Chaotic Motion in Velocity Control System of Induction Motor

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Abstract

This paper is concerned with the occurrence of chaotic motion in velocity control system of induction motor. Though the control of induction motor is very difficult, it is currently used by realization of so-called vector control method. The reason is that the induction motor is superior to a dc motor in maintenance. But, the stability of the velocity control system is not established, neither is robustness. The dynamics of the induction motor is represented by nonlinear equations with three inputs and three outputs. Though precise parameters are necessary for the vector control method, it is difficult to obtain the precise values. Therefore, there are often identified errors in parameters and then the effect of the nonlinear terms remains. On this occasion, it is known that chaotic motions may occur in the system. In this paper, the existence of chaotic motion in such system can be verified using the concept of Lyapunov exponent, and the generating conditions of chaotic motion are found, when the desired value and load conditions are given.

1. Introduction

The induction motor is superior to a dc motor in maintenance. The control method which is widely used for the induction motor is a vector control.⁽¹⁾⁻⁽³⁾ Though the precise parameter values are necessary for a vector control, it is generally difficult to identify these parameters. The existence of identification errors for parameters disturbs linearization in the vector control and the effect of nonlinear terms remains. Then, it may be thought that chaotic motion occurs. The induction motor is driving source with nonlinear characteristics. Many reports of chaos⁽⁴⁾⁻⁽⁸⁾ occurring in mechanical systems concern with external force, for example, such as a pendulum or a nonlinear restoring force. In this case, it's presumed that the driving source has a constant torque. Then, there are few reports with respect to the occurrence of chaos based on the effect of nonlinear characteristics in the driving source.⁽⁹⁾⁽¹⁰⁾ In this research, the dynamics of the induction motor is treated considering driving source, in which the velocity control system is composed using vector control method. At this time, the responses are examined in cases of periodic input and step signal with periodic input, and the occurrence of chaotic motion verified using Lyapunov exponent. Also, in these cases, the generating conditions of

chaos are shown for parameters of desired value and load conditions.

2. Modeling of Induction Motor

2.1 Circuit Equation of Induction Motor

Symbols used in this paper are denoted below.

i : stator current, Φ : rotor flux,
 ω_r : rotor angular velocity, v : stator voltage,
 ω_e : electrical angular velocity, R : resistance,
 L : coil inductance, M : mutual inductance, J : inertia,
 p : number of pole pairs, D : viscosity constant
 $\sigma = 1 - M^2/L_s L_r$: leakage constant,
 $T_r = L_r/R_r$: rotor time constant

The block diagram of velocity control system is shown in Fig.1. The circuit equations of induction motor are given as,

$$\dot{i}_s = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r}\right)i_s + \omega_e i_{\delta s} + \frac{M}{\sigma L_s L_r} \frac{1}{T_r} \Phi_r + \frac{M}{\sigma L_s L_r} \omega_r \Phi_{\delta r} + \frac{v_s}{\sigma L_s} \quad (1)$$

$$\dot{i}_{\delta s} = -\omega_e i_s - \left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r}\right)i_{\delta s} - \frac{M}{\sigma L_s L_r} \omega_r \Phi_r + \frac{M}{\sigma L_s L_r} \frac{1}{T_r} \Phi_{\delta r} + \frac{v_{\delta s}}{\sigma L_s} \quad (2)$$

$$\dot{\Phi}_r = \frac{M}{T_r} i_s - \frac{1}{T_r} \Phi_r + (\omega_e - \omega_r) \Phi_{\delta r} \quad (3)$$

$$\dot{\Phi}_{\delta r} = \frac{M}{T_r} i_{\delta s} - (\omega_e - \omega_r) \Phi_r - \frac{1}{T_r} \Phi_{\delta r} \quad (4)$$

$$\dot{\omega}_r = \frac{pM}{JL_r} (i_{\delta s} \Phi_r - i_s \Phi_{\delta r}) - \frac{D}{J} \omega_r \quad (5)$$

where “ $\dot{}$ ” shows time differential.

In the construction of velocity control is shown of Fig.1, the parameter M/T_r is an important factor for vector control of induction motor. As this parameter includes R_r which changes under the influence of heat etc., it may be considered that an identification error occurs through the difference between the true value inside a motor and the control value. Letting this parameter error be set with K_e , the equations of velocity control are given as follows.

$$\dot{i}_s^* = \frac{\Phi_r^*}{M} - \Phi_{\delta r} \{G_{\omega}(s)(\omega_r^* - \omega_r)\} \quad (6)$$

$$\dot{i}_{\delta s}^* = \frac{\Phi_{\delta r}^*}{M} + \Phi_r \{G_{\omega}(s)(\omega_r^* - \omega_r)\} \quad (7)$$

$$v_s = G_v(s)(i_s^* - i_s) \quad (8)$$

$$v_{\delta s} = G_{\delta}(s)(i_{\delta s}^* - i_{\delta s}) \quad (9)$$

Fig.1 Block Diagram of Velocity Control System

$$\omega_e - \omega_r = K_e \frac{M}{T_r} \{G_\omega(s)(\omega^*_r - \omega_r)\} \quad (10)$$

where “*” shows desired value.

$G_\gamma(s)$, $G_\delta(s)$ and $G_\omega(s)$ are PI controller, with integral time T_i and proportional gain K_e , as follows.

$$G(s) = K \left(1 + \frac{1}{T_i s} \right) \quad (11)$$

2.2 Observer

Though the values Φ_γ and Φ_δ are required to perform

$$x_1 = i_{\gamma s}, \quad x_2 = i_{\delta s}, \quad x_3 = \Phi_{\gamma r}, \quad x_4 = \Phi_{\delta r}, \quad x_5 = \omega_r, \quad x_6 = \theta_e, \quad x_7 = \int (i^*_{\gamma s} - i_{\gamma s}) dt, \quad x_8 = \int (i^*_{\delta s} - i_{\delta s}) dt, \\ x_9 = \int (\omega^*_r - \omega_r) dt, \quad x_{10} = \hat{i}_{ds}, \quad x_{11} = \hat{i}_{qs}, \quad x_{12} = \hat{\Phi}_{dr}, \quad x_{13} = \hat{\Phi}_{qr}$$

$$\dot{x}_1 = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r} \right) x_1 + \left\{ x_5 + K_e K_\omega \frac{M}{T_r} \left(\omega^*_r - x_5 + \frac{x_9}{T_{i\omega}} \right) \right\} x_2 + \frac{M}{\sigma L_s L_r} \frac{1}{T_r} x_3 + \frac{M}{\sigma L_s L_r} x_4 x_5 \\ + \frac{K_\gamma}{\sigma L_s} \left\{ \frac{\Phi^*_{\gamma r}}{M} - K_\omega \left(\omega^*_r - x_5 + \frac{x_9}{T_{i\omega}} \right) (-x_{12} \sin x_6 + x_{13} \cos x_6) - x_1 + \frac{x_7}{T_{i\gamma}} \right\} \quad (13)$$

$$\dot{x}_2 = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r} \right) x_2 - \left\{ x_5 + K_e K_\omega \frac{M}{T_r} \left(\omega^*_r - x_5 + \frac{x_9}{T_{i\omega}} \right) \right\} x_1 + \frac{M}{\sigma L_s L_r} \frac{1}{T_r} x_4 - \frac{M}{\sigma L_s L_r} x_3 x_5 \\ + \frac{K_\delta}{\sigma L_s} \left\{ \frac{\Phi^*_{\delta r}}{M} - K_\omega \left(\omega^*_r - x_5 + \frac{x_9}{T_{i\omega}} \right) (x_{12} \cos x_6 + x_{13} \sin x_6) - x_2 + \frac{x_8}{T_{i\delta}} \right\} \quad (14)$$

vector control, it's difficult to measure directly. Then, the observer is composed. In order to make a setup of observer gain easy, the fixed d-q coordinate is considered. The equations of observer are as follows.

$$\dot{\hat{x}} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \hat{x} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ k_1 \end{bmatrix} (y - \hat{y}), \quad (12)$$

Here, the variables are,

$$\hat{x} = \begin{bmatrix} \hat{i}_{ds} & \hat{i}_{qs} & \hat{\Phi}_{dr} & \hat{\Phi}_{qr} \end{bmatrix}^T, \quad u = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T \\ y = \begin{bmatrix} i_{ds} & i_{qs} \end{bmatrix}^T \quad \text{and} \quad \hat{y} = \begin{bmatrix} \hat{i}_{ds} & \hat{i}_{qs} \end{bmatrix}^T$$

Also, each coefficient save given as,

$$a_{11} = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r} \right) I, \quad a_{12} = \frac{M}{\sigma L_s L_r} \frac{1}{T_r} I + \frac{M}{\sigma L_s L_r} \omega_r J, \\ a_{21} = \frac{M}{T_r} I, \quad a_{22} = -\frac{1}{T_r} I - \omega_r J, \quad b_1 = \frac{1}{\sigma L_s} I \\ k_1 = \frac{M}{T_r} I, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

where “^” is an estimated value and “T” a transpose. Also, the transform matrix C_1 from γ - δ to d-q coordinate and C_2 from d-q to γ - δ coordinate are respectively,

$$C_1 = \begin{bmatrix} \cos \theta_e & -\sin \theta_e \\ \sin \theta_e & \cos \theta_e \end{bmatrix} \quad \text{and} \quad C_2 = \begin{bmatrix} \cos \theta_e & \sin \theta_e \\ -\sin \theta_e & \cos \theta_e \end{bmatrix}.$$

In this control system, first, Eqs.(6) to (10) are calculated by giving $\Phi^*_{\gamma r}$, $\Phi^*_{\delta r}$ and ω^*_r as desired values. At result, $v_{\gamma s}$ and $v_{\delta s}$ are obtained as outputs. Also, ω_e is obtained by adding ω_r to $\omega_e - \omega_r$. Second, $v_{\gamma s}$, $v_{\delta s}$ and ω_e are, as input values, given to the induction motor, and the outputs $i_{\gamma s}$, $i_{\delta s}$ and ω_r detected here are feedback to the control block. Finally, after coordinate conversion of C_1 , $v_{\gamma s}$, $v_{\delta s}$, $i_{\gamma s}$ and $i_{\delta s}$ are transferred to the observer and then the observer outputs Φ_{dr} and Φ_{qr} , after coordinate conversion of C_2 , feedback to control block. The differential equations for this control system are given as Eqs.(13)~(25).

$$\dot{x}_3 = \frac{M}{T_r} x_1 - \frac{1}{T_r} x_3 + K_e K_\omega \frac{M}{T_r} \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) x_4 \quad (15)$$

$$\dot{x}_4 = \frac{M}{T_r} x_2 - K_e K_\omega \frac{M}{T_r} \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) x_3 - \frac{1}{T_r} x_4 \quad (16)$$

$$\dot{x}_5 = \frac{pM}{JL_r} (x_2 x_3 - x_1 x_4) - \frac{D}{J} x_5 \quad (17) \quad \dot{x}_6 = x_5 + K_e K_\omega \frac{M}{T_r} \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) \quad (18)$$

$$\dot{x}_7 = \frac{\Phi^*_{\dot{r}}}{M} - K_\omega \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) (-x_{12} \sin x_6 + x_{13} \cos x_6) - x_1 \quad (19)$$

$$\dot{x}_8 = \frac{\Phi^*_{\dot{\delta}}}{M} + K_\omega \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) (x_{12} \cos x_6 + x_{13} \sin x_6) - x_2 \quad (20)$$

$$\dot{x}_9 = \omega^*_{\dot{r}} - x_5 \quad (21)$$

$$\begin{aligned} \dot{x}_{10} = & - \left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r} \right) x_{10} + \frac{M}{\sigma L_s L_r} \frac{1}{T_r} x_{12} + \frac{M}{\sigma L_s L_r} x_5 x_{13} \\ & + \frac{K_\gamma}{\sigma L_s} \left\{ \frac{\Phi^*_{\dot{r}}}{M} - K_\omega \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) (-x_{12} \sin x_6 + x_{13} \cos x_6) - x_1 + \frac{x_7}{T_{iy}} \right\} \cos x_6 \\ & - \frac{K_\delta}{\sigma L_s} \left\{ \frac{\Phi^*_{\dot{\delta}}}{M} - K_\omega \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) (x_{12} \cos x_6 + x_{13} \sin x_6) - x_2 + \frac{x_8}{T_{id}} \right\} \sin x_6 \end{aligned} \quad (22)$$

$$\begin{aligned} \dot{x}_{11} = & - \left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r} \right) x_{11} - \frac{M}{\sigma L_s L_r} x_5 x_{12} + \frac{M}{\sigma L_s L_r} \frac{1}{T_r} x_{13} \\ & + \frac{K_\gamma}{\sigma L_s} \left\{ \frac{\Phi^*_{\dot{r}}}{M} - K_\omega \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) (-x_{12} \sin x_6 + x_{13} \cos x_6) - x_1 + \frac{x_7}{T_{iy}} \right\} \sin x_6 \\ & + \frac{K_\delta}{\sigma L_s} \left\{ \frac{\Phi^*_{\dot{\delta}}}{M} - K_\omega \left(\omega^*_{\dot{r}} - x_5 + \frac{x_9}{T_{i\omega}} \right) (x_{12} \cos x_6 + x_{13} \sin x_6) - x_2 + \frac{x_8}{T_{id}} \right\} \cos x_6 \end{aligned} \quad (23)$$

$$\dot{x}_{12} = -\frac{1}{T_r} x_{12} - x_5 x_{13} + \frac{M}{T_r} (x_1 \cos x_6 - x_2 \sin x_6) \quad (24)$$

$$\dot{x}_{13} = x_5 x_{12} - \frac{1}{T_r} x_{13} + \frac{M}{T_r} (x_1 \sin x_6 + x_2 \cos x_6) \quad (25)$$

3. Simulation

Parameters of velocity control are given in Table 1. Also, $\omega^*_{\dot{r}}$ is given as follows.

$$\omega^*_{\dot{r}} = U + \Omega \cos 2\pi f t \quad (26)$$

This equation means the case of step input with a periodic signal. Numerical calculations are performed by Runge-Kutta method, in which a sampling time is 0.001s and all initial values are set as 0. Identification error K_e , parameters of $\omega^*_{\dot{r}}$, U , Ω , f , and load parameters D , J are set arbitrarily.

3.1 Calculation of Lyapunov Exponent

The calculation procedure of Lyapunov exponent is as

follows. As there are many variables, an approximated method is used.⁽¹¹⁾ The solution of differential equations (13)~(25) are expressed by vector $\mathbf{x}$ in 13-dimensional phase space. These differential equations are transformed to difference equations. The orbits of solution are given as x_n , η^1_n , η^2_n , if the initial condition are set as x_0 , $x_0 + e^1$ and $x_0 + e^2$. As a sampling time $\|e^1\|$ and $\|e^2\|$ is very small, the deviations ε^1_n and ε^2_n between x_n and η^1_n , x_n and η^2_n are given respectively as,

$$\varepsilon^1_n = \eta^1_{n+1} - x_{n+1} \quad (27)$$

$$\varepsilon^2_n = \eta^2_{n+1} - x_{n+1} \quad (28)$$

Accordingly, Lyapunov exponents λ_1 and λ_2 are defined by

$$\lambda_1 = \lim_{n \rightarrow \infty} \frac{1}{n\Delta t} \sum_{j=0}^{n-1} \log \frac{\|\varepsilon^1_j\|}{\|e^1\|} \quad (29)$$

$$\lambda_2 = \lim_{n \rightarrow \infty} \frac{1}{n\Delta t} \sum_{j=0}^{n-1} \log \frac{\|\varepsilon^1_j \times \varepsilon^2_j\|}{\|e^1 \times e^2\|} - \lim_{n \rightarrow \infty} \frac{1}{n\Delta t} \sum_{j=0}^{n-1} \log \frac{\|\varepsilon^1_j\|}{\|e^1\|} \quad (30)$$

Through the calculations of Eqs. (29) and (30), it may be judged as chaos, when the maximum Lyapunov exponent is positive.

3.2 Chaos by Periodic Input

First, the desired input of Eq. (26) is set as $U=0$, $\Omega=20.0$ and $f=12.0$. Also, the load parameters are $J=0.0050$ and $D=0.0016$. According to the change of K_e , the bifurcation map is shown in Fig.2 and Lyapunov exponent in Fig.3. Lyapunov exponent is the quantity that the two close orbits leave with time in phase space. When this exponent is positive, it is judged to be chaos. From Fig.2, the bifurcation map ω_r shows the complicated branches after $K_e=1.25$ and then from Fig.3, Lyapunov exponent is positive in the range of complicated branches. Then, the chaos is well judged from Lyapunov exponent of Fig.3.

Table 1 Motor Constants and Control Parameters

R_s	0.5Ω	p	2
R_r	1.0Ω	K_v	10.0Ω
L_s	0.105H	T_{ir}	0.01s
L_r	0.105H	K_δ	10.0Ω
M	0.100H	T_{id}	0.01s
Φ^*_{vr}	0.25Vs	K_ω	15.0Ω
$\Phi^*_{\delta r}$	0.0Vs	$T_{i\omega}$	0.05s

Fig.2 Bifurcation ω_r by K_e

Fig.3 Lyapunov Exponent

Next, the generating conditions are considered, when the identification error is $K_e=1.30$. Figures 4 and 5 show the maps of behaviors of ω_r by the parameter variation of both desired value and load, respectively. Fig.4 shows the range of chaotic behaviors by the variations of Ω and f , when $J=0.0050$ and $D=0.0016$. Similarly, Fig.5 shows the range of chaos based on the variation of J and D , when $\Omega=20.0$ and $f=12.0$.

From these figures, the generating conditions of chaos become obvious. In Fig.4, chaos occurs in the range of $\Omega=17.8\sim30.0$, $f=5.0\sim17.5$, and periodic solution exists in the range of $f=17.5\sim25.0$ close to this chaos range. In Fig.5, chaos solution exists in the range of $J=0.004\sim0.025$, $D=0.0\sim0.11$, and 1-cycle solution appears into the lower right of Fig.5. Fig.6 shows the bifurcation map of ω_r by the

Fig.4 Generating Map of Chaos (by the variation of Ω and f)

Fig.5 Generating Map of Chaos (by the variation of J and D)

Fig.6 Bifurcation of Route (A) in Fig.4

variation of f , when $\Omega=20.0$. After chaos generates at $f=5.0$, 1-cycle solution and the chaos are mutually repeated. After periodic solution ends at $f=16.0$, it appears 1-cycle solution. From Figure 4 and 5, it is known that the behaviors of ω_r change by parameter variation and chaotic motion appears, as mentioned above.

3.3 Chaos by Step Input with Periodic Signal

From the practical viewpoint, we'll consider the behavior with step input $U=75.0$ and periodic signal with $\Omega=15.0$, $f=12.0$. In Fig.7, the bifurcation map with maximum and minimum value of ω_r is shown by changing K_e . Also, Lyapunov exponent is drawn at the same time in Fig.8. The behavior becomes unstable more than $K_e=1.84$. The bifurcation map and Lyapunov exponent also correspond well. Comparing with the case of periodic case, it is remarkable point that an unstable solution appears by the addition of step input.

When $K_e=1.30$ is chosen in the range of chaos, the map of classified behavior ω_r is shown in Fig.9 by changing parameters of load. It's set with $\Omega=15.0$, $f=12.0$. Fig.9 is different from the case of periodic input. The range of chaos exits widely, and each range of chaos, 2-cycle and 1-cycle solution exist in the lower right of this map. The bifurcation map of ω_r is shown in Fig.10 where $J=0.0050$. After it changes to chaos suddenly at $D=0.028$, it becomes 2-cycle. It changes to chaos again and 5-cycle window exists close to $D=0.12$ and 3-cycle window close to $D=0.13$. Finally, to observe the chaotic behavior, an example of both the time response and Poincaré map are shown in Fig.11.

The parameters are chosen as $U=0.0$, $\Omega=20.0$, $f=14.0$, $J=0.0050$, $D=0.0016$. It's judged that this angular velocity

Fig.8 Lyapunov Exponent

Fig.7 Bifurcation ω_r by K_e

Fig.9 Classification of Response

Fig.10 Bifurcation on Route (B) in Fig.9

Fig.11 Time Response and Poincaré Map

ω_r is chaos from time response of angular velocity and Poincaré map distribution.

4. Analysis When Cause of Chaos Occurrence

It's considered that chaotic behavior generates because there is something causes at variables. Then, we examined variables which are obtained from Eqs.(1)~(4). The results are shown in Fig.13~Fig.14. Here, $U=0$, $\Omega=20.0$, $f=12.0$, $J=0.0050$, $D=0.0016$ and $K_e=1.30$.

Figure 13 and 14 show comparison of desired values of i_{ds} ,

i_{ys} and values of Eqs.(1) and (2). These figures show values of Eqs.(1) and (2) follow desired values. Fig.15 shows time response of $\Phi_{\delta r}$, $\Phi_{\gamma r}$ which are obtained from Eqs.(3) and (4). Though these desired values are $\Phi_{\delta r}^*=0.0$, $\Phi_{\gamma r}^*=0.25$, values of Eqs.(3) and (4) don't follow desired value at all. When $K_e=1.1$ is chosen as an example which chaos doesn't occur, the time response at this time is shown at Fig.16.

Fig.13 Time Response

Fig.14 Time Response

Fig.15 Time Response

Fig.16 Time Response

The mentioned above, it's verified fluxes may not follow these desired values when chaos occurs in this system.

5. Conclusion

In this research, the analysis has been studied on the chaotic motion of induction motor when identification error exists. Consequently, the following results are verified.

1. The existence of chaotic motion has been clarified using Lyapunov exponent.
2. Based on the parameter change of both desired input and load, the solution behaviors have been classified and generating conditions of chaos found.
3. It's verified fluxes may not follow these desired values when chaos occurs in this system.

It is considered first and second analyses the mentioned above are useful to prevent occurrence chaos in this system. The third analysis is left to future study why fluxes don't follow these desired values when chaos occurs.

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