Blind equalization of SIMO FIR systems by using circular Levinson algorithm

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abstract

An approach to blind equalization of single input and multiple output (SIMO) FIR systems is proposed. The approach utilizes the periodic linear prediction of SIMO FIR systems and exploits only the second-order statistics of the output signals. A joint order estimation and blind equalization can be achieved by the periodic linear prediction of a SIMO FIR system. With the circular Levinson algorithm, the periodic linear prediction problem can be efficiently solved. To improve the performance, a modified algorithm is developed. Computer simulations are presented to see the performance of the proposed approach.

1 Introduction

Recently, there have been a lot of works on the blind equalization based on the second-order statistics. See, e.g., [1], [2], and the references therein. The underlying idea of blind equalization based on second-order statistics is that, by fractional sampling and/or by utilizing an antenna array, a digital communication system can be modeled as a single input and multiple output (SIMO) FIR system, or equivalently, as a scalar linear periodically time-variant (LPTV) system. The properties of SIMO FIR systems or the cyclostationarity of the output of LPTV systems enable us to blindly identify the unknown system and to blindly equalize its output (c.f. [3], [4]).

Many algorithms for the blind identification with second-order statistics have been proposed, e.g., the least-squares approach by Xu and et al. [5], the subspace approach by Moulines et al. [2]. Once the channel is estimated by a blind identification algorithm, the equalizer is constructed based on the estimated channel

coefficients. However, most of them fail in blind identification unless the true order of the unknown system is available. As it is often the case in parametric estimation methods, it is not easy to estimate the order of the channel.

For blind equalization, direct (one-step) equalizer design, that avoids the step of the estimation of the channel parameters, may be attractive. For one thing, it is computationally effective. Another is that it is robust to the order of the equalizer. The linear prediction approach, proposed by Slock [6], directly equalize the outputs of the unknown system and can be applied to the case where the order of the equalizer is overdetermined [7]. Giannakis and Halford [8] [9] proposed another approach to direct equalizer design based on the zero forcing conditions and the minimum mean square error criterion. They developed adaptive direct blind equalization algorithms, which are computationally effective and are able to track (slowly) time-varying channels. However, they can not apply to the case where there is a delay between the transmitter and the receiver (, strictly speaking, if the delay is not a multiple of the symbol keying period).

We propose a novel approach to direct equalizer design. We introduce the periodic linear prediction of the outputs of a SIMO FIR system, in which linear prediction is performed for the output of each subchannel. It is shown that, without noises, the output of a SIMO FIR channel is equivalent to a scalar periodic AR process and hence the blind equalization can be achieved by solving the Yule Walker equations for the periodic linear prediction. We develop an algorithm to solve the periodic linear prediction problem based on the circular Levinson algorithm [10]. We also present a

procedure for the determination of the length of equalizer. Computer simulations are presented to evaluate the performance of the proposed algorithms.

2 Problem formulation

Let us consider a single input and M outputs (SIMO) FIR model given by

$$\mathbf{y}(n) = \sum_{m=0}^L \mathbf{h}(m)x(n-m) + \mathbf{v}(n), \quad (1)$$

where $\mathbf{y}(n) = [y_0(n), \dots, y_{M-1}(n)]^T$ is the output of an M -channel system with impulse response:

$$\mathbf{h}(n) = [h_0(n), \dots, h_{M-1}(n)]^T, \quad (2)$$

$\{x(n)\}$ is the scalar input symbol of the system, and $\mathbf{v}(n) = [v_0(n), \dots, v_{M-1}(n)]^T$ is the additive noise vector. This model results from oversampling digital communication systems and/or from exploiting antenna arrays (c.f. [3]). We note that, by putting

$$y(Mn+i) = y_i(n) \quad (3)$$

for $i = 0, \dots, M-1$ and $n = 0, 1, \dots$, and similarly for $v(n)$ and $h(n)$, the SIMO FIR system is shown to be equivalent to an SISO system given by

$$y(n) = \sum_{m=0}^L h(n-Mm)x(m) + v(n). \quad (4)$$

The above system is a linear periodically time-variant (LPTV) system with period M , because the impulse response from the input $x(n)$ to the output $y(n)$ is periodic in time with period M . It is well-known that the output $y(n)$ of a LPTV system with period M driven by a stationary input is a cyclostationary process. The correlation function of a cyclostationary process with period M is periodic in m and n such as

$$R(m+M, n+M) = R(m, n), \quad (5)$$

where

$$R(m, n) := E[y(m)y^*(n)], \quad (6)$$

and $E[\cdot]$ denotes the expectation operator and the asterisk $*$ stands for the complex conjugation.

The goal of the paper is to retrieve the original input signal up to a scaling factor only from the second-order statistics of the corresponding output signal without prior knowledge of the system and the input symbols.

In the sequel, let us denote the z -transform of a sequence by a corresponding capital letter. Then, let us make the following assumptions.

(A1) The components of $\mathbf{H}(z)$ have no common zero.

(A2) Let the leading coefficient (i.e. the first nonzero element) of the sequence $\{h(n)\}$ be $h(d)$, i.e., $h_d(0)$. Then assume $d < M$.

(A3) The input signal $x(n)$ is a zero mean stationary process with the correlation function

$$E[x(n+\tau)x^*(n)] = \delta(\tau)\sigma_x^2, \quad (7)$$

where $\delta(k)$ stands for the Kronecker delta.

(A4) The additive noise vector is white with zero-mean and variance $\sigma_v^2 I$. It is independent of the input.

3 Periodic Linear Prediction

Let us define the linear prediction error of order j for the output of the k th subchannel as

$$\epsilon_j(Mn+k) = y(Mn+k) - \sum_{m=1}^j a_k^*(m)y(Mn+k-m). \quad (8)$$

Now let us find the optimal taps that minimize the error variance defined as

$$\sigma_{k,j}^2 := E[|\epsilon_j(Mn+k)|^2], \quad (9)$$

for each $k = 0, 1, \dots, M-1$. The minimization leads to Yule-Walker equations given by

$$\mathbf{R}_{k,j} [1, -a_k(1), \dots, -a_k(j)]^T = [\sigma_{k,j}^2, 0, \dots, 0]^T, \quad (10)$$

where

$$\mathbf{R}_{k,j} = \begin{bmatrix} R(k, k) & \cdots & R(k, k-j) \\ \vdots & \ddots & \vdots \\ R(k-j, k) & \cdots & R(k-j, k-j) \end{bmatrix}. \quad (11)$$

We note that the above linear prediction is different from the usual one in the sense that the optimal taps and the variances of the resultant errors vary periodically in time. Thus we refer to the above linear prediction as *periodic linear prediction*.

Let us give the following basic lemma without proof.

Lemma 1: Under Assumptions (A1), (A2) and (A3), if there is no additive noise, then there exist a set of integers $(p_0, p_1, \dots, p_{M-1})$ and the prediction filters such that

$$y(Mn+k) - \sum_{m=1}^j a_k^*(m)y(Mn+k-m)$$

$$= \begin{cases} h_d(0)x(n) & \text{for } j \geq p_d, \text{ if } k = d, \\ 0 & \text{for } j \geq p_k, \text{ otherwise,} \end{cases} \quad (12)$$

This lemma means that the blind equalization can be achieved by solving the Yule-Walker equations for the subchannel d and an order $j \geq p_d$.

We also get from (12) that

$$\sigma_{k,j}^2 = \begin{cases} |h_d(0)|^2 \sigma_x^2 & \text{for } j \geq p_d, \text{ if } k = d, \\ 0 & \text{for } j \geq p_k, \text{ otherwise.} \end{cases} \quad (13)$$

This enables us to determine the *delay* d of the scalar channel $\{h(n)\}$ (, or equivalently, the subchannel d of the SIMO system $\{h(n)\}$).

Lemma 1 also implies that the periodic FIR system (12) is periodically autoregressive (AR) such as

$$y(Mn+k) = \sum_{m=1}^{p_k} a_k^*(m)y(Mn+k-m) + \epsilon_{p_k}(Mn+k). \quad (14)$$

Thus we can estimate the orders $\{p_k\}$ of the periodic AR system by using the theory of the AR model fitting [10].

Let us consider the case where there is a white additive noise and suppose the optimal minimum mean squared error (MMSE) filter $F(z)$ of order j that minimizes

$$E[|x(n) - \sum_{m=0}^j f^*(m)y(Mn+d-m)|^2]. \quad (15)$$

The minimization yields

$$\mathbf{R}_{d,j} [f(0), f(1), f(2), \dots, f(j)]^T = [h_d(0)\sigma_x^2, 0, \dots, 0]^T. \quad (16)$$

It is easy to see from (10) and (16) that the d th linear prediction filter is proportional to the MMSE filter. This means that the d th linear prediction filter is optimal in the mean-squared sense even if the order is underestimated or overestimated and hence our approach is robust to the order estimation.

We note that Giannakis and Halford [8] [9] gave the optimum equalizers based on (16) when $h_0(0) \neq 0$. Thus their optimal equalizer is equivalent to ours when $h_0(0) \neq 0$. However, they did not consider the case where $h_0(0) = 0$.

4 Algorithms for the periodic linear prediction approach

With prediction filters of sufficient length, say p , and the given d , we only have to solve a set of Yule Walker equations, which requires about $O(p^3)$ operations. If we

estimate the delay, we have to compute M sets of Yule Walker equations. This requires about $O(Mp^3)$ operations. If we would like to obtain an equalizer of reasonable length, we have to solve M sets of Yule Walker equations for each order to determine the length of the equalizer. This requires about $O(Mp^4)$ operations with direct computations, where p is the maximal order of the prediction filters. Sakai [10] proposed the circular Levinson algorithm to efficiently solve M sets of Yule Walker equations, which requires only $O(Mp^2)$ operations. Unfortunately, our computer simulations showed that the circular Levinson algorithm is not stable in some scenarios. Thus we give the following alternative algorithm.

Let us define

$$\mathbf{a}_{k,j} = [1, -a_{k,j}(1), \dots, -a_{k,j}(j)]^T, \quad (17)$$

for $k = 0, \dots, M-1$, where we denote the explicit dependence of $a_k(n)$ on j as $a_{k,j}(n)$. Then (10) is written as

$$\mathbf{R}_{k,j} \mathbf{a}_{k,j} = [\sigma_{k,j}^2, 0, \dots, 0]^T. \quad (18)$$

As in [10], let us introduce auxiliary vectors:

$$\mathbf{b}_{k,j} = [-b_{k,j}(j), \dots, -b_{k,j}(1), 1]^T, \quad (19)$$

for $k = 0, \dots, M-1$, which satisfy

$$\mathbf{R}_{k,j} \mathbf{b}_{k,j} = [0, \dots, 0, \tau_{k,j}^2]^T. \quad (20)$$

Let us define the initial values as

$$\alpha_{k,0} = \beta_{k,0} = 1, \quad \sigma_{k,0}^2 = \tau_{k,0}^2 = R(k, k), \quad (21)$$

for $k = 0, \dots, M-1$.

At order j , let us solve the equations given by

$$\mathbf{R}_{-1,j} \mathbf{b}_{-1,j} = [0, \dots, 0, \tau_{-1,j}^2]^T, \quad (22)$$

where $\mathbf{R}_{-1,j}$ is defined as

$$\mathbf{R}_{-1,j} = \begin{bmatrix} R(-M, -M) & \cdots & R(-M, -M-j) \\ \vdots & \ddots & \vdots \\ R(-M-j, -M) & \cdots & R(-M-j, -M-j) \end{bmatrix}. \quad (23)$$

Then, as in [10], for $k = 0, \dots, M-1$, we have the following equations for the order update:

$$\mathbf{a}_{k,j+1} = \begin{bmatrix} \mathbf{a}_{k,j} \\ 0 \end{bmatrix} - \frac{\Delta_{k,j}}{\tau_{k-1,j}^2} \begin{bmatrix} 0 \\ \mathbf{b}_{k-1,j} \end{bmatrix} \quad (24)$$

$$\sigma_{k,j+1}^2 = \sigma_{k,j}^2 - \frac{|\Delta_{k,j}|^2}{\tau_{k-1,j}^2} \quad (25)$$

$$\mathbf{b}_{k,j+1} = \begin{bmatrix} 0 \\ \mathbf{b}_{k-1,j} \end{bmatrix} - \frac{\Delta_{k,j}^*}{\sigma_{k,j}^2} \begin{bmatrix} \mathbf{a}_{k,j} \\ 0 \end{bmatrix} \quad (26)$$

$$\tau_{k,j+1}^2 = \tau_{k-1,j}^2 - \frac{|\Delta_{k,j}|^2}{\sigma_{k,j}^2} \quad (27)$$

where

$$\Delta_{k,j} = \sum_{n=0}^j R(k-j-1, k-n) \alpha_{k,j}(n). \quad (28)$$

This algorithm compute the optimal filters and the resultant error variances successively both in order and subchannel. Hence it is matched to our requirement for the estimation of the orders and delay. Since solving (22) requires about $O(j^3)$ computations at order j , the computational complexly of the above algorithm is about $O(p^4)$.

We remark that, in the circular Levinson algorithm, $\mathbf{b}_{-1,j}$ and $\tau_{-1,j}^2$ in (22) are set as

$$\mathbf{b}_{-1,j} = \mathbf{b}_{M-1,j}, \quad \tau_{-1,j}^2 = \tau_{M-1,j}^2, \quad (29)$$

in place of solving (22). This resulted from the periodicity of the correlations function [10] (c.f. (5)). However, in practice, the sample correlation function does not exhibit the pure periodicity. This may be a reason why the circular Levinson algorithm was not stable in some of our computer simulations.

To estimate the orders and the delay, we propose the following procedure based on (13).

Step 1 Determine the order of the k th subchannel by

$$\hat{p}_k = \arg \min_j (N \log \hat{\sigma}_{k,j}^2 + p(j)), \quad (30)$$

where $\hat{\sigma}_{k,j}^2$ is the error variance of the k th prediction filter of order j when the sample correlation function estimated from N observations is used and $p(j)$ is an appropriate penalty function.

Step 2 Put the maximal order to be $\hat{p}_{max} = \max_k \hat{p}_k$.

Step 3 Estimate the delay by $\hat{d} = \arg \max_k \hat{\sigma}_{k,p_{max}}^2$.

5 Simulation results

The performance of proposed algorithms was assessed by Monte-Carlo experiments in deferent scenarios. For each experiment 100 independent Monte-Carlo simulations were performed.

The input symbols were drawn from a 4-QPK constellation. The additive noises were i.i.d. zero-mean

Fig. 1. Averaged ISI for the first example where p denotes the order of equalizer and the number of symbols is 200.

Fig. 2. Comparison of the proposed algorithm and the circular Levinson algorithm.

circular Gaussian. We set the penalty function in (30) for the estimation of the order to be $\log N$, where N is the number of symbols used for the equalization. As a performance measure of equalization, the intersymbol interference (ISI) was employed, which is defined by

$$\frac{\sum_n |f(n)|^2 - \max_n |f(n)|^2}{\max_n |f(n)|^2} \quad (31)$$

where $f(n)$ is the composite equalized channel impulse response. For simplicity of comparison, we normalized the channel by $h_d(0) = 1$. The signal-to-noise ratio (SNR) is defined as

$$10 \log \frac{E[|x(n)|^2]}{E[|v(n)|^2]}. \quad (32)$$

We considered the first example in [5], in which the number of subchannels was set as $M = 4$ and the order of the channels was chosen as $L = 3$. If there is no

Data/SNR(dB)	0	5	10	15	20	25	30
100	0.93	1	1	1	1	1	1
200	0.97	1	1	1	1	1	1
300	1	1	1	1	1	1	1
400	1	1	1	1	1	1	1

TABLE I

RATES OF THE ESTIMATED DELAY FOR THE FIRST EXAMPLE.

order/SNR(dB)	10	15	20	25	30
4	0	0	0	0	0
5	0	0.01	0.25	0.85	0.99
6	0	0	0.11	0.10	0.01
7	0.55	0.15	0.02	0	0
8	0.41	0.82	0.60	0.05	0

TABLE II

RATES OF THE ESTIMATED ORDERS FOR THE FIRST EXAMPLE
WHERE THE NUMBER OF SYMBOLS IS 200.

additive noise, the delay and the order of equalizer are given by $d = 1$ and $p = 5$, respectively.

Figure 1 shows the averaged ISI of the proposed algorithm in dB ($10 \log \text{ISI}$) where the number of symbols was set to be 200 and the values of SNR were varied from 0–40 dB. The true delay was assumed to be known. The ISI of the equalizers of order 4, 5, and 8 are plotted in Figure 1. There are no significant differences among the performance of the equalizers. Thus we can say that our approach is robust to the underdetermination ($p = 4$) and overdetermination ($p = 8$) of the order. Figure 2 compares the performances of the proposed algorithm and the circular Levinson algorithm. They have almost same performances. We note that the circular Levinson algorithm requires only one “shifting” operation given by (29).

Table I shows the rate of the succeeded delay estimation of the proposed procedure where the number of symbols was varied from 100–400 and the values of SNR were varied from 0–30 dB. From this table, we can conclude that our procedure for the delay estimation is effective. We list in Table II the rates of the estimated orders for the true subchannel ($d = 1$) in the above simulation where the number of symbols was fixed to 200. We can see that the order is almost correctly estimated for the values of SNR over 25 dB, while it is overestimated for the values of SNR under 20 dB.

We repeated the above simulations for a three-ray

Fig. 3. Averaged ISI for the second example where p denotes the order of equalizer and the number of symbols is 200.

Fig. 4. Comparison of the proposed algorithm and the circular Levinson algorithm for the second example.

multipath channel given by

$$h(t) = p(t) - 0.8p(t - 0.5T) + 0.4p(t - 3T) \quad (33)$$

where T is the baud period and $p(t)$ is the raised-cosine pulse limited in $6T$ with roll-off factor 0.11. The continuous received signal was oversampled by the factor $M = 2$. If there is no additive noise, the delay and the order of equalizer are given by $d = 1$ and $p = 15$, respectively.

Figure 3 shows the plots of the averaged ISI of equalizers of order 10, 12, and 14 for given true delay. Figure 2 compares the performances of the proposed algorithm and the circular Levinson algorithm. The circular Levinson algorithm has poor performances. This may result from many “shifting” operations (29) in the circular Levinson algorithm.

Table III shows the rate of the succeeded delay estimation. In this case, a value of SNR over 20dB is

Data/SNR(dB)	0	5	10	15	20	25	30
100	0	0	0	0	0.74	1	1
200	0	0	0	0	0.83	1	1
300	0	0	0	0	0.91	1	1
400	0	0	0	0	0.97	1	1

TABLE III

RATES OF THE ESTIMATED DELAY FOR THE SECOND EXAMPLE.

order/SNR(dB)	20	25	30	35	40
10	0.04	0.02	0.02	0	0.01
11	0.08	0.07	0.12	0.09	0.13
12	0.69	0.89	0.81	0.86	0.80
13	0.02	0.02	0.03	0.05	0.05
14	0	0	0	0	0.01

TABLE IV

RATES OF THE ESTIMATED ORDERS FOR THE SECOND EXAMPLE
WHERE THE NUMBER OF SYMBOLS IS 200

necessary for the delay estimation. Table IV illustrates the rates of the estimated orders. This suggests that, for values of SNR over 20 dB, the order $p = 12$ is valid to equalize the channels. Although the delay estimation fails below 15 dB as shown in Table III, the ISI is still small as can be seen from Figure 3. This implies that more efficient technique for the delay estimation is required and could be developed.

6 Conclusions

We have proposed a new approach to blind equalization of SIMO FIR systems. Our approach is based on the periodic linear prediction of SIMO FIR systems. The equalizer is directly designed by solving the Yule-Walker equations of the periodic linear prediction. An algorithm to solve the periodic linear prediction and a procedure for the order estimation have been provided. Computer simulations have been presented to demonstrate the potential of the proposed approach.

One of the salient features of our approach is that it is robust to the order estimation. On the other hand, the difficulty of our approach is that it needs the delay estimation. Simulation results showed that our procedure for the delay estimation does not work well for long channels under low SNR. More effective procedure should be developed. The theoretical analysis of our approach and fast algorithms based on the periodic linear will be presented in the future.

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