Towards A New Generation of Adaptive Control

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Abstract

This paper presents a novel approach to the design of adaptive controllers for systems with unknown degrees and uncertain relative degrees. First, we consider the case where the degree of controlled systems are unknown but the relative degrees are known exactly, and show that the proposed adaptive controllers (Controller I) stabilize unknown systems of unknown degrees and adaptively construct internal models of reference signals and unknown disturbances in the control loop. Second, we consider the case where even relative degrees are partially unknown. It is shown that the second proposed controllers (Controller II) also have the same capability as Controller I without exact knowledge of relative degrees. The asymptotic stability of both adaptive schemes is proved without any assumption on sufficient excitation of signals.

1. Introduction

In most of the studies of adaptive control, it is assumed that upper bounds on degrees and exact knowledge of relative degrees of controlled systems are known. This is because the knowledge of those indices is needed to determine the order and structure of the adaptive controllers with sufficient freedom of achieving control objectives [1, 10]. However, that assumption sometimes becomes too restrictive, since it is difficult to obtain reasonable estimation of those indices a priori for unknown practical processes. Hence, the study of adaptive control for processes with unknown degrees and unknown relative degrees has been of great importance from both theoretical and practical point of view.

The purpose of this paper to present a novel approach to the design of adaptive servo controllers for unknown systems with unknown degrees. In this paper, the controlled systems are influenced by unknown disturbances and are to follow reference signals. Both disturbances and reference signals are described as outputs of unknown linear systems with finite dimensions whose upper bounds are known. Based on those assumptions, first, we consider the case where the relative degrees of controlled systems are known exactly, and show that the proposed servo controllers (Controller I) stabilize unknown systems of unknown degrees and adaptively construct internal models of reference signals and disturbances in the control loop. Second, we consider the case where even relative degrees are partially unknown, that is, those are known to be r, r + 1, or r + 2 with known r. It is shown that the second proposed controllers (Controller II) have the same capability as Controller I with no exact knowledge of relative degrees. In both cases, the asymptotic stability of tracking errors is assured without any assumption on sufficient excitation of reference signals.

2. Problem Statement

We consider a single-input single-output linear system influenced by deterministic disturbances as a controlled system:

\[
\frac{dx(t)}{dt} = Ax(t) + bu(t) + gd_1(t),
\]
\[
y(t) = c^T x(t) + d_2(t),
\]

where \( x(t) \in \mathbb{R}^n \), \( A \in \mathbb{R}^{n \times n} \), and \( b, c, g \in \mathbb{R}^n \). \( d_1(t) \) and \( d_2(t) \) are disturbances. For that controlled system (1), (2), only the input \( u(t) \) and the output \( y(t) \) are assumed to be measurable, but system parameters in \( A, b, c, g \) are unknown, and the state \( x(t) \) and disturbances \( d_1(t) \), \( d_2(t) \) are unmeasurable. Also, the degrees \( n \) is unknown. The following assumptions are introduced.

Assumption 1: (1-1) The system (1), (2) is stabilizable and observable. 1-2) The zeros of \( c^T(sI - A)^{-1}b \) lie in \( \mathbb{C}^- \). 1-3) The sign of the high frequency gain \( b_0 \) of \( c^T(sI - A)^{-1}b \) is known. In the following context, it is assumed that \( b_0 > 0 \) without loss of generality. 1-4) Unmeasurable disturbances \( d_1(t) \) and \( d_2(t) \) are uniformly bounded, and satisfy \( A_d(s)d_1(t) = 0 \) (1 = 1, 2), where \( A_d(s) \) is a monic polynomial of \( s \in \mathbb{C}^- \). Although \( A_d(s) \) is unknown, an upper bound on the degree of \( A_d(s) \) is known, which is denoted by \( n_d \) (\( n_d \geq \deg A_d(s) \)).

The assumptions on the relative degrees of \( c^T(sI - A)^{-1}b \) are as follows for Controller I and Controller II, respectively:

Assumption 2 (Controller I): The relative degree of \( c^T(sI - A)^{-1}b \), denoted as \( n^* \), is known exactly.

Assumption 3 (Controller II): The relative degree of \( c^T(sI - A)^{-1}b \), denoted as \( n^* \), is partly unknown. That is, it is known to be \( n^* = r, r + 1, \) or \( r + 2 \), with known \( r \).

A reference signal \( y_r(t) (\in \mathbb{R}^1) \) which the output of the process \( y(t) \) must follow, is introduced. For \( y_r(t) \), it is assumed that

Assumption 4: \( y_r(t) \) is measurable and uniformly bounded, and satisfy \( A_d(s)y_r(t) = 0 \).
Then the problem of this paper can be stated as follows: Given a linear system (1) (2) with unknown parameters, an unknown degrees \( n \), an unmeasurable state \( x(t) \), unmeasurable disturbances \( d_1(t) \), \( d_2(t) \), and a known relative degree \( (\text{for Controller I}) \), or a partly unknown relative degree \( n^* = r + 1 \) or \( r + 2 \) (for Controller II), and given a measurable reference signal \( y_r(t) \) and a known upper bound \( n_d \) on the degree of the unknown polynomial \( A_d(s) \), determine an adaptive servo controller such that the overall system is stabilized and the tracking error

\[
e(t) = y(t) - y_r(t),
\]

converges to zero asymptotically.

### 3. Adaptive Servo Controller I

First, we consider the case where the degrees of controlled systems are unknown, but the relative degrees are exactly known.

#### 3.1. System Representation

Before constructing adaptive servo controllers (Controller I), we derive an input-output representation of the error system by utilizing the zero dynamics of \( c^T(sI - A)^{-1}b \) and generating models of \( y_r(t) \) and a known upper bound \( n_d \) on the degree of the unknown polynomial \( A_d(s) \).

**Lemma 1**: On Assumption 1, 2 and 4, the error system is represented as follows:

\[
\frac{d}{dt}[e(t)] + \lambda e(t) = \mathcal{L}(e(t)) + b_0 u_{f_1} - 1(t) + \Theta^T \omega_1(t) + e(t), \quad \omega_1(t) = [u_{f_2} - 1(t), \ldots, u_{f_n} - n_d(t)]^T, \quad u_{f_1}(t) = \frac{1}{(s + \lambda)^n} u(t),
\]

where \( \lambda > 0 \) is a design parameter, \( \Theta \) and \( b_0 \) are unknown parameters, and \( e(t) \) is an exponentially decaying term. \( \mathcal{L}(e(t)) \) is defined by

\[
\mathcal{L}(e(t)) \equiv G_0(s)e(t), \quad (G_0(s) \in \mathbb{H}_\infty).
\]

#### 3.2. Design of Adaptive Controller I

Based on the representation in Lemma 1, the design procedure of the controller I is now presented. It consists of \( n \) steps derived from "backstepping techniques [2]."

**Step 1)** Define \( z_1(t) \) by

\[
z_1(t) \equiv e(t) = y(t) - y_r(t),
\]

and take the time derivative of it.

\[
z_1(t) = -\lambda z_1(t) + \mathcal{L}(z_1(t)) + \Theta^T \omega_1(t) + b_0 u_{f_n} - 1(t) + e(t).
\]

If \( n^* = 1 \), then \( u_{f_n} - 1(t) \equiv u(t) \). In that case, the design procedure is completed by setting \( u(t) \equiv \alpha_1(t) \), and \( z_2(t) \equiv 0 \) in the following context. Otherwise, when \( n^* \geq 2 \), introduce new variables \( z_2(t) \) and rewrite above equation.

\[
z_2(t) \equiv u_{f_2} - 1(t) - \alpha_1(t),
\]

\[
z_1(t) = -\lambda z_1(t) + b_0 z_2(t) + \mathcal{L}(z_1(t)) + b_0 \{ \alpha_1(t) + p \phi(t) \} + \phi(t) \equiv \Theta^T \omega(t),
\]

where \( p \equiv 1/b_0 \). \( \alpha_1(t) \) is a virtual control input determined in the following way:

\[
\alpha_1(t) = -\dot{\phi}(t) - \dot{\phi}(t) - \dot{z}_1(t) - z_1(t),
\]

\[
\phi(t) = \Theta^T \omega(t),
\]

**Step 2)** Take the time derivative of \( z_2(t) \).

\[
z_2(t) = -\lambda u_{f_n} - 1(t) + u_{f_n} - 2(t) - \alpha_1(t),
\]

\[
\alpha_1(t) = \beta_1(t) + \gamma_1(t) \{ b_0 u_{f_n} - 1(t) + \Theta^T \omega_1(t) + W(t) \} + \gamma_2(t) \nu(t),
\]

\[
\gamma_1(t) = \frac{\partial \alpha_1}{\partial \beta_1},
\]

\[
\beta_1(t) = \gamma_1(t) \{ -\lambda z_1(t) \} + \frac{\partial \alpha_1}{\partial \gamma_1},
\]

\[
\gamma_2(t) = \frac{\partial \alpha_1}{\partial \gamma_2},
\]

\[
W(t) = \mathcal{L}(z_1(t)) + \epsilon(t),
\]

\[
\nu(t) = \frac{\partial \nu(t)}{\partial \gamma_2},
\]

\[
\nu(t) = \frac{\partial \nu(t)}{\partial \gamma_2},
\]

If \( n^* = 2 \), then \( u_{f_n} - 2(t) \equiv u(t) \). In that case, the design procedure is completed by setting \( u(t) \equiv \alpha_1(t) \), and \( z_3(t) \equiv 0 \) in the following context. Otherwise, when \( n^* \geq 3 \), new variables \( z_3(t) \) and \( \alpha_2(t) \) are introduced.

\[
z_3(t) \equiv u_{f_n} - 1(t) - \alpha_3(t).
\]

Then the next relation is obtained.

\[
z_2(t) = -\lambda u_{f_n} - 1(t) + z_3(t) + \alpha_3(t),
\]

\[
\alpha_3(t) = \gamma_3(t) \{ -\lambda z_1(t) \} + \frac{\partial \alpha_3}{\partial \gamma_3}(t),
\]

\[
\gamma_3(t) = \frac{\partial \gamma_3}{\partial \alpha_3}(t),
\]

\[
W(t) = \mathcal{L}(z_1(t)) + \epsilon(t),
\]

\[
\nu(t) = \frac{\partial \nu(t)}{\partial \gamma_3}(t),
\]

\[
\nu(t) = \frac{\partial \nu(t)}{\partial \gamma_3}(t),
\]
For \( z_i(t) \), the following way:

\[
\omega_{i-1}(t) = [u_{fn-*i-1}(t), \omega_{i-2}(t)]^T, \\
\gamma_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial z_2} - \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial z_j} \gamma_{j-1}(t), \\
\gamma_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \theta} \gamma_{i-1}(t), \\
\gamma_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \beta} \gamma_{i-1}(t), \\
\gamma_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \delta} \gamma_{i-1}(t).
\]

Then we have the relation of \( z_i(t) \) and \( \alpha_i(t) \) as follows:

\[
\dot{z}_i(t) = -\lambda u_{fn-*i-1}(t) + \alpha_i(t) - \beta_i(t),
\]

\[
\begin{align*}
&\dot{\beta}_i(t) = -\gamma_i(t)\dot{\omega}_i(t) + b_i(t) \gamma_i(t) u_{fn-*i-1}(t) - z_i(t) \\
&\dot{\gamma}_i(t) = -k_{z1} z_i(t) - k_{z2} \gamma_i(t) Z_2(t) + \gamma_i(t) \gamma_{i-1}(t) \tau_i(t) \tau_{i-1}(t) + \alpha_i(t) \gamma_{i-1}(t),
\end{align*}
\]

where \( \tau_{i-1}(t) \) are auxiliary signals defined by

\[
\tau_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \theta} \gamma_{i-1}(t),
\]

\[
\tau_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \beta} \gamma_{i-1}(t),
\]

\[
\tau_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \delta} \gamma_{i-1}(t).
\]

Step \( n \) Define \( z_n(t) \) and take the time derivative of it.

\[
z_n(t) \equiv u_{fn-1}(t) - \alpha_n(t),
\]

\[
z_n(t) = -\lambda u_{fn-1}(t) + u(t) - \alpha_n(t),
\]

\[
\dot{\alpha}_n(t) = \beta_n(t),
\]

\[
\dot{\gamma}_n(t) = \gamma_n(t) \dot{\omega}_n(t) + \Theta_n(t) u_{fn-1}(t) + W(t),
\]

\[
\dot{\beta}_n(t) = -k_{zn1} z_n(t) - k_{zn2} \gamma_n(t) \gamma_{n-1}(t) Z_2(t) + \gamma_n(t) \gamma_{n-1}(t) \tau_n(t) + \alpha_n(t) \gamma_{n-1}(t),
\]

\[
\dot{\gamma}_n(t) = -k_{zn1} z_n(t) - k_{zn2} \gamma_n(t) \gamma_{n-1}(t) Z_2(t) + \gamma_n(t) \gamma_{n-1}(t) \tau_n(t) + \alpha_n(t) \gamma_{n-1}(t).
\]

For \( z_i(t) \), new variables \( z_{i-1}(t) \) and \( \alpha_i(t) \) are introduced in the following way:

\[
z_{i+1}(t) \equiv u_{fn-i-1}(t) - \alpha_i(t).
\]

Then we have the relation of \( z_i(t) \) and \( \alpha_i(t) \) as follows:

\[
\dot{z}_i(t) = -\lambda u_{fn-i-1}(t) + \alpha_i(t) - \beta_i(t),
\]

\[
\begin{align*}
&\dot{\beta}_i(t) = -\gamma_i(t)\dot{\omega}_i(t) + b_i(t) \gamma_i(t) u_{fn-i-1}(t) - z_i(t) \\
&\dot{\gamma}_i(t) = -k_{z1} z_i(t) - k_{z2} \gamma_i(t) Z_2(t) + \gamma_i(t) \gamma_{i-1}(t) \tau_i(t) \tau_{i-1}(t) + \alpha_i(t) \gamma_{i-1}(t),
\end{align*}
\]

where \( \tau_{i-1}(t) \) are auxiliary signals defined by

\[
\tau_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \theta} \gamma_{i-1}(t),
\]

\[
\tau_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \beta} \gamma_{i-1}(t),
\]

\[
\tau_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \delta} \gamma_{i-1}(t).
\]

Step \( n+1 \) (Stability Analysis): For stability analysis, a positive function \( V(t) \) (Lyapunov function candidate) is introduced.

\[
V(t) = \frac{1}{2} \sum_{i=1}^{n+1} z_i(t)^2 + \frac{b_0}{2g_{11}} \{k_1 - k_1(t)\}^2 + \frac{b_0}{2g_{12}} \{p - \hat{p}(t)\}^2
\]

\[
+ \frac{1}{2} \{\Theta - \hat{\Theta}(t)\}^T G_{13}^{-1} \{\Theta - \hat{\Theta}(t)\}
\]

\[
+ \frac{1}{2g_2} \{b_0 - \hat{b_0}(t)\}^2.
\]

We take the time derivative of \( V(t) \) by using representations of \( z_1(t) \sim z_n(t) \) and adaptive laws of \( k_1(t), \hat{p}(t), \hat{\Theta}(t), \hat{b_0}(t), \) and integrate it again. Then \( V(t) \) is evaluated by

\[
V(t) = V(0) \leq \{L_1(K_2) - b_0 k_1\} \int_0^t z_1(t)^2 dt - \sum_{j=2}^{n+1} k_{j1} \int_0^t z_j(t)^2 dt + L_2(K_2),
\]

where

\[
K_2 \equiv \sum_{j=2}^{n+1} \frac{1}{k_{j2}},
\]

\[
L_1(K_2) \equiv -\lambda + \frac{1}{2C} + CM_1 + \frac{M_2}{2} K_2,
\]

\[
L_2(K_2) \equiv CM_2 + \frac{M_2}{2} K_2,
\]

and \( M_1, M_2 \) are unknown positive constants determined from system parameters and initial conditions, and \( C \) is an arbitrary positive constant. Here we set \( k_1 \) such that

\[
k_1 > \frac{L_1(K_2) + \delta}{b_0} \quad (\delta > 0),
\]

then the following inequality is obtained

\[
V(t) + \delta \int_0^t z_1(t)^2 dt + \sum_{j=2}^{n+1} k_{j1} \int_0^t z_j(t)^2 dt \leq V(0) + L_2(K_2) < \infty
\]

and it follows that the overall system is uniformly bounded and that the state variables \( z_i(t) \) converge to zero asymptotically.

\[
\lim_{t \to \infty} z_i(t) = 0, \quad (1 \leq i \leq n^*).
\]

Theorem 1 (Controller I): Consider a controlled system (1), (2). Suppose that Assumption 1, 2 and 4 can
be met. Then it follows that all the signals in the resulting control system are uniformly bounded, and that the state variables $z_i(t)$ ($1 \leq i \leq n^*$) (where $z_i(t) = e(t)$) converge to zero asymptotically.

3.3. Remark

The basic concept of the present method (Controller I) are division of the controlled system into $n^*$ hierarchical subsystems ($S_i \sim S_{n^*}$) and recursive stabilization of each subsystems whose outputs are $z_i(t)$ ($1 \leq i \leq n^*$), via virtual control inputs $\hat{u}_i(t)$. The adaptive stabilizing controllers for each subsystems are easily constructed by utilizing the strictly positive realness of transfer functions $\frac{1}{s^r}$. Then combining all subsystems into one adaptive scheme, the proposed adaptive servo controllers for unknown degrees are constructed (Fig.1).

4. Adaptive Servo Controller II

Next, we consider the case where even the relative degrees are partially unknown, that is, the relative degrees are known to be $r$, $r+1$, or $r+2$ with known $r$.

4.1. System Representation

For uncertain relative degrees $n^* = r+2$, $r+1$, $r$, different representations of controlled systems are introduced in the following lemma.

Lemma 2 : On Assumption 1, 3 and 4, the error system is represented as follows:

1) when $n^* = r+2$

$$
\frac{d}{dt} E(t) + \lambda E(t) = \mathcal{L}(e(t)) + b_0 u_{fr}(t) + \Theta_1^T \omega_1(t) + \epsilon(t),
$$

(58)

$$
E(t) = \mathcal{L}(e(t)) + b_0 u_{fr+1}(t) + \Theta_2^T \omega_2(t) + \epsilon(t),
$$

(59)

2) when $n^* = r+1$

$$
E(t) = \mathcal{L}(e(t)) + b_0 u_{fr}(t) + \Theta_1^T \omega_1(t) + \epsilon(t),
$$

(60)

3) when $n^* = r$

$$
E(t) = \mathcal{L}(e(t)) + b_0 u_{fr-1}(t) + \Theta_0^T \omega_0(t) + \epsilon(t),
$$

(61)

$$
e(t) = \mathcal{L}(e(t)) + b_0 u_{fr}(t) + \Theta_1^T \omega_1(t) + \epsilon(t),
$$

(62)

where

$$
\mathcal{L}(e(t)) = \frac{d}{dt} e(t) + \lambda e(t),
$$

$$
\omega_0(t) = [u_{fr}(t), u_{fr+1}(t), \ldots, u_{fr+n_{-d}-1}(t)]^T,
$$

$$
\omega_1(t) = [u_{fr+1}(t), u_{fr+2}(t), \ldots, u_{fr+n_{-d}+1}(t)]^T,
$$

$$
\omega_2(t) = [u_{fr+2}(t), u_{fr+3}(t), \ldots, u_{fr+n_{-d}+1}(t)]^T,
$$

and $u_{fi}(t)$ is defined previously. $\Theta_0, \Theta_1, \Theta_2, b_0$ are unknown system parameters (vectors with proper dimensions and a scalar), $\lambda$ is an arbitrary positive constant, and $\epsilon(t)$ and $L(e(t))$ are defined in the same way as Lemma 1.

4.2. Design of Adaptive Controller II

Based on those different representations of controlled systems, the adaptive controllers (Controller II) are constructed for uncertain relative degrees $n^* = r+2$, $r+1$, $r$. The design procedure consists of $r+1$ steps.

Step 1) Define state variables $z_1(t)$ and $z_2(t)$ by

$$
z_1(t) \equiv e(t) = y(t) - y_r(t),
$$

(67)

$$
z_2(t) \equiv u_{fr}(t) - \alpha_1(t),
$$

(68)

where $\alpha_1(t)$ is the virtual input determined as follows:

$$
\alpha_1(t) = -\hat{k}_1(t) z_1(t) - \hat{f}_1(t) \omega_1(t),
$$

(69)

$$
\hat{k}_1(t) = \hat{k}_1(t) + k_1(t),
$$

$$
\hat{k}_1(t) = \frac{1}{2} g_{11} z_1(t)^2,
$$

(70)

$$
\hat{k}_{12}(t) = g_{12} z_1(t),
$$

(71)

$$
\hat{f}_1(t) = \hat{f}_1(t) + \hat{f}_1(t),
$$

$$
\hat{f}_1(t) = \hat{f}_1(t) + \hat{f}_1(t),
$$

(72)

$$
\hat{k}_1(t) = \hat{k}_1(t) + \hat{k}_1(t),
$$

(73)

$$
\hat{f}_1(t) = \hat{f}_1(t) + \hat{f}_1(t),
$$

(74)

$$
\hat{f}_1(t) = \hat{f}_1(t) + \hat{f}_1(t),
$$

(75)

$$
\hat{k}_1(t) = \hat{k}_1(t) + \hat{k}_1(t),
$$

(76)

$$
\hat{k}_1(t) = \hat{k}_1(t) + \hat{k}_1(t),
$$

(77)

$$
\hat{f}_1(t) = \hat{f}_1(t) + \hat{f}_1(t),
$$

(78)

$$
\hat{f}_1(t) = \hat{f}_1(t) + \hat{f}_1(t),
$$

(79)
\( \gamma_1(t) = \frac{\partial \alpha_1}{\partial \hat{K}_1} G_1 v_1(t) + \frac{\partial \alpha_1}{\partial Z_1}, \quad (80) \)

\( \hat{K}_1(t) = [\hat{k}_1(t), \hat{\phi}_1(t)]^T, \quad (81) \)

\( v_1(t) = [z_1(t), \omega_1(t)]^T, \quad (82) \)

\( G_1 = \text{block diag}(g_{11}, g_{12}), \quad (83) \)

\( k_2 = \begin{cases} 0 & (n^* = r) \\ 1 & (n^* = r + 1, r + 2), \end{cases} \quad (84) \)

\( \hat{\phi}_0(t) = [\hat{\phi}_0(t), \hat{\phi}_1(t), \ldots, \hat{\phi}_n(t)]^T, \quad (85) \)

\( \Phi_0 = \begin{cases} \theta_0 / b_0 & (n^* = r) \\ 0 & (n^* = r + 1, r + 2), \end{cases} \quad (86) \)

where \( k_{20}(t), \hat{k}_2(t), \Phi_0(t) \) are tuning parameters, and \( k_{20}(t) \) and \( \Phi_0(t) \) are also current estimates of \( k_20(t) \) and \( \Phi_0(t) \), respectively. \( k_22 \) is an arbitrary positive constant.

**Step 1** (\( 3 \leq i \leq r + 1 \)):

For \( z_i(t) \), determine a state variable \( x_i+1(t) \) and a virtual input \( \alpha_i(t) \) in the following way:

\( z_i(t) \equiv u_{fr-i+2}(t) - \alpha_{i-1}(t), \quad (87) \)

\( x_i+1(t) \equiv u_{fr-i+1}(t) - \alpha_i(t), \quad (88) \)

\( \alpha_i(t) = \lambda \alpha_{fr-i+1}(t) + \beta_{i-1}(t) - z_i(t) \quad \)

\(+ \gamma_{i-1}(t) \omega_0(t) - k_i z_i(t) - k_{i2} \gamma_{i-1}(t)^2 z_i(t) \quad \)

\(+ \gamma_{2i-1}(t) r_i(t) + \alpha_i(t), \quad (89) \)

\( k_{i1}, k_{i2} > 0, \quad \beta_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial K_i} \hat{K}_i(t) + \frac{\partial \alpha_{i-1}}{\partial u_f} u_f^{(r+i+3)}(t) \quad \)

\(- \frac{\partial \alpha_{i-1}}{\partial Z_i} \gamma_{i-1}(t) \quad \)

\(+ \sum_{j=1}^{l-i} \frac{\partial \alpha_{i-1}}{\partial Z_j} \gamma_{j-1}(t), \quad (90) \)

\( \gamma_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial K_1} G_1 v_1(t) + \frac{\partial \alpha_{i-1}}{\partial Z_1} - \sum_{j=2}^{l-i} \frac{\partial \alpha_{i-1}}{\partial Z_j} \gamma_{j-1}(t), \quad (91) \)

\( \gamma_{32}(t) = 0, \quad (92) \)

\( \gamma_{33}(t) = \frac{\partial \gamma_{32}}{\partial B_0}, \quad (93) \)

\( \gamma_{31}(t) = \frac{\partial \alpha_{i-1}}{\partial B_0} - \sum_{j=4}^{l-i} \frac{\partial \alpha_{i-1}}{\partial Z_4} \gamma_{j-1}(t), \quad (94) \)

\( (5 \leq i \leq r + 1), \quad \hat{K}_2 = [k_{20}(t), \hat{k}_2(t), \Phi_0(t)]^T, \quad (95) \)

\( \hat{\omega}_0(t) = [u_{fr-1}(t), \omega_0^{(r+i+3)}(t)]^T, \quad (96) \)

\( u_f^{(r+i+3)}(t) = [u_{fr-i+1}(t), u_{fr-i+4}(t), \ldots, u_{fr+n-1}(t)]^T, \quad (97) \)

\( \tau_{33}(t) = -G_3 \gamma_2(t) \omega_0(t) z_2(t), \quad (98) \)

\( \gamma_{31}(t) = \gamma_{31}(t) - G_3 \gamma_{1-i}(t) \omega_0(t) z_i(t), \quad (99) \)

\( G_3 = G_3^T > 0, \quad (100) \)

where \( B_0(t) \) is a tuning vector and also a current estimate of \( B_0, K_1 \), and \( k_2 \) are arbitrary positive constants, and \( \alpha_i(t) \) are auxiliary signals defined in the final step. In **Step r+1**, the actual control input \( u(t) \) is obtained in the following:

\( u(t) = \alpha_{r+1}(t). \quad (101) \)

**Step r + 2**:

In the final step, the design procedure is completed by determining adaptive laws of \( \hat{B}_0(t) \) and auxiliary signals \( \alpha_i(t) \) (\( 4 \leq i \leq r + 1 \)) as follows:

\( \hat{B}_0(t) = r \tau_{31}(t), \quad (102) \)

\( \hat{\alpha}_i(t) = - \sum_{k=4}^{i-1} \gamma_{nk-1}(t) G_3 \gamma_{n-1}(t) \omega_0(t) z_k(t), \quad (103) \)

\( (5 \leq i \leq r + 1), \quad \hat{\alpha}_4(t) = 0. \quad (104) \)

**Step r + 3** (**Stability Analysis**):

For stability analysis, we should also note that the equivalent representations of adaptive laws of \( \hat{k}_1(t) \) (70) and \( \hat{\phi}_1(t) \) (71) are given in the following forms:

\( \hat{k}_1(t) = g_{11} z_1(t) \gamma_1(t), \quad (105) \)

\( \hat{\phi}_1(t) = G_{12} \omega_1(t) z_1(t), \quad (106) \)

\( Z_1(t) \equiv \alpha_1(t) + \lambda \gamma_1(t). \quad (107) \)

However those cannot be realized in the actual adaptive schemes, since unknown \( \gamma_1(t) \) is included in \( Z_1(t) \). Thus, Eq.(70) and (71) are utilized as adaptive laws of \( \hat{k}_1(t) \) and \( \hat{\phi}_1(t) \) in the construction of control systems.

Then, stability analysis can be carried out by utilizing different representations of controlled systems, considering equivalent representations of adaptive laws (105)–(107), and introducing three types of Lyapunov functions for each relative degrees \( n^* = r + 2, r + 1, r \) (For detail, see Ref. [7]).

**Theorem 2**:

Consider a controlled system (1), (2). Suppose that Assumption 1, 3 and 4 can be met. Then it follows that all the signals in the resulting control system are uniformly bounded, and that the state variables \( z_i(t) \) (\( 1 \leq i \leq r + 1 \)) (where \( z_1(t) = c(t) \)) converge to zero asymptotically.

**4.3. Remarks**

Similar to the previous case (Controller I), the controlled system is divided into \( r + 1 \) hierarchical subsystems and recursive stabilization of each subsystems are carried out by utilizing virtual control inputs \( \alpha_i(t) \) (\( 1 \leq i \leq r + 1 \)). However, owing to uncertain relative degrees \( n^* = r + 2, r + 1, r \), the relative degree of the first subsystem is also uncertain.

For that part, we see \( Z_1(t) \) as an output and \( \alpha_1(t) \) as a (virtual) control input, and stabilize \( Z_1(t) \) via \( \alpha_1(t) \) by utilizing strictly positive realness of the transfer function \( \frac{1}{s + r} \) of \( Z_1(t) \) and \( \alpha_1(t) \) for \( n^* = r + 2 \), strictly positive realness of "1" for \( n^* = r + 1 \), and strictly positive realness of "(s + \lambda)" for \( n^* = r \), respectively. Recursive stabilization of \( z_i(t) \) (\( 2 \leq i \leq r + 1 \)) via \( \alpha_i(2) \leq i \leq r + 1 \) is similar to Controller I, since relative degrees of \( z_i(t) \) and \( \alpha_i(t) \) (\( i \geq 2 \)) are 1, but additional consideration is needed because of the peculiar structure of \( \alpha_1(t) \) and \( Z_1(t) \) (Fig.2).

5. Towards a New Generation of Adaptive Control

In the present paper, we showed recent results of our studies on the design of robust adaptive control for systems with
The unstructured uncertainty of Type I can change the degrees but also the relative degrees to some extent ($n^* = r \sim r + 2$). For that Type II uncertainties, simple high-gain feedback schemes cannot be directly applied. However, by considering the fact that the relative degrees of strictly positive real transfer functions are $1$, $0$, and $-1$, that is, transfer functions $\frac{1}{s^2 + \lambda}$, $1$, $s + \lambda$ $(\lambda > 0)$ are all strictly positive real, we can construct globally asymptotically stable adaptive control systems for unknown degrees and uncertain relative degrees $r \sim r + 2$ (Controller II). Those two controllers (Controller I and Controller II) did show certain stable region and robust performance of adaptive controllers under unstructured uncertainties.

In the next several years, the discussion on robust performances of adaptive control systems become more important topics, and the relation to other control schemes, such as classical quadratic optimal control and recent robust and (nonlinear) $H_{\infty}$ control schemes, will be fully discussed. Even in the present stage, it can be shown that the modified version of Controller I is suboptimal to certain nonlinear but meaningful quadratic cost functions, and that another modified version of it is also suboptimal to certain $H_{\infty}$ or $L_2$-normed cost functions, which will be reported in the future. The discussion on the performance of adaptive control may lead to the new generation of adaptive control or the end of the traditional adaptive control (just like "Evangelion"). However, it is certain that a new methodology of adaptation scheme will emerge in order to solve many existing problems in the field of control theory and technology.

6. References