k)e_i(n)] = 0$ for all k , we obtain the optimal subband filters

$$G_0(z^2) = \frac{z}{2} \left[(1+z^{-1}) \frac{S_{dx}(z)}{S_{xx}(z)} - (1-z^{-1}) \frac{S_{dx}(-z)}{S_{xx}(-z)} \right], \quad (11)$$

$$G_1(z^2) = \frac{z}{2} \left[(1+z^{-1}) \frac{S_{dx}(-z)}{S_{xx}(-z)} - (1-z^{-1}) \frac{S_{dx}(z)}{S_{xx}(z)} \right]. \quad (12)$$

Substituting (11) and (12) into (8) again, we finally get $W(z) = S_{dx}(z)/S_{xx}(z)$. This is just the Wiener filter for the fullband ADF. Hence, aliasing effects are eliminated independently of the frequency response of nonideal analysis filters.

3 STABILITY OF ADAPTATION ALGORITHM

Here we examine the convergence condition of the LMS algorithm (3). Defining the *learning* (at time n) and *optimal* weight vectors of dimension N as $\mathbf{w}(\lfloor n/2 \rfloor)$ (the argument $n/2$ is due to decimation) and $\mathbf{w}_{opt}$, respectively, we have the expression of the error signal as

$$e(n) = [\mathbf{w}_{opt} - \mathbf{w}(\lfloor \frac{n}{2} \rfloor)]^T \mathbf{x}(n) + v(n) \quad (13)$$

where $\mathbf{x}(n) = [x(n) \ x(n-1) \ \dots \ x(n-N+1)]^T$ and the additive noise $v(n)$ is uncorrelated with $x(n)$. The subband signals are given by $x_i(n) = \sum_m h_i(m)x(2n-m)$ and a similar expression for $e_i(n)$. Assuming that $\tilde{\mathbf{y}}$ and $\bar{\mathbf{y}}$ denote the even and odd tap vectors of $\mathbf{y}$, respectively, we reexpress (3) as

$$\begin{aligned} \mathbf{g}_i(n+1) &= \mathbf{g}_i(n) + \mu \sum_{lm} h_i(l)h_i(m)\tilde{\mathbf{x}}(2n-l) \\ &\quad [\mathbf{w}_{opt} - \mathbf{w}(n - \lfloor \frac{m+1}{2} \rfloor)]^T \mathbf{x}(2n-m) \\ &\quad + \mu \sum_{lm} h_i(l)h_i(m)\tilde{\mathbf{x}}(2n-l)v(2n-m) \end{aligned} \quad (14)$$

where $\mu := \mu_0 = \mu_1$. Using (7), we obtain

$$\tilde{\mathbf{w}}(n) = \frac{1}{2}[\mathbf{g}_0(n) + \mathbf{g}_1(n)], \quad \bar{\mathbf{w}}(n) = \frac{1}{2}[\mathbf{g}_0(n) - \mathbf{g}_1(n)]. \quad (15)$$

Defining the weight-error vector as $\epsilon(n) = \mathbf{w}(n) - \mathbf{w}_{opt}$ and using (14) and (15), we have

$$\begin{aligned} \tilde{\epsilon}(n+1) &= \tilde{\epsilon}(n) \\ &\quad - \mu \sum_{lm} a_0(l, m)\tilde{\mathbf{x}}(2n-l)\mathbf{x}^T(2n-m) \\ \epsilon(n - \lfloor \frac{m+1}{2} \rfloor) & \\ &\quad + \mu \sum_{lm} a_0(l, m)\tilde{\mathbf{x}}(2n-l)v(2n-m) \end{aligned} \quad (16)$$

where $a_i(l, m) = 1/2\{h_0(l)h_0(m) + (-1)^i h_1(l)h_1(m)\}$ ($i = 0, 1$). Assuming that the input is statistically independent of the weight-error, it follows that

$$\begin{aligned} E[\tilde{\epsilon}(n+1)] &= E[\tilde{\epsilon}(n)] \\ &\quad - \mu \sum_{lm} a_0(l, m)\{\tilde{\mathbf{R}}_{lm}E[\tilde{\epsilon}(n - \lfloor \frac{m+1}{2} \rfloor)] \\ &\quad + \bar{\mathbf{R}}_{lm}E[\bar{\epsilon}(n - \lfloor \frac{m+1}{2} \rfloor)]\} \end{aligned} \quad (17)$$

where $\tilde{\mathbf{R}}_{lm} = E[\tilde{\mathbf{x}}(2n-l)\tilde{\mathbf{x}}^T(2n-m)]$ and $\bar{\mathbf{R}}_{lm} = E[\bar{\mathbf{x}}(2n-l)\bar{\mathbf{x}}^T(2n-m)]$. A similar equation is derived for $E[\bar{\epsilon}(n)]$. Combining these updating equations, finally a characteristic equation is given by

$$\begin{aligned} |(z-1)\mathbf{I} \\ + \mu \sum_{lm} \left[\frac{a_0(l, m)\tilde{\mathbf{R}}_{lm}}{a_1(l, m)\tilde{\mathbf{R}}_{lm}} \mid \frac{a_0(l, m)\bar{\mathbf{R}}_{lm}}{a_1(l, m)\bar{\mathbf{R}}_{lm}} \right] z^{-\lfloor \frac{m+1}{2} \rfloor}| \\ = 0. \end{aligned} \quad (18)$$

If all the roots of (19) are inside the unit circle, the means of the tap coefficients converge. In general, it is difficult to find the stabilizing parameter μ analytically.

4 MULTI-BAND DELAYLESS SUBBAND ADF

We consider a 4-band delayless subband ADF with tree structure. Here we define the subband taps in the i -th band as $g_i(k)$ ($i = 0, 1, 2, 3$; $k = 0, 1, \dots, N/4 - 1$). Reducing this scheme to the 2-band one, as in the Section 2 the subband taps are transformed as

$$w_i(k) = \frac{1}{2}\{g_{2i}(\lfloor \frac{k}{2} \rfloor) + (-1)^k g_{2i+1}(\lfloor \frac{k}{2} \rfloor)\} \quad (19)$$

for $i = 0, 1$, $k = 0, 1, \dots, N/2 - 1$ where $w_0(k)$ and $w_1(k)$ are the taps in the lowpass and highpass bands, respectively. Next, we transform $w_i(k)$ into the fullband taps $w(k)$ using

$$w(k) = \frac{1}{2}\{w_0(\lfloor \frac{k}{2} \rfloor) + (-1)^k w_1(\lfloor \frac{k}{2} \rfloor)\} \quad (20)$$

for $k = 0, 1, \dots, N - 1$. From (20) and (21), the transformation of the subband taps into the full-

band ones is expressed as

$$\begin{bmatrix} w(4k) \\ w(4k+1) \\ w(4k+2) \\ w(4k+3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} g_0(k) \\ g_2(k) \\ g_1(k) \\ g_3(k) \end{bmatrix} \quad (21)$$

for $k = 0, 1, \dots, N/4 - 1$. We note that the matrix at the right-hand side of (21) is the 4×4 Hadamard matrix and the order of band in the subband tap vector is changed appropriately.

In an M -band case where $M = 2^p$ ($p = 1, 2, \dots$), repeating the above operation recursively, the subband taps are transformed into the fullband one as

$$w_f(k) = \frac{1}{M} \mathbf{H}(p) w_s(k) \quad (22)$$

for $k = 0, 1, \dots, N/M - 1$ where

$$w_f(k) = [w(Mk) \ w(Mk+1) \ \dots \ w(Mk+M-1)]^T, \quad (23)$$

$$w_s(k) = [g_{j(0)}(k) \ g_{j(1)}(k) \ \dots \ g_{j(M-1)}(k)], \quad (24)$$

$$\mathbf{H}(p+1) = \left[\begin{array}{c|c} \mathbf{H}(p) & \mathbf{H}(p) \\ \hline \mathbf{H}(p) & -\mathbf{H}(p) \end{array} \right], \quad \mathbf{H}(1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (25)$$

where $j(k)$ is the bit-reversal number of k . The Hadamard transformation requires $M \log M$ multiplications for each block and a total of $N \log M$ per subband input sample. Since $M \leq N$ generally, computational complexity in the proposed method is less than in the frequency sampling method.

5 CONCLUSION

We have reviewed recent topics on the delayless schemes for subband ADF. The scheme using the Hadamard transform has been described in detail. We are now investigating the possibility of wavelet-packet type nonuniform decomposition of frequency bands and the use of the affine projection type algorithm for accelerating the convergence.

References

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