Blind Identification for IIR Model to Communication Applications

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Abstract

This paper deals with a blind system identification for an IIR model in a time domain. Based on an output over-sampling scheme, which can extract information about both the unknown input signal and system structure, an SIMO model with a common denominator polynomial in its transfer function can be obtained, then the blind identification problem can be solved by using the proposed algorithm. It can be applied to identification of a nonminimum phase IIR model without the restriction of an FIR system.

1 Introduction

Blind identification is an important problem arising in wide areas including digital communication, seismology, image processing and signal restoration since it may be impractical to obtain the source signal or the training sequence. In a digital communication literature, when the intersymbol interference (ISI) of transmitted sequence through the digital communication channel is significant, equalization should be performed to remove the distortion, which requires the knowledge of the channel coefficients. Classically a channel can be identified if a training sequence is available, however, the transmission of training sequence may reduce the transmission efficiency significantly. Thus an attractive interest is that the unknown channel is identified just through a channel output and some prior properties of the transmitted sequence, *i.e.*, it called blind identification or equalization.

In the last two decades many approaches for blind problem were developed, which include the high-order statistics based algorithms (HOS) [1, 2], the maximum likelihood (ML) approach [3], and recently attractive approaches based on the second-order cyclostationary statistics (SOCS) [4, 5, 6, 7]. However, the existing HOS approaches exhibit somewhat slow convergence and have heavy computational load, while the ML approaches require the statistics of an unknown input signal. The approaches based on SOCS have a considerably modest computation, however they have almost

been restricted availability to an FIR model. Though an over-sampling scheme based approach to the blind identification for an IIR model has been proposed [8], it seems that a high sampling rate is needed to ensure the identifiability. Another approach based on second-order cyclostationary is also presented [7], however, it is only applicable in frequency domain.

The purpose of this paper is to propose a novel blind identification approach to identify an IIR model. We will demonstrate that the over-sampled IIR model can be represented by an SIMO model description with a common denominator polynomial in its transfer function, and making use of it we identify the unknown IIR model from the only accessible over-sampled output.

2 Problem Statement

A linear discrete-time IIR channel model considered in this paper is illustrated in Figure 1. Let $\{s_m\}$ be the source sequence, $\{w(\cdot)\}$ is a white observation noise with zero mean and a finite variance, and is independent of the source sequence $\{s_m\}$. $\{z(\cdot)\}$ and $\{y(\cdot)\}$ are noise-free and noise-corrupted output signal respectively. Though it can also be modeled by an FIR model, it may need long length of the impulse response to guarantee its modeling accuracy. Therefore it is expected to obtain the IIR model identification in some practical applications. Now the problem we deal with is how to identify the IIR model parameters just from the noise corrupted output $\{y(\cdot)\}$.

Figure 1: A typical digital communication system

An input sequence consider here is given by

$$u(t) = \begin{cases} s_m, & t = mT \\ 0, & t \neq mT \end{cases} \quad (1)$$

where T is the symbol period. In many cases, the sequence $\{s_m\}$ can be considered as an i.i.d sequence with zero mean and a known variance σ_s^2 . The proposed algorithm can also be extended to the case in which the input signal has duration time T with a zero-order holder.

The system output is over-sampled and it becomes to be a cyclostationary signal [4]. The sampling interval Δ satisfies $\Delta = T/p$, where the integer p is referred to an over-sampling rate. Under the sampling interval Δ , the IIR model called Δ -model can be characterized by

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) + w(k) \quad (2)$$

where

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{b_1q^{-1} + \dots + b_nq^{-n}}{1 + a_1q^{-1} + \dots + a_nq^{-n}}$$

Here n is the model order and $q^{-1} \triangleq e^{-\Delta s}$ is denoted as a backward shift operator. Meanwhile the system input and output samples are denoted as $u(k)$ and $y(k)$ respectively, and $w(k)$ is a white additive observation noise.

The model in (2) can describe a linear system efficiently, however, we will use an alternative model description for the blind identification.

3 Model Description for Identification

Consider the state-space representation of the channel model, which can be written as follows:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) \\ y(k) = \mathbf{c}\mathbf{x}(k) + w(k) \end{cases} \quad (3)$$

where

$$\mathbf{A} = \begin{bmatrix} -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & \ddots & 0 \\ \vdots & \vdots & 0 & 1 \\ -a_n & 0 & \dots & 0 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \mathbf{c} = [1 \quad 0 \quad \dots \quad 0] \quad (4)$$

In the over-sampling scheme, we define some new variables corresponding to the over-sampling rate p as follows.

$$\begin{cases} \mathbf{x}_j(m) = \mathbf{x}(mp+j) \\ u_j(m) = u(mp+j) \\ z_j(m) = z(mp+j) = \mathbf{c}\mathbf{x}_j(m) \\ w_j(m) = w(mp+j) \\ y_j(m) = y(mp+j) = z_j(m) + w_j(m) \end{cases} \quad (5)$$

where $j = 1, \dots, p$. From the property of the transmitted sequence described in (1), $u_j(m)$ satisfies that

$$\begin{cases} u_1(m) = \dots = u_{p-1}(m) = 0 \\ u_p(m) = s_{m+1} \end{cases} \quad (6)$$

By using the property of the channel input given above, (3) can be written as

$$\begin{aligned} \mathbf{x}_1(m+1) &= \mathbf{x}((m+1)p+1) = \mathbf{A}^p\mathbf{x}(mp+1) \\ &\quad + \mathbf{b}s_{m+1} = \mathbf{A}^p\mathbf{x}_1(m) + \mathbf{b}s_{m+1} \\ \mathbf{x}_2(m+1) &= \mathbf{x}((m+1)p+2) = \mathbf{A}^p\mathbf{x}(mp+2) \\ &\quad + \mathbf{A}\mathbf{b}s_{m+1} = \mathbf{A}^p\mathbf{x}_2(m) + \mathbf{A}\mathbf{b}s_{m+1} \\ &\quad \dots \quad \dots \quad \dots \\ \mathbf{x}_p(m+1) &= \mathbf{x}((m+1)p+p) = \mathbf{A}^p\mathbf{x}(mp+p) \\ &\quad + \mathbf{A}^{p-1}\mathbf{b}s_{m+1} = \mathbf{A}^p\mathbf{x}_p(m) + \mathbf{A}^{p-1}\mathbf{b}s_{m+1} \end{aligned} \quad (7)$$

Let $z^{-1} \triangleq e^{-sT} = q^{-p}$, (7) can be written by the following expression

$$\begin{aligned} \mathbf{x}_j(m) &= z^{-1}\mathbf{A}^p\mathbf{x}_j(m) + \mathbf{A}^{j-1}\mathbf{b}s_m \\ &= (\mathbf{I} - z^{-1}\mathbf{A}^p)^{-1}\mathbf{A}^{j-1}\mathbf{b}s_m \end{aligned} \quad (8)$$

From the definition in (5), $y_j(m)$ can be given by

$$y_j(m) = \mathbf{c}(\mathbf{I} - z^{-1}\mathbf{A}^p)^{-1}\mathbf{A}^{j-1}\mathbf{b}s_m + w_j(m) \quad (9)$$

Define that

$$G(z^{-1}) = \det(\mathbf{I} - z^{-1}\mathbf{A}^p) = 1 + \sum_{i=1}^n g_i z^{-i} \quad (10a)$$

$$\begin{aligned} H_j(z^{-1}) &= \mathbf{c} \cdot \text{adj}(\mathbf{I} - z^{-1}\mathbf{A}^p) \cdot \mathbf{A}^{j-1}\mathbf{b} \\ &= \sum_{i=1}^n h_{j,i} z^{-i+1} \end{aligned} \quad (10b)$$

then (3) can be written in a compact form as

$$y_j(m) = \frac{H_j(z^{-1})}{G(z^{-1})}s_m + w_j(m) \quad (11)$$

(11) is an alternative representation of (2) with observation interval T , hence it can be denoted as T -model. Meanwhile, it can also be considered as a SIMO model description with input s_m and output $y_j(m)$. Summarizing the above results, we have following Theorem.

Theorem 1 The input signal to a linear channel is given in (1), where the emission period is T . When the channel output is over-sampled at interval $\Delta = T/p$, where p is an over-sampling rate, then the channel model given by (2) can also be described by a single-input p -output rational transfer function model with a common denominator polynomial $G(z^{-1})$, which is given in (10a). The p numerator polynomials $H_j(z^{-1})$ are given in (10b).

Furthermore, for $h_{1,1}$ we have following result.

Corollary The coefficient $h_{1,1}$ of $H_1(z^{-1})$ satisfies that $h_{1,1} = b_1 \geq 0$.

On the other hand, the parameters of Δ -model can be determined uniquely from the T -model parameters.

Substituting the definition of (5) into (2) leads to

$$\sum_{i=0}^n (a_i y_{\beta(k-i)}(\alpha(k-1-i))) = \sum_{i=1}^n (b_i s_{\alpha(k-i)}) + \sum_{i=0}^n (a_i w_{\beta(k-i)}(\alpha(k-1-i))) \quad (12)$$

where $\alpha(x)$ is an integer part of the quotient x/p , $\beta(x)$ is the remainder of division x/p , and $k = mp+1, \dots, (m+1)p$. Substituting (11) leads to

$$\sum_{i=0}^n (a_i H_{\beta(k-i)}(z^{-1}) z^{\alpha(k-1-i)-\alpha(k-1)}) = G(z^{-1}) \sum_{i=1}^n (b_i z^{\alpha(k-i)-\alpha(k-1)}) \quad (13)$$

If $G(z^{-1})$ and $H_j(z^{-1})$ are known, we can obtain a set of linear equations about a_i and b_i by making the coefficients equal with respect to the order of z^{-1} . Thus the Δ -model parameters can be determined by solving an over-determined linear equation set.

Following the results above, a new blind identification can be established in the next section.

4 Channel Identification

We consider the case for $p = 2$ for the sake of space, but it can be extended into a general case of $p > 2$.

4.1 Estimation of Numerator Polynomials

The SIMO representation of the channel is given in (11). For $j = 1, 2$, we have that

$$y_1(m) = \frac{H_1(z^{-1})}{G(z^{-1})} s_m + w_1(m) \quad (14a)$$

$$y_2(m) = \frac{H_2(z^{-1})}{G(z^{-1})} s_m + w_2(m) \quad (14b)$$

By multiplying $H_2(z^{-1})$ and $H_1(z^{-1})$ to (14a) and (14b) respectively, we have

$$\begin{aligned} H_2(z^{-1}) (y_1(m) - w_1(m)) \\ = H_1(z^{-1}) (y_2(m) - w_2(m)) \end{aligned} \quad (15)$$

Define a matrix $\mathbf{Y}_{1,2}(L)$ as

$$\mathbf{Y}_{1,2}(L) = [\mathbf{Y}_1(L) \quad -\mathbf{Y}_2(L)] \quad (16)$$

where

$$\mathbf{Y}_j(L) = \begin{bmatrix} y_j(L) & \cdots & y_j(1) \\ \vdots & \cdots & \vdots \\ y_j(N+L-1) & \cdots & y_j(N) \end{bmatrix}_{N \times L}$$

where $L \geq n$. From the definitions in (5), it can also be expressed by

$$\mathbf{Y}_{1,2}(L) = \mathbf{Z}_{1,2}(L) + \mathbf{W}_{1,2}(L) \quad (17)$$

where $\mathbf{Z}_{1,2}(L)$, $\mathbf{W}_{1,2}(L)$ have the same structure as $\mathbf{Y}_{1,2}(L)$ with just replacing $y_1(\cdot)$ and $y_2(\cdot)$ by $z_1(\cdot)$, $z_2(\cdot)$ and $w_1(\cdot)$, $w_2(\cdot)$ respectively.

Theorem 2 If $H_1(z^{-1})$ and $H_2(z^{-1})$ are coprime, and the transmitted sequence $\{s_m\}$ satisfies the persistently exciting (PE) condition, then

$$\text{Rank}(\mathbf{R}_{1,2}^Z(L)) = L + n - 1 \quad (18)$$

where $\mathbf{R}_{1,2}^Z(L) = (\mathbf{Z}_{1,2}^T(L) \mathbf{Z}_{1,2}(L)) / N$.

Proof: From the SIMO channel description, we have

$$\begin{aligned} \mathbf{z}_{1,2}^T(m) &= [z_1(m+L), \dots, z_1(m+1), \\ &\quad -z_2(m+L), \dots, -z_2(m+1)]^T \\ &= \mathbf{H}_{1,2} \mathbf{s}(m) \end{aligned} \quad (19)$$

where

$$\begin{aligned} \mathbf{H}_{1,2} &= \begin{bmatrix} \mathbf{H}_1 \\ -\mathbf{H}_2 \end{bmatrix} \\ \mathbf{H}_1 &= \begin{bmatrix} h_{1,1} & \cdots & h_{1,n} \\ & \ddots & \\ & & h_{1,1} & \cdots & h_{1,n} \end{bmatrix} \\ \mathbf{s}(m) &= \begin{bmatrix} \frac{s_{m+L}}{G(z^{-1})} \\ \vdots \\ \frac{s_{m-n+1}}{G(z^{-1})} \end{bmatrix} \end{aligned}$$

where the dimension of $\mathbf{H}_{1,2}$ is $2L \times (L + n - 1)$. If $H_1(z^{-1})$ and $H_2(z^{-1})$ are coprime, then

$$\text{Rank}(\mathbf{H}_{1,2}) = L + n - 1 \quad (20)$$

Let

$$\mathbf{S}(m) = \mathbf{s}(m) \mathbf{s}^T(m)$$

Then the matrix $\mathbf{R}_{1,2}^Z(L)$ can also be expressed by

$$\mathbf{R}_{1,2}^Z(L) = \mathbf{H}_{1,2} \left(\sum_{m=0}^{N-1} \mathbf{S}(m) \right) \mathbf{H}_{1,2}^T \quad (21)$$

Since the transmitted sequence $\{s_m\}$ satisfies the persistently exciting (PE) condition, the matrix $\sum_{m=0}^{N-1} \mathbf{S}(m)$ will be a definite positive matrix, and (18) can be obtained. $\square$

However, the observations are corrupted by observation noise, therefore we can only obtain $\mathbf{R}_{1,2}^Y(L)$. Since the observation noise $w(\cdot)$ is a white stationary noise and independent of the input signal, then we have that

$$\lim_{N \rightarrow \infty} \mathbf{R}_{1,2}^Y(L) = \lim_{N \rightarrow \infty} \mathbf{R}_{1,2}^Z(L) + \sigma_w^2 \mathbf{I} \quad (22)$$

Then the EVD of $\mathbf{R}_{1,2}^Y(L)$ can be obtained as

$$\mathbf{R}_{1,2}^Y(L) = [\mathbf{U}_s \quad \mathbf{U}_\perp] \begin{bmatrix} \mathbf{\Gamma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_\perp \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^T \\ \mathbf{U}_\perp^T \end{bmatrix} \quad (23)$$

And from Theorem 2, a column rank of $R_{1,2}^Z(L)$ is $L + n - 1$, therefore the eigenvectors can be divided into two subspaces [5], one is the signal subspace U_s determined by the columns of $H_{1,2}(L)$, and the other is noise subspace $U_\perp$, where the noise subspace $U_\perp$ is orthogonal to the signal subspace U_s , i.e., $U_\perp \perp H_{1,2}(L)$, therefore, we have

$$U_\perp^T H_{1,2}(L) = 0 \quad (24)$$

where the noise subspace has a dimension of $2L \times (L - n + 1)$. Let the parameters of numerator polynomials $H_1(z^{-1})$, $H_2(z^{-1})$ be

$$h_{1,2} = [h_{1,1} \ \cdots \ h_{1,n} \ h_{2,1} \ \cdots \ h_{2,n}]^T$$

Based on the orthogonal property given in (24), $h_{1,2}$ and the noise subspace $U_\perp(:, l)$ has following relationship

$$U_{\perp,12}^{(l)} h_{1,2} = 0 \quad (25)$$

where

$$U_{\perp,12}^{(l)} = \begin{bmatrix} U_{\perp,1}^{(l)} & U_{\perp,2}^{(l)} \\ U_{\perp,1}^{(l)} & U_{\perp,2}^{(l)} \\ \vdots & \vdots \\ U_{\perp,1}^{(l)} & U_{\perp,2}^{(l)} \end{bmatrix}$$

$$U_{\perp,1}^{(l)} = \begin{bmatrix} U_\perp(1, l) & \vdots & U_\perp(L, l) \\ U_\perp(L+1, l) & \vdots & U_\perp(2L, l) \end{bmatrix}$$

Then parameters estimate of $h_{1,2}$ with a scalar ambiguity η can be given by

$$\hat{h}_{1,2} = \left[\eta (\Psi_l^T \Psi_l)^{-1} \Psi_l^T \Psi_l \right] \quad (26)$$

where

$$\Psi_l = U_{\perp,12}^T(:, 2:2n), \quad \psi_l = U_{\perp,12}^T(:, 1) \quad (27)$$

From Theorem 2, $\eta = h_{1,1} = b_1$. Denote

$$\bar{h}_{1,2} = h_{1,2}/\eta$$

Then $\hat{\bar{h}}_{1,2}$ can be given by

$$\hat{\bar{h}}_{1,2} = \left[(\Psi_l^T \Psi_l)^{-1} \Psi_l^T \Psi_l \right] \quad (28)$$

where the ambiguity scalar η will be estimated later.

For the situation $p > 2$, the estimate of $H_j(z^{-1})$ is given in Section 4.3.

4.2 Denominator Parameters Estimation

Let the auto-covariance $R_j(\tau)$ be denoted as

$$R_j(\tau) = E \{ y_j(m) y_j(m - \tau) \}$$

then it follows from (11) that

$$E \left\{ \left(y_j(m) + \sum_{i=1}^n g_i y_j(m - i) \right) y_j(m - \tau) \right\} = E \left\{ \left(\sum_{i=1}^n h_{j,i} s(m - i) + \sum_{i=0}^n g_i w_j(m - i) \right) y_j(m - \tau) \right\} \quad (29)$$

Denote the impulse response sequence of $1/G(z^{-1})$ as

$$\frac{1}{G(z^{-1})} = \sum_{i=0}^{\infty} \gamma_i z^{-i} \quad (30)$$

Then $y_j(m)$ can also be written as

$$y_j(m) = \sum_{i=1}^{\infty} \lambda_{j,i} s(m - j) + w_j(m) \quad (31)$$

where

$$\begin{bmatrix} \lambda_{j,1} \\ \lambda_{j,2} \\ \lambda_{j,3} \\ \lambda_{j,4} \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{j,1} & \vdots & \vdots & \vdots & \vdots \\ h_{j,n} & \ddots & h_{j,1} & \vdots & \vdots \\ & h_{j,n} & \ddots & h_{j,1} & \vdots \\ & & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \end{bmatrix}$$

From the independence property of the sequence $\{s(m)\}$, (29) can be written as

$$E \left\{ \left(y_j(m) + \sum_{i=1}^n g_i y_j(m - i) \right) y_j(m - \tau) \right\} = \sum_{i=1}^{\max(0, n - \tau + 1)} h_{j,i + \tau} \lambda_{j,i} \sigma_s^2 + g_\tau \sigma_w^2 \quad (32)$$

where $\tau \geq 0$. For $\tau = 0, \dots, n - 1$, we have that

$$(\sigma_w^2 \mathbf{J} - \mathbf{R}_j) \mathbf{g} + \Omega_j \boldsymbol{\gamma} = \mathbf{r}_j - \sigma_w^2 \mathbf{e}_1 \quad (33)$$

where

$$\mathbf{R}_j = \begin{bmatrix} R_j(-1) & R_j(-2) & \cdots & R_j(-n) \\ R_j(0) & R_j(-1) & \cdots & R_j(-n+1) \\ \vdots & \vdots & \ddots & \vdots \\ R_j(n-2) & \cdots & R_j(0) & R_j(-1) \end{bmatrix}$$

$$\Omega_j = \begin{bmatrix} h_{j,1} & \cdots & h_{j,n} \\ \vdots & \ddots & \vdots \\ h_{j,n} & \cdots & h_{j,1} \end{bmatrix} \begin{bmatrix} h_{j,1} \\ \vdots \\ h_{j,n} \end{bmatrix} \sigma_w^2$$

$$\mathbf{r}_j = [R_j(0) \ \cdots \ R_j(-n+1)]^T \quad (34)$$

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{e}_1 = [1 \ 0 \ \dots \ 0]^T$$

Let $\bar{\mathbf{R}}_j = \mathbf{R}_j - \sigma_w^2 \mathbf{J}$, $\bar{\mathbf{r}}_j = \mathbf{r}_j - \sigma_w^2 \mathbf{e}_1$, then for $j = 1, 2$

$$\begin{bmatrix} \bar{\mathbf{R}}_1 & \Omega_1 \\ \bar{\mathbf{R}}_2 & \Omega_2 \end{bmatrix} \begin{bmatrix} \mathbf{g} \\ \gamma \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{r}}_1 \\ \bar{\mathbf{r}}_2 \end{bmatrix} \quad (35)$$

We have obtained the estimate $\hat{H}_j(z^{-1}) = \eta \hat{\hat{H}}_j(z^{-1})$. Then the estimate of Ω_j can be obtained by

$$\hat{\Omega}_j = \eta^2 \hat{\hat{\Omega}}_j \quad (36)$$

where the structure of $\hat{\hat{\Omega}}_j$ is the same as Ω_j but replacing $\hat{h}_{j,i}$ by $\hat{\hat{h}}_{j,i}$ and it can be calculated by using $\hat{\hat{H}}_j(z^{-1})$ as similar to (34). Denote $\bar{\gamma} = \eta^2 \gamma$, then (35) can be rewritten as

$$\begin{bmatrix} \bar{\mathbf{R}}_1 & \hat{\hat{\Omega}}_1 \\ \bar{\mathbf{R}}_2 & \hat{\hat{\Omega}}_2 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{g}} \\ \bar{\gamma} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{r}}_1 \\ \bar{\mathbf{r}}_2 \end{bmatrix} \quad (37)$$

Then g_i can be estimated from (37). Notice that $\gamma_0 = 1$, then η can be calculated by

$$\hat{\eta} = \sqrt{\hat{\gamma}_0} \quad (38)$$

Moreover, the parameters of Δ -model can also be calculated by using the approach given in Section 3.

4.3 Channel Identification for $p > 2$

Let us construct a matrix $\mathbf{Y}$ as follows.

$$\mathbf{Y}(L) = \begin{bmatrix} \mathbf{Y}_1 & -\mathbf{Y}_2 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{Y}_1 & \mathbf{0} & -\mathbf{Y}_3 & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{Y}_1 & \mathbf{0} & \dots & \mathbf{0} & -\mathbf{Y}_p \\ \hline \mathbf{0} & \mathbf{Y}_2 & -\mathbf{Y}_3 & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_2 & \mathbf{0} & \mathbf{0} & -\mathbf{Y}_p \\ \hline & & \vdots & & \\ \hline \mathbf{0} & \dots & \mathbf{0} & \mathbf{Y}_{p-1} & -\mathbf{Y}_p \end{bmatrix}^T$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & -\mathbf{H}_2 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{H}_1 & \mathbf{0} & -\mathbf{H}_3 & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{H}_1 & \mathbf{0} & \dots & \mathbf{0} & -\mathbf{H}_p \\ \hline \mathbf{0} & \mathbf{H}_2 & -\mathbf{H}_3 & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 & \mathbf{0} & \mathbf{0} & -\mathbf{H}_p \\ \hline & & \vdots & & \\ \hline \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}_{p-1} & -\mathbf{H}_p \end{bmatrix}$$

$$\Psi(m) = \begin{bmatrix} s(m) & & \\ & \ddots & \\ & & s(m) \end{bmatrix}$$

And denote $\Sigma(m) = \Psi(m)\Psi(m)^T$, then we have

$$\mathbf{R}^y(L) = \mathbf{H} \left(\sum_{m=0}^{N-1} \Sigma(m) \right) \mathbf{H}^T + \sigma_w^2 \mathbf{I} \quad (39)$$

Notice that $\mathbf{H}$ has dimension $(p-1)pL/2 \times (L+n-1)p$, while $\sum_{m=0}^{N-1} \Sigma(m)$ is a positive definite matrix with dimension $(L+n-1)p \times (L+n-1)p$, then similarly to the case $p = 2$, the parameters of numerator polynomials can be estimated by using subspace method [5]. Then by using the approach proposed in Section 4.2, the common denominator parameters can also be estimated just choose $j = 1, \dots, p$.

5 Numerical Simulations

In this paper we consider the transmitted sequence $\{s_m\}$ with emission period $T = 1.0$, variance $\sigma_s^2 = 1.0$, which is an i.i.d signal. We consider the discrete-time communication channel model with sampling interval $\Delta = 0.5$, i.e., $p = 2$. The observation noise w is an white noise with zero mean and a finite variance σ_w^2 and is independent of $\{s_m\}$, and the signal to noise ratio is set to SNR=20dB. We deal with two IIR models with different impulse response shape.

Case 1. The impulse response shape is often encountered with in a communication system. The over-sampled non-minimum phase Δ -model is given by

$$\begin{aligned} A(q^{-1}) &= 0.0545q^{-1} - 0.0179q^{-2} - 0.0460q^{-3} \\ &\quad + 0.2168q^{-4} + 0.1642q^{-5} \\ B(q^{-1}) &= 1 - 1.2479q^{-1} + 0.8574q^{-2} - 0.2902q^{-3} \end{aligned}$$

And the true parameters of T -model transfer function polynomials are given by

$$\begin{aligned} G(z^{-1}) &= 1 + 0.1575z^{-1} + 0.0109z^{-2} - 0.0842z^{-3} \\ H_1(z^{-1}) &= 0.0545z^{-1} - 0.0216z^{-2} + 0.3901z^{-3} \\ &\quad + 0.2037z^{-4} \\ H_2(z^{-1}) &= 0.0501z^{-1} + 0.1599z^{-2} + 0.3774z^{-3} \\ &\quad + 0.0477z^{-4} \end{aligned}$$

By using the observations during 2000T, the estimated parameters could be obtained as follows:

$$\begin{aligned} \hat{H}_1(z^{-1}) &= 0.0527z^{-1} - 0.0233z^{-2} + 0.3905z^{-3} \\ &\quad + 0.2023z^{-4} \\ \hat{H}_2(z^{-1}) &= 0.0479z^{-1} + 0.1579z^{-2} + 0.3779z^{-3} \\ &\quad + 0.0441z^{-4} \\ \hat{G}(z^{-1}) &= 1 + 0.1472z^{-1} + 0.0005z^{-2} - 0.10182z^{-3} \\ \hat{A}(q^{-1}) &= 1 - 1.2591q^{-1} + 0.8736q^{-2} - 0.3054q^{-3} \\ \hat{B}(q^{-1}) &= 0.0526q^{-1} - 0.0184q^{-2} - 0.0453q^{-3} \\ &\quad + 0.2157q^{-4} + 0.1632q^{-5} \end{aligned}$$

Case 2. Consider a discrete-time communication channel model with more general impulse response shape.

Figure 2: The Estimated impulse response.
Solid line: True; Dotted line: Estimate

Its sampling interval is $\Delta = 0.5$, i.e., $p = 2$, then the model is given as

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{2.0q^{-1} - 1.38q^{-2} - 0.756q^{-3}}{1 - 1.9101q^{-1} + 1.5183q^{-2} - 0.4607q^{-3}}$$

The SIMO model transfer function polynomials are given by

$$\begin{aligned} G(z^{-1}) &= 1 - 0.6117z^{-1} + 0.5453z^{-2} - 0.2122z^{-3} \\ H_1(z^{-1}) &= 2.0z^{-1} - 0.3553z^{-2} - 1.7836z^{-3} \\ H_2(z^{-1}) &= 2.4401z^{-1} - 2.6179z^{-2} - 0.3483z^{-3} \end{aligned}$$

The signal-noise ratio $SNR = 25dB$. The estimated parameters of the SIMO transfer function from the observations within $1200T$ are as follows:

$$\begin{aligned} \hat{G}(z^{-1}) &= 1 - 0.6261z^{-1} + 0.5615z^{-2} - 0.2398z^{-3} \\ \hat{H}_1(z^{-1}) &= 1.9915z^{-1} - 0.3861z^{-2} - 1.7570z^{-3} \\ \hat{H}_2(z^{-1}) &= 2.4232z^{-1} - 2.6320z^{-2} - 0.3155z^{-3} \end{aligned}$$

The original model parameters are calculated as follows

$$\frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})} = \frac{2.0105q^{-1} - 1.4914q^{-2} - 0.6529q^{-3}}{1 - 1.9401q^{-1} + 1.5690q^{-2} - 0.4897q^{-3}}$$

The estimated impulse response with sampling interval Δ is shown in Figure 3.

6 Conclusions

We have proposed a novel approach to solve the blind identification problem for an IIR model. By over-sampling the system output, an SISO model can be represented by an SIMO model, which contains the structure information of the system. It can be clarified that if the over-sampling rate $p \geq 2$, an IIR model can be identified by just using an accessible output observations sampled at higher sampling rate than the symbol rate.

Figure 3: The Estimated impulse response.
Solid line: True; Dotted line: Estimate

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