Wavelet Packets Transform based Adaptive Filter

Motonari Inoue Naoki Toyama Jaka Sembiring Kageo Akizuki

Dept. of Electrical, Electronics and Computer Engineering,
Waseda University,3-4-1 Ohkubo,Shinjuku-ku, Tokyo 169-8555, JAPAN
e-mail: inoue@akizuki.elec.waseda.ac.jp

Abstract
In this paper wavelet packets transform is used in an adaptive filtering structure. The coefficients of the adaptive filters are updated by utilizing Recursive Least Square (RLS) algorithm. In order to get an efficient adaptive filter, the input and output will be decomposed by wavelet packet transform and in each band, RLS algorithm will be applied. Further more we will compare our proposed methods to the wavelet transform based one.

1. Introduction
The most common implementation of adaptive filters is based on time-domain form and the application of Least Mean Square (LMS) algorithm for the adaptation of filter weights. When we use the time-domain adaptive filters, in some cases the order will go extra ordinary high, hundreds or thousands orders, for example in echo canceler application. The results is that the calculation will be very complex. Recently, wavelet transform based adaptive filter is suggested to avoid such situation. Such scheme has been proposed in [1] which is analogous to the DFT-based adaptive filtering in [2], [3]. In discrete wavelet analysis [1], [4], [5], [6], signals are represented by a weighted sum of the translated and scaling of mother wavelet. The advantages of wavelet analysis over Fourier analysis is clearly understood, since in Fourier analysis a weighted sum of sinusoids does not adequately represent the time-varying modes of the signals involved. Multi-resolution analysis [5], [6], [7] provides an orthonormal bases of wavelets spanning the space \( L^2(R) \) of square integrable function. These wavelets can be grouped by their scaling constant into disjoint subsets spanning proper and orthogonal subspaces of \( L^2(R) \). These subspaces that correspond to different scales are said to represent signals at different resolution levels. By using wavelet transform, we can get an efficient adaptive filter calculation. To reduce the calculation further, in other words to achieve a more efficient procedure, in this paper we suggest the wavelet packets transform based adaptive filter. First, we will show the basic theory of adaptive filter and discrete wavelet packets. In section 3, we will focus on finding the wavelets packets transform based adaptive filter. Then in section 4, we will simulate the estimation of an unknown system using adaptive filter of wavelet and wavelet packets. And we will conclude our results in section 5.

2. Basic Theory
2.1 Adaptive filter
Basically adaptive filter can be seen as an estimation procedure of parameter of the unknown system, relying on the input and output of such system. It is used in many applications, such as echo canceler, noise canceler, adaptive noise controller etc. Let us consider a general adaptive filter structure shown in Fig.1. We assume that the construction of the unknown system is the FIR model. One method to minimize the error \( e(n) \) is by utilizing Recursive Least Square (RLS) algorithm. In order to derive this algorithm, we start with the famous equation of Wiener Hopf. From our assumption that the system is FIR model, we can write the unknown system and the estimated system as

\[
\begin{align*}
y(n) &= \sum_{i=0}^{M-1} a(i) x(n-i) \quad (1) \\
y(n) &= \sum_{i=0}^{N-1} h(i) x(n-i) \quad (2)
\end{align*}
\]

where \( a(i) \) is the weight of the unknown system and \( h(i) \) is the weight of the adaptive filter. Assume \( e(n) \) as the error of \( y(n) \) and \( \tilde{y}(n) \). In order to find the optimal filter weight \( h(i) \), we use the mean square of \( e(n) \).

\[
J = E[e^2(n)] = E[(y(n) - \tilde{y}(n))^2] \quad (3)
\]

Substituting (2) in (3) will give

\[
J = h^T R h - 2 h^T p + \sigma^2 \quad (4)
\]

where

\[
\begin{align*}
R &= E[u(n)u^T(n)] \\
p &= E[u(n)y(n)] \\
\sigma^2 &= E[y^2(n)]
\end{align*}
\]

By taking partial differential of both side of eq.(4), we can get

\[
\frac{\partial J}{\partial h} = 2Rh - 2p \quad (5)
\]

To minimize the mean square error \( J \), we assume the left hand side of equation (5) is zero, so finally we can get Wiener Hopf equation as follows.
By solving Wiener Hopf equation recursively one can obtain Recursive Least Square (RLS) algorithm. This algorithm can be represented by the following set of equations.

\[ h(n+1) = h(n) + k(n+1)[y(n+1) - x^T(n+1)h(n)] \]

\[ k(n+1) = \frac{P(n)[z(n+1)-P(n)]}{1 + x^T(n+1)P(n)z(n+1)} \]

\[ P(n+1) = \left[ 1 - k(n+1)z^T(n+1) \right]P(n) . \]

where \( z^T(n) = [u(n), u(n-1), \ldots, u(n-N)] \)

This Recursive Least Square (RLS) algorithm has total amount of calculation of order \( 2N^2 + 6N \).

Fig 1. Configuration of Adaptive Filter

2.2 Discrete Wavelet Packets

(a) Discrete Wavelet Packet

In short, wavelet packets are particular linear combination of wavelet. A wavelet packet \( W_f \) is a square integrable modulated waveform, well localized in both time and frequency. Let \( x = x(t) \) be a function in \( L^2(R) \), and let \( \{x_p : p \in Z \} \) be the coefficient of its projection onto space \( \sigma^2 \Omega_0 \), which is equivalent to \( \nu_0 \) in [6]. It has been derived in [8] that

\[ x_p = \int \sigma^{1/2} W_0(2^{-L}t - p) x(t) d t . \]

The \( L^2 \) approximation function given by this projection will be denoted by \( P_L x(t) = \sum_p x_p \sigma^{1/2} W_0(2^{-L}t - p) \). If \( x_p \) is the coefficient of \( x(t) \) in \( \sigma^2 \Omega_0 \), then the coefficient of \( x(t) \) in any space \( \sigma^k \Omega_n \) for \( 0 \leq s \leq L \) and \( 0 \leq f < 2^{L-s} \), \( L \) is the highest level, can be derived as follows:

\[ x_p^s = \int \sigma^{1/2} W_f(2^{-s}t - p) x(t) d t \]  \hspace{1cm} \( p \in Z, 0 \leq s \leq L, 0 \leq f < 2^{L-s} \)

The \( \{2^{-s/2}W_f(2^{-s}t - p) \} \) is a collection of functions called wavelet packets and forms an orthonormal basis for \( L^2(R) \). The wavelet packets bases of \( L^2(R) \) is any orthonormal basis selected from amongst the functions \( \{2^{-s/2} W_f(2^{-s}t - p) \} \). The coefficients above form a recursion relation,

\[ x_p^{s+1} = F_0 x_p^s \]  \hspace{1cm} (8a)

\[ x_p^{s+2} = F_1 x_p^{s+1} \]  \hspace{1cm} (8b)

where \( F_0 \) and \( F_1 \) define an operation from \( i^s(Z) \) to \( i^{s+1}(Z) \).

Fig 2. Complete wavelet packets structure

It is clear from equation (7) and (8) that wavelet packets form a library of functions as a complete tree structure, see Fig 2. Each row is completed from the row above it by one application of either \( F_0 \) or \( F_1 \), denoted as \( s \) and \( d \) respectively.

3. Wavelet Packet based Adaptive Filter

The wavelet transform based adaptive filter has been described in [9], [10], [11]. In short, wavelet transform is the combination of the low pass filters and high pass filters as in Fig.3. On the input side as well as on the desired signal, we compute coefficients through sub-band decomposition and decimation. And then we apply the adaptive algorithm on the coefficients. It allows us to get smaller adaptive filter taps and consequently to reduce the calculation. Using wavelet transform with four bands decomposition, there exists adaptive filters with \( N/2 \), \( N/4 \), \( N/8 \) taps on every band, because the input and desired signal are decimated by wavelet transform, see Fig.3. As a result the amount of calculation is now can be reduced to \( 11N^2/16 + 6N \), while as in section 2 the amount of calculation of RLS algorithm is \( 2N^2 + 6N \). In order to get more efficient calculation, we introduce the wavelet packets transform based adaptive filter. Similar to wavelet transform, wavelet packets transform is also a combination of low and high pass filters, see Fig.4. In ordinary wavelet transform we decompose only the low
frequency components, not in the high, see Fig.3, so that the adaptive filter taps is not reduced in the high frequency side. On the contrary, in wavelet packets transform method, we make a complete tree structure, as in Fig.2, i.e. we decompose both low and high frequency components, so that the taps of filter are smaller than the wavelet counterpart.

The configuration of wavelet packets based adaptive filter is shown in Fig.5, with $H_i$ and $F_i$ are decomposition and reconstruction filters, respectively. $\downarrow D_i$ is decimator, whereas $\uparrow D_i$ is interpolator. By using these filters, we decompose the input and desired signal according to wavelet packets tree structure as in Fig.4. We can reduce further the tap length of the wavelet packets based adaptive filter ($\psi_i$) by this decomposition. Let $u_i$ and $y_i$ be a decomposed input and a decomposed desired signal, respectively. Substituting $u_i$ and $y_i$ in $u$ and $y$ into the RLS algorithm will give the wavelet packets based adaptive filter. Wavelet packets transform with $M$ bands decomposition produce $M$ components of adaptive filter ($\psi_i$) with $N/M$ taps, so by noting that the total amount of RLS algorithm in the previous section is $2N^2 + 6N$, then the amount of calculation now can be derived as 

$$M\left[2\left(\frac{N}{M}\right)^2 + 6\left(\frac{N}{M}\right)\right] = 2N^2/M + 6N.$$ 

For example in four band wavelet packets decompositions, there exists four adaptive filters in which the length of each taps is $N/4$. It is easy to show that we can have more efficient calculation, down to $N^2/2 + 6N$ in complexity.

4. Simulation

In this section, we present the results of our computer simulations to verify some arguments we have described in the previous sections. In our simulations, the adaptive filter is used in the system identification configuration, see Fig.2. First we set the data condition for our simulations.

Data condition

Unknown system:

FIR model of 256 orders

Input: colored signal (variance: 1)

Measurement noise: white noise

(variance: 0.6)

The input signal and output (desired) signal of the unknown systems can be seen in Fig.6 and Fig.7.
First, to verify the effectiveness of the wavelet packets transform based adaptive filter, as a comparison we also give a simulation result with wavelet transform. We use four bands decomposition in adaptive filter using wavelet transform. In this example, we use four adaptive filters with $256/2=128$ taps, $256/4=64$ taps, $2 \times 256/8=32$ taps. The amount of calculation is shown in Table 1. Now we will show how to reduce the computation further. We use the wavelet packets transform based adaptive filter of four bands decomposition. As we have described in previous sections, in this example there exists four adaptive filters with $256/4=64$ taps each. The following figures are square error of output (Fig. 8 and Fig. 9), filter weight convergence (Fig. 10 and Fig. 11) and output of adaptive filters (Fig. 12 and Fig. 13). The total amount of calculation is shown in Table 1.

Table 1: The calculation of the adaptive filter

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelet transform based</td>
<td>46592</td>
</tr>
<tr>
<td>Wavelet packets transform based</td>
<td>34304</td>
</tr>
</tbody>
</table>

Fig. 7 Output of the unknown system

Fig. 8 Filter weight convergence (wavelet packets transform based)

Fig. 9 Filter weight convergence (wavelet transform based)

Fig. 10 Square error of output (wavelet packets transform based)

Fig. 11 Square error of output (wavelet transform based)

Fig. 12 Output of adaptive filter (wavelet packets transform based)
The advantage of using wavelet packets method is clear from these figures. The total amount of calculation of adaptive filter using wavelet packets is smaller compare to the one using wavelet, see Table.1. Moreover, the square error of wavelet packets based adaptive filter is very small it is comparable to wavelet transform method. Finally we can conclude that it is possible to reduce the amount of calculation and at the same time keeping the accuracy of adaptive filter, see Fig.10 and Fig.11.

5. Conclusion
In this paper we have investigated the possibility of reducing the amount of computation of adaptive filter using wavelet packets decomposition. We started the basic theory of adaptive filter and discrete wavelet packets. Then we constructed adaptive filter using wavelet packets decomposition and described its advantages. To verify the effectiveness of wavelet packets transform method, we presented the simulation results and we compared the results to the wavelet transform method. We can conclude that wavelet packets transform based adaptive filter is superior to wavelet transform method in term of calculation. And at the same time, it gives us sufficient accuracy and steady convergence.

Reference


Fig.13 Output of adaptive filter
(wavelet transform based)