Symbol Reliability Estimation Using the Bidirectional Viterbi Decoding Algorithm

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Abstract

In this paper, we first review the Soft-BIVA based on the *bidirectional Viterbi algorithm (BIVA)*. Next, we apply the Soft-BIVA to QLI codes under the assumption that a Scarce State Transition Viterbi decoder is used. Then the symbol reliability values are derived by applying the Soft-BIVA to the main decoder. Moreover, it is shown that in the case of QLI codes, the symbol reliability is also obtained using *bidirectional syndrome decoding* in the syndrome trellis.

1 Introduction

We restrict the following discussion to a binary rate-1/n convolutional code specified by a polynomial generator matrix $G(D)$ to preserve clarity. Let $u=\{u_k\}$ be a transmitted information sequence. Also denote by $z=\{z_k\}$ the received sequence corresponding to a convolutionally coded sequence $w(D)=u(D)G(D)$. Then *symbol-by-symbol MAP decoding* is accomplished by maximizing the joint probability $\mu(u_k, z)$ for each time k . The scheme is named the **BCJR** algorithm after the inventors [1]. This BCJR algorithm requires a “backward” as well as a forward recursion and consequently storage of all data. Moreover, it has to deal with not path metrics but the exact path probabilities. Hence its computational complexity is high. However, it can generate reliability information on the decoded symbols. At the beginning the BCJR algorithm did not attract much attention for the reason of not having a particular advantage with respect to the bit error rate compared with the *Viterbi algorithm (VA)* [2]. But in 1993 the “*turbo-codes*” were proposed [3] and the situation has changed. Actually, reliability information on the decoded symbols plays an essential role in turbo decoding. Since then the BCJR algorithm has been attracting a great deal of attention [4], [5]. In particular,

reduction of the computational complexity of the original BCJR algorithm is one of the main problems to be solved.

On the other hand, apart from the above circumstances, it has been known that a convolutionally coded sequence corresponding to a frame of information bits terminated by tail bits can be decoded from both directions simultaneously (i.e., *bidirectional Viterbi decoding* [6], [7]). Since in this method the states on the *maximum-likelihood (ML)* path are identified by joining the forward and backward decoders, the authors thought that bidirectional Viterbi decoding must have close relation to the BCJR algorithm which also needs both forward and backward recursions. As a result, we have shown in [8] that an approximate value of the *log-likelihood ratio* $\Lambda_k \equiv \ln[\mu(u_k=0 \mid z)/\mu(u_k=1 \mid z)]$ (i.e., reliability information on u_k) can be obtained using bidirectional Viterbi decoding.

Now, we divide the time- k states $\{s_k\}$ in the trellis into two groups:

$$\{s_k^+ = (u_{k-\nu+1}, \dots, u_{k-1}, u_k=0)\}, \tag{1a}$$

$$\{s_k^- = (u_{k-\nu+1}, \dots, u_{k-1}, u_k=1)\}, \tag{1b}$$

where ν is the memory length of $G(D)$. If the forward and backward decoders join at time k , the best path $x^*(s_k)$ which passes through each state s_k in the trellis is determined. We denote by $m^*(s_k)$ the path metric (i.e., *log-likelihood*) of $x^*(s_k)$. Then it is shown that

$$\rho_k^* \equiv \max_{u_{k-\nu+1}, \dots, u_{k-1}} \{m^*(s_k^+)\} - \max_{u_{k-\nu+1}, \dots, u_{k-1}} \{m^*(s_k^-)\} \tag{2}$$

is a good approximation to Λ_k . In the following, we call the above scheme the *bidirectional Viterbi decoding algorithm with soft-outputs (Soft-BIVA)*. (A similar idea is given in [9].)

In this paper, we first state the basic properties of the Soft-BIVA. In particular, we examine the relation between the Soft-BIVA and the *Soft-Output Viterbi Algorithm* (SOVA) proposed by Hagenauer et al. [10]. Next, we apply the Soft-BIVA to *Quick Look-In (QLI)* codes under the assumption that a *Scarce State Transition (SST)* Viterbi decoder [11] is used. Then the symbol reliability values are derived by applying the Soft-BIVA to the main decoder. Moreover, it is shown that in the case of QLI codes, the symbol reliability is also obtained using *bidirectional syndrome decoding* in the syndrome trellis. Here note that the syndrome sequence must be generated in time-reversed order for backward syndrome decoding. It is shown that the time-reversed syndrome sequence can be generated using a generator matrix of the “*reciprocal dual code*” [12] of the original code.

In the following, a BPSK modulation/demodulation is assumed and a coded sequence is transmitted symbol by symbol over a memoryless AWGN channel. The j -th received data can be modeled as $z_j = y_j \sqrt{(2E_s/N_0)} + n_j$, where $y_j = \pm 1$ depending on whether the code symbol is 0 or 1, n_j denotes a zero-mean unit variance Gaussian random variable, E_s is the signal energy per coded bit, and N_0 is one-sided power spectral density of the noise. Terminated data transmission is also assumed.

2 Review of the Soft-BIVA

2.1 Soft-BIVA

For evaluating the symbol reliability, all the paths which pass through $s_k^\pm$ are taken into consideration in the BCJR algorithm. On the other hand, the Soft-BIVA estimates the symbol reliability using only the best path which passes through $s_k^\pm$. (Note that the Soft-BIVA is based on the Viterbi algorithm.) Now let

$$\beta_0 \equiv \sum_{u_{k-\nu+1}, \dots, u_{k-1}} \mu(z | x^*(s_k^+)), \quad (3a)$$

$$\beta_1 \equiv \sum_{u_{k-\nu+1}, \dots, u_{k-1}} \mu(z | x^*(s_k^-)). \quad (3b)$$

Then it is shown that $\ln|\beta_0/\beta_1|$ is approximated by ρ_k^* [8].

Here let $s_k^\wedge = (u_{k-\nu+1}^\wedge, \dots, u_{k-1}^\wedge, u_k^\wedge)$ be the state at time k which the ML path passes through, where the ML path is determined when the forward and backward decoders join. Since $m^*(s_k^\wedge) = \max_{s_k} m^*(s_k)$, ρ_k^* is expressed as

$$\rho_k^* = \Delta_k^* \cdot (1 - 2u_k^\wedge), \quad (4)$$

where

$$\Delta_k^* \equiv m^*(s_k^\wedge) - \max_{u_{k-\nu+1}, \dots, u_{k-1}} \{ m^*(u_{k-\nu+1}, \dots, u_{k-1}, 1 - u_k^\wedge) \} \quad (\geq 0). \quad (5)$$

Remark : If $\{s_k^+\} = \phi$ or $\{s_k^-\} = \phi$, we set $\Delta_k^* = \infty$, where ϕ is the empty set.

Under these conditions, the Soft-BIVA is defined as follows [8]:

<Soft-BIVA>

[Evaluate the metric of the best path $x^*(s_k)$ which passes through each state s_k of the trellis at time k , where $x^*(s_k)$ is determined when the forward and backward decoders join at time k . Then identify the ML-state $s_k^\wedge = (u_{k-\nu+1}^\wedge, \dots, u_k^\wedge)$ which the ML path passes through at time k . At the same time, calculate the reliability of the symbol $u_k^\wedge$ using Δ_k^* .]

In the Soft-BIVA, only the best path which passes through each state of the trellis is considered. Here, we examine the variation of ρ_k^* in the case where the second best paths are also taken into consideration in addition to the best paths. Let

$$\begin{aligned} \beta_0' &\equiv \sum_{u_{k-\nu+1}, \dots, u_{k-1}} \{ \mu(z | x^*(s_k^+)) + \mu(z | x^{**}(s_k^+)) \}, \\ \beta_1' &\equiv \sum_{u_{k-\nu+1}, \dots, u_{k-1}} \{ \mu(z | x^*(s_k^-)) + \mu(z | x^{**}(s_k^-)) \}, \end{aligned} \quad (6a, b)$$

where $x^{**}(s_k)$ denotes the second best path which passes through s_k . Next, we evaluate the difference $\rho^{(1)} \equiv \ln|\beta_0'/\beta_1'| - \ln|\beta_0/\beta_1|$. Let $\Delta^\pm$ be

$$\Delta^\pm \equiv \max_{u_{k-\nu+1}, \dots, u_{k-1}} \{ m^*(s_k^\pm) \} - \max_{u_{k-\nu+1}, \dots, u_{k-1}} \{ m^{**}(s_k^\pm) \} \quad (\geq A > 0), \quad (7)$$

where $m^{**}(s_k)$ denotes the metric of the second best path $x^{**}(s_k)$. Then it is shown that if e^{-A} is sufficiently small, $\rho^{(1)}$ is approximated by

$$\rho^{(1)} \doteq \exp[-\Delta^+] - \exp[-\Delta^-]. \quad (8)$$

Hence, if the above assumption holds, we can conclude that the difference $\rho^{(1)}$ is small.

2.2 Relation between Soft-BIVA and SOVA

In this section, we discuss the relation between the Soft-BIVA and the SOVA. In the SOVA, the reliability of the symbol u_k on the ML path is evaluated by the process in which the ML path has survived through competitions with the discarded paths [10]. Actually, the reliability of the symbol u_k is defined as

Δ_k^H ≡ min_{1 ≤ d ≤ δ_1} Δ_k^d, (9)

where Δ_k^d denotes the metric difference between the ML path and the discarded path at time k+d (≤ k+δ_1, δ_1 is the truncation length of a survivor).

Remark : If the discarded path at time k+d passes through a state (⋯, u_k) at time k, we set Δ_k^d=∞.

Here, we assume that δ_1 is large enough so that all survivors have been merged with sufficiently high probability (we rewrite δ_1 as δ). Then it is shown that the inequality:

Δ_k^* ≤ Δ_k^H (10)

holds [8]. Furthermore, let s_k^* ≡ (u_{k-ν+1}^*, ⋯, u_{k-1}^*, 1-u_k) be the state at which Δ_k^* is attained and take notice of the best path x^# which passes through the state s_k^*. We also assume that x^# has merged with the ML path at k+d' (≤ k+δ). In this case, if x^# has contended with the ML path at time k+d', then it is shown that Δ_k^*=Δ_k^H holds [8].

In the SOVA, the probability of decision error P_e ≡ μ(u_k ≠ u_k) is approximated by P_e^H=1/(1+exp[Δ_k^H]). Similarly, it is shown that P_e is approximated by P_e^*=1/(1+exp[Δ_k^*]) in the Soft-BIVA. Then it follows that P_e^H ≤ P_e^* from the relation Δ_k^* ≤ Δ_k^H.

2.3 Simulation Results

Simulations were conducted to evaluate the relation between the Soft-BIVA and the SOVA. A rate-1/2 code with generator matrix G_1=(1+D^2, 1+D+D^2) was used. For applying the SOVA(or Soft-BIVA), the conventional integer code symbol metrics m_j' (0 ~ 7) were used according to the formula m_j' = 2 × z_j^* × y_j + 3.5, where z_j^* is the median of the interval to which the received data z_j belongs. We set δ_1=36. The length of an information sequence is about 10^7. Then it has been found that the percentage of Δ_k^*=Δ_k^H is 60 ~ 85% for E_b/N_0 between 0 and 6dB [8]. Table 1 shows the average Δ_k^H and Δ_k^* for correct and incorrect VA outputs (̄ denotes the average). It is shown that the Soft-BIVA generates slightly smaller

symbol reliability values compared with the SOVA. Simulations were also conducted to compare P_e^H and P_e^* with P_e, where P_e was calculated using the BCJR algorithm. Table 2 shows their averages for incorrect VA outputs (cf., [5]). It is confirmed that the estimate P_e^* derived from the Soft-BIVA is nearer to P_e compared with P_e^H.

Table 1 Average Δ_k^H and Δ_k^* for correct and incorrect VA outputs

E_b/N_0 (dB)	Δ_k^H (SOVA)		Δ_k^* (Soft-BIVA)	
	correct	incorrect	correct	incorrect
0.0	11.64	3.75	9.75	2.92
0.5	12.81	3.61	10.96	2.84
1.0	14.11	3.44	12.34	2.74
1.5	15.53	3.27	13.85	2.64
2.0	17.04	3.11	15.48	2.55
2.5	18.61	2.90	17.17	2.41
3.0	20.20	2.73	18.90	2.30
3.5	21.78	2.51	20.63	2.18
4.0	23.32	2.32	22.31	2.05

Table 2 Average P_e^H, P_e^* and P_e for incorrect VA outputs

E_b/N_0 (dB)	P_e^H	P_e^*	P_e
0.0	0.2276	0.2477	0.2769
0.5	0.2272	0.2448	0.2666
1.0	0.2283	0.2433	0.2595
1.5	0.2286	0.2412	0.2540
2.0	0.2293	0.2399	0.2442
2.5	0.2323	0.2411	0.2367
3.0	0.2341	0.2413	0.2392
3.5	0.2354	0.2406	0.2391

Remark : For evaluating P_e^H(P_e^*), the exact log-likelihoods m_j are necessary. Since m_j is expressed as m_j = √(2E_s/N_0) × z_j × y_j + η_j, integer metrics were transformed to the log-likelihoods by the formula Δ_k^H(Δ_k^*) × √(2E_s/N_0)/2. Hence, some errors due to metric quantization are contained in P_e^H and P_e^*. However, they are expected to be small.

3 Application of Soft-BIVA to QLI Codes

3.1 SST Viterbi Decoding of QLI Codes [11], [13]

In this section, we apply the Soft-BIVA to a general rate-1/2, memory length ν , QLI code specified by a polynomial generator matrix:

$$G=(g^{(1)}(D), g^{(2)}(D)) \quad (g^{(1)}(D)+g^{(2)}(D)=D^L, 1 \leq L \leq \nu-1). \quad (11)$$

Of course, the results in the preceding section also apply to QLI codes. For QLI codes, however, in addition to conventional Viterbi decoding, SST Viterbi decoding which consists of two decoding steps (i.e., pre-decoding + main decoding) can be applied [11], [13].

Let "Hard" mean hard-decision. Then $\text{Hard}(z(D))$ is expressed as $\text{Hard}(z(D))=i(D)G+e(D)$, where $i(D)$ and $e(D)$ are the original information sequence and the error sequence, respectively. Now consider SST Viterbi decoding of G . In this scheme, first, the information sequence $i(D)$ is roughly estimated by

$$\text{Hard}(z(D))G^{*-1}=i(D)D^L+e(D)G^{*-1}, \quad (12)$$

where $G^{*-1}=(1, 1)^T$ (T means transpose) corresponds to pre-decoding. Next, the pre-decoded sequence is re-encoded and then added to the L -delayed received sequence $z(D)D^L$. The result $z^\#(D)$ becomes the input to the main decoder. In particular, the hard-decision input to the main decoder is expressed as

$$\begin{aligned} \text{Hard}(z^\#(D)) &= (\text{Hard}(z(D))G^{*-1})G + \text{Hard}(z(D))D^L \\ &= (e(D)G^{*-1})G + e(D)D^L \cdots (*) \\ &= e(D)(H^T, H^T) = (\zeta(D), \zeta(D)), \end{aligned} \quad (13)$$

where $H^T=(g^{(2)}(D), g^{(1)}(D))^T$ is the syndrome former corresponding to G and $\zeta(D)$ represents the syndrome sequence. In the main decoder, $u(D) \equiv e(D)G^{*-1}$ (i.e., $u_k=e_k^{(1)}+e_k^{(2)}$) is decoded using the Viterbi algorithm (cf., (*)). Finally, it is added to the output from the pre-decoder, i.e., $\text{Hard}(z(D))G^{*-1}=i(D)D^L+e(D)G^{*-1}$, to get the original information sequence $i(D)D^L$.

3.2 Application of Soft-BIVA to the Main Decoder

Let $\{i_k\}$ be the original information sequence. Denote by $\{u_k=e_k^{(1)}+e_k^{(2)}\}$ the apparent information sequence for the main decoder. Then the final time- k output from the decoder is expressed as $i_{k-L}=(i_{k-L}+u_k)+u_k$. Since $(i_{k-L}+u_k)$

is the output from the pre-decoder (i.e., uniquely determined from the received data), we have

$$\begin{aligned} & |\ln[\mu(i_{k-L}=0|z)/\mu(i_{k-L}=1|z)]| \\ &= |\ln[\mu(u_k=0|z)/\mu(u_k=1|z)]|. \end{aligned} \quad (14)$$

That is, the reliability of the original symbol i_{k-L} is identical to that of u_k . By the way, since $u_k=e_k^{(1)}+e_k^{(2)}$ is the apparent information symbol for the main decoder, its reliability can be obtained by applying the Soft-BIVA to the main decoder. Note that the application of the Soft-BIVA is justified by the following facts:

- 1) In the Soft-BIVA, the *path* metric of the best path which passes through $s_k^\pm$ is used for symbol reliability estimation.
- 2) For SST Viterbi decoding of QLI codes, metrics can be computed using the *conventional* method by regarding $\{u_k\}$ as the original information sequence [14].

If these assumptions did not hold, we could not apply the Soft-BIVA to the main decoder in an SST Viterbi decoder.

4 Symbol Reliability Estimation Using Bidirectional Syndrome Decoding

It is known that the structure of the main decoder in an SST Viterbi decoder for QLI codes is closely related to syndrome decoding [13]. Then it is quite natural to expect that the reliability of the symbol i_{k-L} can be obtained using bidirectional syndrome decoding.

Again we take a QLI code specified by $G=(g^{(1)}(D), g^{(2)}(D))$ ($g^{(1)}(D)+g^{(2)}(D)=D^L$). The corresponding syndrome former is given by $H^T=(g^{(2)}(D), g^{(1)}(D))^T$. We assume that the degree of $g^{(n)}(D)$ ($n=1, 2$) is ν . Also let $\sigma(k)=(\sigma_1(k), \sigma_2(k), \dots, \sigma_\nu(k))$ be the time- k state representation of an *observer canonical form* [15] of the syndrome former H^T . Then the least significant bit $\sigma_\nu(k)$ is expressed as $\sigma_\nu(k)=e_k^{(1)}+e_k^{(2)}$.

Since $\sigma_\nu(k)$ is the least significant bit of the state $\sigma(k)$ in the syndrome trellis of H^T , its reliability can be estimated using bidirectional syndrome decoding like the Soft-BIVA. That is, the reliability of $e_k^{(1)}+e_k^{(2)}$ is obtained by applying bidirectional syndrome decoding.

Next, we investigate what is corresponding to the reliability of $e_k^{(1)}+e_k^{(2)}$ in the original information

sequence. It is known that there exists a one-to-one correspondence between the states of an encoder and those of the corresponding syndrome former [15]. Let an encoder physical state be $(i_{k-\nu+1}, \dots, i_{k-L}, \dots, i_{k-1}, i_k)$. This state is represented by a polynomial ψ in D^{-1} :

$$\psi = i_k + i_{k-1}D^{-1} + \dots + i_{k-L}D^{-L} + \dots + i_{k-\nu+1}D^{-\nu+1}. \quad (15)$$

Next, we compute the corresponding abstract state:

$$v \equiv (v_1, v_2) = (\psi G)Q, \quad (16)$$

where “ Q ” is the projection operator that truncates sequences to start at time 1:

$$v_1 = (\psi g^{(1)}(D))Q, \quad (17a)$$

$$\begin{aligned} v_2 &= (\psi g^{(2)}(D))Q = (\psi (g^{(1)}(D) + D^L))Q \\ &= v_1 + (\psi D^L)Q \\ &= v_1 + i_k D^L + i_{k-1} D^{L-1} + \dots + i_{k-L+1} D. \end{aligned} \quad (17b)$$

Finally, in order to transform the resulting abstract state to the syndrome former state (i.e., *dual state*), we compute vH^T . After some calculations we have

$$\begin{aligned} vH^T &= (v_1, v_2)(g^{(2)}(D), g^{(1)}(D))^T \\ &= v_1(g^{(1)}(D) + D^L) \\ &\quad + (v_1 + i_k D^L + i_{k-1} D^{L-1} + \dots + i_{k-L+1} D)g^{(1)}(D) \\ &= v_1 D^L + (i_k D^L + i_{k-1} D^{L-1} + \dots + i_{k-L+1} D)g^{(1)}(D) \\ &= \dots + i_{k-L} D^\nu. \end{aligned} \quad (18)$$

That is, the coefficient of D^ν is equal to i_{k-L} . Hence, the physical state of the syndrome former is expressed as $\sigma(k) = (\sigma_1(k), \sigma_2(k), \dots, i_{k-L})$.

It has been shown that the state $(i_{k-\nu+1}, \dots, i_{k-L}, \dots, i_k)$ of the encoder corresponds to the state $(\sigma_1(k), \sigma_2(k), \dots, i_{k-L})$ of the syndrome former. This fact means that the reliability of $\sigma_\nu(k) = e_k^{(1)} + e_k^{(2)}$, which can be obtained using bidirectional syndrome decoding, is identical to that of i_{k-L} . This is consistent with the result of Section 3.

5 Structure of Bidirectional Syndrome Decoding

In this section, we examine the structure of bidirectional syndrome decoding introduced in the preceding section. In the following, we deal with a binary rate $(n-1)/n$, memory length ν , convolutional code specified by a polynomial generator matrix $G(D)$. By calculating an

invariant-factor decomposition [16], we get the corresponding syndrome former H^T . From the assumption of the code rate, H^T has the form $H = (h_1(D), h_2(D), \dots, h_n(D))$. If both G and H are “*minimal*”, their overall constraint lengths are equal (i.e., ν) [15], [16]. Then the maximum degree of $h_i(D)$ is ν . Hence, an observer canonical form of the syndrome former H^T contains ν memory elements.

Now, consider bidirectional syndrome decoding based on the syndrome trellis of H^T [17]. It is natural that backward decoding is also performed based on the syndrome trellis of H^T . In this case, however, it becomes necessary to generate the syndrome sequence in time-reversed order for backward decoding. Because in order for the backward decoder to go backward, the corresponding syndrome bits must be delivered to it in advance.

Now, with each convolutional code C a reciprocal dual code $C^\perp$ can be associated. A generator matrix H^* of $C^\perp$ is obtained in the following way [12]:

(1) Substitute D^{-1} for D in the generator matrix $H(D)$ of the dual code $C^\perp$.

(2) Then multiply the j -th row of the resulting matrix with $D^{\gamma(j)}$, where $\gamma(j)$ is the degree of the j -th row of $H(D)$.

Then it is shown that the time-reversed syndrome sequence is generated using H^{*T} .

Let $(e_1, \dots, e_k, \dots, e_N, e_{N+1}, \dots, e_{N+\nu})$ be an error sequence, where $e_k = (e_k^{(1)}, \dots, e_k^{(n)})$ and $(e_{N+1}, \dots, e_{N+\nu})$ are $\nu \times n$ “0” error symbols which force the syndrome former H^T into the all-zero state. Then, the syndrome sequence $\zeta^+ \equiv (\zeta_1, \dots, \zeta_{k-1}, \zeta_k)$ is generated when the error symbols $e^+ \equiv (e_1, \dots, e_{k-\nu+1}, \dots, e_{k-1}, e_k)$ are inputted to the syndrome former H^T . Similarly, the syndrome sequence $\zeta^- \equiv (\zeta_{N+\nu}, \dots, \zeta_{k+2}, \zeta_{k+1})$ is generated when the error symbols $e^- \equiv (e_N, \dots, e_k, e_{k-1}, \dots, e_{k-\nu+1})$ are inputted to the syndrome former H^{*T} .

Note that the two syndrome sequences ζ^+ and ζ^- are disjoint and the corresponding two error sequences e^+ and e^- have the intersection $(e_{k-\nu+1}, \dots, e_{k-1}, e_k)$. Therefore, it has been shown that if the syndrome bits are generated once and only once using the forward syndrome former H^T and the backward syndrome former H^{*T} , then the corresponding two error sequences have the intersection

of $\nu \times n$ error symbols. This property is considered to be dual to that for bidirectional (Viterbi) decoding based on a code trellis [6].

6 Conclusions

In this paper, we reviewed the Soft-BIVA based on the bidirectional Viterbi algorithm and stated its basic properties. Next, we applied the Soft-BIVA to general QLI codes. For QLI codes, in addition to conventional Viterbi decoding, SST Viterbi decoding can be applied. It has been shown that for SST Viterbi decoding of QLI codes, the reliability of the original information symbol i_{k-L} can be obtained from that of the apparent information symbol u_k for the main decoder. Furthermore, noting the close relation between SST Viterbi decoding and syndrome decoding for QLI codes, it has been shown that the reliability of u_k can be obtained using bidirectional syndrome decoding. It has also been found that the syndrome sequence must be generated in time-reversed order for backward syndrome decoding and for this purpose a generator matrix of the reciprocal dual code of the original code is required.

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