Identification of Multivariable State Feedback Systems Using the Indirect Identification method

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Abstract

In this paper, we propose an indirect identification method to obtain the system matrices of the plant under a state feedback with a full order observer (or Kalman filter). First it is shown that the plant transfer function can be factored into a right coprime form such that each factor can be identified by applying 4SID (specially PO-MOESP) identification methods. Then it is shown that the estimated plant model is twice of the true order of the plant due to the existence of the full order observer in the closed loop system. Finally a balanced model reduction method for unstable systems is applied for reducing the estimated plant model to a half order. A numerical example is given to show the effectiveness of the proposed method.

1 Introduction

Many plants operate under feedback controllers which are often designed based on their nominal mathematical models, including, naturally, modeling errors. Also due to unstable behavior of the plant as well as efficiency of operation, experimental data can only be obtained under so-called closed loop conditions. Recently, many research efforts are directed toward to develop identification methods for closed-loop systems [1] [2].

One of difficulties identifying the plant dynamics in a closed loop condition is the fact that the control signal is correlated with the process and the measurement noises. Therefore, open-loop identification methods can not apply directly to closed-loop systems. One of methods of solving this problem is an indirect identification method. The characteristic of this method is that a measurable reference signal is utilized as an input. Especially Hansen [3] and Van den Hof [4] proposed closed

loop identification methods based on coprime factorization of the plant which is one of indirect solutions. The coprime factorization of a stable or an unstable rational matrix is composed of two stable rational matrices. Therefore we can identify the two factors by any open loop identification methods. Additionally it is also known that a right coprime factorization of the plant by the state space representation exists [5]. On the basis of this fact, we already have proposed an identification method of MIMO state feedback system applying the 4SID method [6]. However our scheme did not take account of an observer. In most of real systems, only the part of state variables can be observed. To realize state feedback, it is necessary to estimate the rest of them by an observer.

In this paper, we propose a new method to identify the system matrices of the plant using an indirect identification method under a state feedback with a full order observer. We first examine a method to calculate the system matrices of the plant using a right coprime factorization in the state space representation. Next we show an extended state space representation which includes dynamics of the full order observer and applying the 4SID (specially, PO-MOESP) method for identifying the system matrices of the plant. Since we obtain the estimated model having the two times order of the true plant model due to the existence of the full order observer in the closed-loop system, we apply the balanced model reduction method for unstable system models such that the final estimated model becomes the same order of the plant model. Finally, we show the effectiveness of the proposed method and the applicability of the model reduction method through numerical experiments.

2 Problem Statement

Consider a linear discrete time-invariant system with a state feedback and an observer (or Kalman Filter) shown in Fig. 1.

Fig.1 : State feedback system with observer

Here, $\hat{x}$ is the state vector estimated by the full order observer based on the nominal model (A_0, B_0, C_0, E_0) which includes the modeling error. K_0 is a feedback gain designed by the optimal regulator by solving LQ or LQG problems using the nominal model, and $r(t)$ is a reference signal which satisfies PE condition. Let us denote the matrix transfer function between u and y as P , namely

$$y = Pu. \quad (1)$$

Then we assume that the plant is described by the following state space model:

$$x(t+1) = Ax(t) + Bu(t) + Fw(t) \quad (2)$$

$$y(t) = Cx(t) + Eu(t), \quad (3)$$

where $x(t) \in \mathcal{R}^n$ is a state vector, $u(t) \in \mathcal{R}^m$ is an input signal, $y(t) \in \mathcal{R}^l$ is an output signal, and $w(t) \in \mathcal{R}^n$ is an unmeasurable zero mean white noise which is statistically uncorrelated to $r(t)$. Also in Fig. 1, $u(t)$ is given by

$$u(t) = r(t) - K_0 \hat{x}(t). \quad (4)$$

As shown in Fig.1, $u(t)$ and $y(t)$ are corrupted by measurement noises $v_1(t)$ and $v_2(t)$, respectively. Namely, the input and output data are given by

$$\tilde{y}(t) = y(t) + v_1(t) \quad (5)$$

and

$$\tilde{u}(t) = u(t) + v_2(t), \quad (6)$$

where $v_1(t)$ and $v_2(t)$ are zero mean white noises, and are statistically uncorrelated to $r(t)$.

In this paper, we present a scheme to estimate the system matrices of P in (1) under these circumstances shown in Fig.1.

3 A Right Factorization of the Plant

Let us denote the matrix transfer functions between $r(t)$ and $y(t)$ and between $r(t)$ and $u(t)$, as N ($: l \times m$) and D ($: m \times m$) respectively. Namely

$$y = Nr \quad (7)$$

$$u = Dr. \quad (8)$$

Then N and D are realized by (A, B, C, E) and $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{E})$, respectively, where we can show $\tilde{E} = I_m$ ($: m \times m$ identity matrix). Since $\tilde{E}^{-1}$ always exists due to the existence of a state feedback, we can show that the inverse system D^{-1} exists [5] [7] and is given by

$$D^{-1} = \tilde{C}(zI - \tilde{A})^{-1}\tilde{B} + \tilde{E}, \quad (9)$$

where $\tilde{A} = \tilde{A} - \tilde{B}\tilde{E}^{-1}\tilde{C}$, $\tilde{B} = \tilde{B}\tilde{E}^{-1}$, $\tilde{C} = -\tilde{E}^{-1}\tilde{C}$, $\tilde{E} = \tilde{E}^{-1} = I_m$. From (1), (7) and (8), we have the following identity,

$$P = ND^{-1}. \quad (10)$$

Therefore substituting the state space representations of N and D^{-1} into (10), P is given by

$$\begin{aligned} P &= \left[\begin{array}{c|c} A & B \\ \hline C & E \end{array} \right] \times \left[\begin{array}{c|c} \tilde{A} - \tilde{B}\tilde{E}^{-1}\tilde{C} & \tilde{B}\tilde{E}^{-1} \\ \hline -\tilde{E}^{-1}\tilde{C} & \tilde{E}^{-1} \end{array} \right] \\ &= \left[\begin{array}{c|c} \left[\begin{array}{cc} A & -B\tilde{E}^{-1}\tilde{C} \\ 0 & \tilde{A} - \tilde{B}\tilde{E}^{-1}\tilde{C} \end{array} \right] & \left[\begin{array}{c} B\tilde{E}^{-1} \\ \tilde{B}\tilde{E}^{-1} \end{array} \right] \\ \hline \left[\begin{array}{cc} C & -E\tilde{E}^{-1}\tilde{C} \end{array} \right] & E\tilde{E}^{-1} \end{array} \right]. \end{aligned} \quad (11)$$

By using the reference signal $r(t)$ and available noisy corrupted data $\tilde{u}(t)$ and $\tilde{y}(t)$, we can estimate N and D by applying the 4SID method [8] [9], especially PO-MOESP method[8] as the next section.

4 Identification Method Based on a Right Coprime Factorization

4.1 An extended order state space model

In this section, we show an extended state space model of N and D . As shown in (2), (3) and (5), the state space representation from u to $\tilde{y}$ is

$$x(t+1) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{F}w(t), \quad (12)$$

$$\tilde{y}(t) = \mathcal{C}x(t) + \mathcal{E}u(t) + v_1(t). \quad (13)$$

Also a full order observer $\hat{x}(t)$ for state of the plant is given by

$$\hat{x}(t+1) = F\hat{x}(t) + G\tilde{y}(t) + Ju(t), \quad (14)$$

where

$$F = \mathcal{A}_0 - G\mathcal{C}_0, \quad J = \mathcal{B}_0 - G\mathcal{E}_0.$$

Then from (4), (6), $\tilde{u}$ is given by

$$\begin{aligned} \tilde{u}(t) &= u(t) + v_2(t) \\ &= r(t) - K_0\hat{x}(t) + v_2(t). \end{aligned} \quad (15)$$

Here we will derive an extended state space model with the input r and the output $\tilde{u}$ and $\tilde{y}$ as follows.

From (12) and (14), we have

$$\begin{aligned} x(t+1) &= \mathcal{A}x(t) + \mathcal{B}(r(t) - K_0\hat{x}(t)) + \mathcal{F}w(t). \end{aligned} \quad (16)$$

Also, from (13), (14) and (15), we have

$$\begin{aligned} \hat{x}(t+1) &= F\hat{x}(t) + G(\mathcal{C}x(t) + \mathcal{E}u(t) + v_1(t)) \\ &\quad + J(r(t) - K_0\hat{x}(t)) \\ &= \hat{x}(t) + G\mathcal{C}x(t) + G\mathcal{E}r(t) - G\mathcal{E}K_0\hat{x}(t) \\ &\quad + Jr(t) - JK_0\hat{x}(t) + Gv_1(t). \end{aligned} \quad (17)$$

Furthermore $\tilde{y}(t)$, $\tilde{u}(t)$ in (13) and (15) can also be written, respectively, as follows.

$$\begin{aligned} \tilde{y}(t) &= \mathcal{C}x(t) + \mathcal{E}u(t) + v_1(t) \\ &= \mathcal{C}x(t) + \mathcal{E}r(t) - \mathcal{E}K_0\hat{x}(t) + v_1(t), \end{aligned} \quad (18)$$

$$\tilde{u}(t) = -K_0\hat{x}(t) + r(t) + v_2(t). \quad (19)$$

From (16)-(19), we have an extended state space model,

$$\begin{aligned} \begin{bmatrix} x(t+1) \\ \hat{x}(t+1) \end{bmatrix} &= \begin{bmatrix} \mathcal{A} & -\mathcal{B}K_0 \\ G\mathcal{C} & F - \mathcal{B}_0K_0 \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathcal{B} \\ J \end{bmatrix} r(t) + \begin{bmatrix} \mathcal{F}w(t) \\ Ev_1(t) \end{bmatrix} \end{aligned} \quad (20)$$

$$\begin{aligned} \begin{bmatrix} \tilde{y}(t) \\ \tilde{u}(t) \end{bmatrix} &= \begin{bmatrix} \mathcal{C} & -\mathcal{E}K_0 \\ 0 & -K_0 \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathcal{E} \\ I_m \end{bmatrix} r(t) + \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}. \end{aligned} \quad (21)$$

Denote $\xi(t) = [x(t)^T \hat{x}(t)^T]^T$. Then (20) and (21) can be rewritten by

$$\xi(t+1) = A_{2n}\xi(t) + B_{2n}r(t) + \begin{bmatrix} \mathcal{F}w(t) \\ Gv_1(t) \end{bmatrix} \quad (22)$$

$$\begin{aligned} \begin{bmatrix} \tilde{y}(t) \\ \tilde{u}(t) \end{bmatrix} &= \begin{bmatrix} C_{2n} \\ \tilde{C}_{2n} \end{bmatrix} \xi(t) + \begin{bmatrix} \mathcal{E} \\ I_m \end{bmatrix} r(t) \\ &\quad + \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}. \end{aligned} \quad (23)$$

From (22), (23), the extended state space model is a deterministic - stochastic system with m inputs, $(l+m)$ outputs and $\xi \in \mathcal{R}^{2n}$. In this paper, we identify the deterministic part $(A_{2n}, B_{2n}, C_{2n}, \tilde{C}_{2n}, \mathcal{E}, I_m)$ in (22), (23) by applying the PO-MOESP method [8].

4.2 Identification of plant dynamics based on a right coprime factorization

After applying the PO-MOESP method, we have estimates

$$\hat{N} : (\hat{A}_{2n}, \hat{B}_{2n}, \hat{C}_{2n}, \hat{\mathcal{E}}) \quad (24)$$

and

$$\hat{D} : (\hat{A}_{2n}, \hat{B}_{2n}, \hat{\tilde{C}}_{2n}, I_m). \quad (25)$$

Let us denote the estimated transfer function $\hat{P}$ and the corresponding system matrices $(\hat{A}, \hat{B}, \hat{C}, \hat{\mathcal{E}})$.

Substituting (24) and (25) into (11), $\hat{P}$ is given by

$$\begin{aligned}\hat{P} &= \left[\begin{array}{c|c} \begin{bmatrix} \hat{A}_{2n} & -\hat{B}_{2n}\hat{C}_{2n} \\ 0 & \hat{A}_{2n} - \hat{B}_{2n}\hat{C}_{2n} \end{bmatrix} & \begin{bmatrix} \hat{B}_{2n} \\ \hat{B}_{2n} \end{bmatrix} \\ \hline \begin{bmatrix} \hat{C}_{2n} & -\hat{E}\hat{C}_{2n} \end{bmatrix} & \hat{E} \end{array} \right] \\ &= \left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{E} \end{array} \right] \quad (26)\end{aligned}$$

Furthermore applying a similarity transformation $x(t) = T\bar{x}(t)$ where

$$T = \begin{bmatrix} I & I \\ 0 & I \end{bmatrix},$$

we can obtain the system matrices of the plant whose order is $2n$ as follows.

$$\begin{aligned}\hat{P} &= \left[\begin{array}{c|c} T^{-1}\hat{A}T & T^{-1}\hat{B} \\ \hline \hat{C}T & \hat{E} \end{array} \right] \\ &= \left[\begin{array}{c|c} \begin{bmatrix} \hat{A}_{2n} & 0 \\ 0 & \hat{A}_{2n} - \hat{B}_{2n}\hat{C}_{2n} \end{bmatrix} & \begin{bmatrix} 0 \\ \hat{B}_{2n} \end{bmatrix} \\ \hline \begin{bmatrix} \hat{C}_{2n} & \hat{C}_{2n} - \hat{E}\hat{C}_{2n} \end{bmatrix} & \hat{E} \end{array} \right] \\ &= \left[\begin{array}{c|c} \hat{A}_{2n} - \hat{B}_{2n}\hat{C}_{2n} & \hat{B}_{2n} \\ \hline \hat{C}_{2n} - \hat{E}\hat{C}_{2n} & \hat{E} \end{array} \right] \quad (27)\end{aligned}$$

where the third equality holds from a well known operation on transfer matrices.

5 Model Reduction Methods for Unstable Systems

In the previous section, it has been demonstrated how to estimate the system matrices of the plant whose order is twice of the true order due to the existence of the observer in the closed loop. In this section, we discuss a method to reduce the order of the estimated model. Our goal is to obtain an reduced order model which decreases the order of the observer. Since we have given an assumption that the plant is not necessary to be stable. In this paper, we examine one of model reduction method for unstable systems introduced in [10] as follows.

[Model reduction for unstable systems]

First, partition an unstable model into a stable part and an unstable part. Next obtain a reduced order model for the stable part. Finally coupling

the reduced order model with the unstable part of the model.

In this paper, we apply the balanced model reduction method [11] for the stable part of the estimated model.

6 Numerical Experiments

In this section, we examine the effectiveness of the proposed method and the applicability of model reduction method shown in Section 5 through numerical experiments.

Consider the following numerical example which comes from an inverse pendulum system with a single input and 2 outputs. The true model is assumed as

$$\begin{aligned}A &= \begin{bmatrix} 1 & -9.81 \times 10^{-5} & 4.45 \times 10^{-2} & -1.06 \times 10^{-6} \\ 0 & 1.03 & 1.33 \times 10^{-2} & 3.62 \times 10^{-2} \\ 0 & -3.81 \times 10^{-3} & 8.07 \times 10^{-1} & -6.65 \times 10^{-5} \\ 0 & 1.19 & 4.60 \times 10^{-1} & 5.15 \times 10^{-1} \end{bmatrix} \quad (28)\end{aligned}$$

$$B = \begin{bmatrix} 5.24 \times 10^{-3} \\ 1.61 \times 10^{-2} \\ 2.02 \times 10^{-1} \\ 6.03 \times 10^{-1} \end{bmatrix} \quad (29)$$

$$C = \begin{bmatrix} 21.7 & 0 & 0 & 0 \\ 0 & 3.34 & 0 & 0 \end{bmatrix} \quad (30)$$

$$E = 0 \quad (31)$$

Also a nominal model which includes a modeling error is given by

$$\begin{aligned}A_0 &= \begin{bmatrix} 1.1 & -1.04 \times 10^{-3} & 1.11 \times 10^{-1} & -3.98 \times 10^{-5} \\ 0 & 1.37 & 9.08 \times 10^{-2} & 7.95 \times 10^{-2} \\ 0 & -1.03 \times 10^{-2} & 4.18 \times 10^{-1} & -5.00 \times 10^{-4} \\ 0 & 2.32 & 6.95 \times 10^{-1} & 2.43 \times 10^{-1} \end{bmatrix} \quad (32)\end{aligned}$$

$$B_0 = \begin{bmatrix} 5.80 \times 10^{-2} \\ 1.50 \times 10^{-1} \\ 5.90 \times 10^{-1} \\ 1.62 \end{bmatrix} \quad (33)$$

$$C_0 = C \quad (34)$$

$$E_0 = E \quad (35)$$

First, we design a controller and a full order observer using the nominal model where we give the weighted matrices Q and R in the quadratic cost function as

$$Q = \text{diag} \begin{bmatrix} 3500 & 800 & 10 & 1 \end{bmatrix},$$

and

$$R = 1.$$

Then we have a feedback gain K_0 as

$$K_0 = \begin{bmatrix} -11.36 & 13.30 & -3.81 \times 10^{-1} & 8.25 \times 10^{-1} \end{bmatrix}. \quad (36)$$

Also the gain G in (14) for designing an observer is given by

$$G = \begin{bmatrix} 5.07 \times 10^{-2} & -3.09 \times 10^{-4} \\ 1.76 \times 10^{-7} & 4.10 \times 10^{-1} \\ 3.09 \times 10^{-7} & -3.07 \times 10^{-1} \\ 8.45 \times 10^{-7} & 6.95 \times 10^{-1} \end{bmatrix}. \quad (37)$$

where the gain G is obtained by solving a LQ problem. Here, a reference signal $r(t)$ is generated from zero mean Gaussian white noise sequences with unit variance. And the process noise $w(t)$ and measurement noises $v_1(t)$ and $v_2(t)$ are generated from mutually independent zero mean white noise sequences with each variance $(0.01)^2$. After stabilizing the plant by the observer and state feedback, we estimate the extended system matrices of the true model by the proposed method using 5,000 as well as 10,000 input-output data. The estimated models are of 8th order. The frequency response functions of estimated models, the nominal model and the true model are shown in Fig.2 (for data 5,000) and Fig.3 (for data 10,000). It is clearly seen that the estimated models by the proposed method are very closed to the true model with comparing to the nominal model.

Next we partition the estimated model in Fig. 3 into a stable and an unstable parts, and apply the balanced model reduction algorithm numerically by using the Matlab programs, where the order of the stable part is 6. Finally we obtain a reduced 4th order model. The frequency responses of reduced order model and true model are shown in Fig. 4.

7 Conclusion

In this paper, we have proposed a method to identify the system matrices of the plant under the state feedback with the full order observer by applying the indirect identification method. Through the numerical experiments, it has been shown that the proposed identification scheme with the model reduction algorithm is applicable for identifying the plant operated under a closed loop condition.

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Fig. 2: Frequency response function (for data 5,000)

Fig. 3: Frequency response function (for data 10,000)

Fig. 4: Frequency response functions of reduced order model and true model