H_∞ Control for Takagi-Sugeno Fuzzy Jump and Sampled-Data Systems

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Abstract

In this paper, the design of H_∞ controllers for Takagi-Sugeno fuzzy jump and sampled-data systems is considered. We first introduce the H_∞ performance for a stable fuzzy jump system and give a sufficient condition for the H_∞ norm being less than a prescribed number. We then consider the H_∞ state feedback control problem and find sufficient conditions for a given controller. We also consider the H_∞ output feedback control problem. In this case, we consider three cases where the premise variables in the if-then rules are given functions, the observation and the state variables. The fuzzy jump system results are applied to sampled-data fuzzy systems.

Notation: We denote $\sum_{i=1}^r \lambda_i(\xi) A_i$ by $\mathcal{A}$, and similar notations will be used.

2 H_∞ Control for Takagi-Sugeno Fuzzy Systems

2.1 H_∞ Performance

Consider the Takagi-Sugeno fuzzy model described by the following fuzzy IF-THEN rules:

$$\begin{aligned} \text{IF} & \quad \xi_1 \text{ is } M_{i1} \text{ and } \dots \text{ and } \xi_q \text{ is } M_{iq} \\ \text{THEN} & \quad \dot{x}(t) = A_i x(t) + B_i w(t), \quad t \neq kh, \\ & \quad x(kh^+) = A_{di} x(kh) + B_{di} w(k), \quad t = kh, \\ & \quad z = \begin{bmatrix} z_{ci}(t) \\ z_{di}(k) \end{bmatrix} = \begin{bmatrix} C_i x(t) \\ C_{di} x(kh) \end{bmatrix}, \quad i = 1, \dots, r \end{aligned}$$

1 Introduction

The Takagi-Sugeno fuzzy dynamic models are systems described by fuzzy if-then rules which give local linear representations of the underlying system ([1], [5], [6]). Such models can even describe or approximate a wide class of nonlinear systems ([2], [7]). Hence it is important to develop system theory for the Takagi-Sugeno fuzzy systems. Asymptotic stability of Takagi-Sugeno fuzzy systems was first studied and sufficient conditions were obtained ([6]). Output feedback stabilization of the fuzzy systems was considered and the separation principle was established in [8], [9]. Furthermore, the H_∞ controllers for the fuzzy continuous-time systems were given in [3].

In this paper, we consider H_∞ control problems for Takagi-Sugeno fuzzy jump systems, and the results are applied for fuzzy sampled-data systems. We first consider the H_∞ performance for a stable fuzzy jump system and obtain sufficient conditions for the H_∞ norm being less than a prescribed number. We then consider the H_∞ state feedback control problem and find sufficient conditions for a given controller. We also consider the H_∞ output feedback control problem. In this case, the premise variables in the if-then rules are important. We consider three cases where they are (i) given functions, (ii) the observation and (iii) the state variables ([9]). In the first two cases, we obtain design methods of H_∞ controllers and give sufficient conditions. In the last case, we give an H_∞ controller for an approximation of the original fuzzy system.

where $x(t) \in \mathbb{R}^n$ is the state with jumps at each sampling time and is left-continuous, $w(t) \in \mathbb{R}^{m_1}$ is the continuous disturbance, $w(k) \in \mathbb{R}^{m_2}$ is the discrete disturbance, $z_c(t) \in \mathbb{R}^{p_1}$ is the continuous controlled output, $z_d(k) \in \mathbb{R}^{p_2}$ is the discrete controlled output. The matrices $A_i, A_{di}, B_i, B_{di}, C_i$ and C_{di} are of appropriate dimensions. The state equations and the controlled output are defined as follows;

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(\xi) (A_i x(t) + B_i w(t)), \quad t \neq kh, \\ x(kh^+) &= \sum_{i=1}^r \lambda_i(\xi) (A_{di} x(kh) + B_{di} w(k)), \quad t = kh, \\ z &= \begin{bmatrix} z_c(t) \\ z_d(k) \end{bmatrix} = \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_i x(t) \\ C_{di} x(k) \end{bmatrix}. \end{aligned} \quad (1)$$

where

$$\lambda_i(\xi(t)) = \frac{w_i(\xi(t))}{\sum_{i=1}^r w_i(\xi(t))}, \quad w_i(\xi(t)) = \prod_{j=1}^p M_{ij}(\xi_j(t)),$$

and $M_{ij}(\cdot)$ are the membership functions of the fuzzy sets of M_{ij} . We assume

$$w_i(\xi(t)) \geq 0, i = 1, \dots, r, \sum_{i=1}^r w_i(\xi(t)) > 0 \forall t.$$

Hence $\lambda_i(\xi(t))$ satisfy

$$\lambda_i(\xi(t)) \geq 0, i = 1, \dots, r, \sum_{i=1}^r \lambda_i(\xi(t)) = 1 \forall t.$$

Definition 2.1 The fuzzy system (1) is input-output stable if $z \in L^2(0, \infty; \mathbb{R}^p)$ for any $w(t) \in L^2(0, \infty; \mathbb{R}^{m_1})$ and $w(k) \in l^2(0, \infty; \mathbb{R}^{m_2})$ where $L^2(0, \infty; \cdot)$ is the space of square integrable functions and $l^2(0, \infty; \mathbb{R}^{m_1})$ is the space of square summable functions.

We wish to find sufficient conditions that the system (1) is exponentially stable and input-output stable with

$$\|z(t)\|_{L_2}^2 + \|z(k)\|_{l_2}^2 \leq d(\|w(t)\|_{L_2}^2 + \|w(k)\|_{l_2}^2) \quad (2)$$

for some $0 < d < \gamma$

where $\|\cdot\|_{L_2}$ is the L^2 -norm and $\|\cdot\|_{l_2}$ is the l^2 -norm. In this case we say that the H_∞ -norm of the system (1) is less than γ . For the usual linear systems the following lemma is known([4]).

Lemma 2.1 Consider the following system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t), t \neq kh, \\ x(kh^+) &= A_d x(kh) + B_d w(k), t = kh, \\ z &= \begin{bmatrix} z_c(t) \\ z_d(k) \end{bmatrix} = \begin{bmatrix} Cx(t) \\ C_d x(kh) \end{bmatrix}. \end{aligned}$$

Then it is exponentially stable and (2) is satisfied if and only if there exists a bounded matrix $X(t) > 0$ which satisfies

$$\begin{aligned} \dot{X}(t) + A^T X(t) + X(t)A + C^T C \\ + \gamma^{-2} X(t) B B^T X(t) \leq -P < 0, t \neq kh, \\ \gamma^2 I - B_d^T X(kh) B_d > 0, \end{aligned} \quad (3)$$

$$\begin{aligned} A_d^T X(kh) A_d - X(kh^-) \\ - A_d^T X(kh) B_d (\gamma^2 I - B_d^T X(kh) B_d)^{-1} B_d^T X(kh) A_d \\ + C_d^T C_d \leq -P_d < 0, t = kh, \end{aligned} \quad (4)$$

where $P > 0$ and $P_d > 0$.

We generalize the sufficiency part of Lemma 2.1 to the system (1).

Lemma 2.2 The inequalities (3)-(4) in Lemma 2.1 can be written as the following inequalities.

$$\begin{aligned} (a) \quad & X^{-1}(kh) - \gamma^{-2} B_d B_d^T > 0, \\ & A_d^T (X^{-1}(kh) - \gamma^{-2} B_d B_d^T)^{-1} A_d \\ & + C_d^T C_d - X(kh^-) < 0. \\ (b) \quad & \begin{bmatrix} X(kh^-) - C_d^T C_d & A_d^T \\ A_d & X^{-1}(kh) - \gamma^{-2} B_d B_d^T \end{bmatrix} > 0. \end{aligned}$$

Using Lemmas 2.1 and 2.2, we can show the following.

Proposition 2.1 Suppose that there exists a common bounded matrix $X(t) > 0$ such that

$$\begin{aligned} \dot{X}(t) + A_i^T X(t) + X(t) A_i + C_i^T C_i \\ + \gamma^{-2} X(t) B_i B_i^T X(t) \leq -P_{1i}, t \neq kh, \end{aligned} \quad (5)$$

$$\begin{aligned} \begin{bmatrix} X(kh^-) - C_{di}^T C_{di} & A_{di}^T \\ A_{di} & X^{-1}(kh) - \gamma^{-2} B_{di} B_{di}^T \end{bmatrix} \\ \geq P_{3i}, t = kh, \end{aligned} \quad (6)$$

for all i and $P_{1i} > 0$, $P_{2i} > 0$ and $P_{3i} > 0$. Then the fuzzy system (1) is stable and input-output stable with (2).

2.2 H_∞ Control with State Feedback

Consider the H_∞ control problem for the Takagi-Sugeno fuzzy models. We first consider the state feedback case. Then a natural assumption is $\xi = x([7])$. But to keep the generality, we will use $\xi = [\xi_1 \dots \xi_q]$ as the premise variables. Consider the following fuzzy rules:

$$\begin{aligned} \text{IF } \xi_1 \text{ is } M_{i1} \text{ and } \dots \text{ and } \xi_q \text{ is } M_{iq} \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_i w(t), t \neq kh, \\ x(kh^+) = A_{di} x(kh) + B_{di} w(k) + B_{2i} u(k), \\ t = kh, \\ z = \begin{bmatrix} C_i x(t) \\ C_{di} x(kh) + D u(k) \end{bmatrix}, i = 1, \dots, r \end{aligned} \quad (7)$$

where $u(k) \in \mathbb{R}^{m_2}$ is the control and matrices given above are of appropriate dimensions. We assume $D^T[C_{di} D] = [0 I] \forall i$. The state equations and the controlled output are defined as follows;

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(\xi) (A_i x(t) + B_i w(t)), t \neq kh, \\ x(kh^+) &= \sum_{i=1}^r \lambda_i(\xi) (A_{di} x(kh) + B_{di} w(k) \\ &\quad + B_{2i} u(k)), t = kh, \\ z &= \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_i x(t) \\ C_{di} x(k) + D u(k) \end{bmatrix}. \end{aligned} \quad (8)$$

For each subsystem (7), we assume that the following rules are given.

$$\begin{aligned} \text{IF } \xi_1 \text{ is } M_{i1} \text{ and } \dots \text{ and } \xi_q \text{ is } M_{iq} \\ \text{THEN } u(k) = F_i x(kh), i = 1, \dots, r. \end{aligned}$$

Based on these rules, we take the following control:

$$u(k) = \sum_{i=1}^r \lambda_i(\xi) F_i x(kh). \quad (9)$$

We use the same weights $\lambda_i(\xi)$ as those for the state equations and the controlled output in (8). Then the closed-loop system is written as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(\xi) (A_i x(t) + B_i w(t)), \quad t \neq kh, \\ x(kh^+) &= \sum_{i=1}^r \sum_{j=1}^r \lambda_i(\xi) \lambda_j(\xi) \{ (A_{di} \\ &\quad + B_{2i} F_j) x(kh) + B_{di} w(k) \}, \quad t = kh, \\ z &= \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_{1i} x(t) \\ (C_{di} + D F_i) x(k) \end{bmatrix}. \end{aligned} \quad (10)$$

We say that the feedback controller (9) is γ -suboptimal if the closed-loop system (10) is stable and input-output stable with (2). We can apply Proposition 2.1 to (10). The feedback controller (9) is γ -suboptimal if there exists a common bounded matrix $X(t) > 0$ such that for all i

$$\begin{aligned} &\dot{X}(t) + A_i^T X(t) + X(t) A_i + C_{1i}^T C_{1i} \\ &\quad + \gamma^{-2} X(t) B_i B_i^T X(t) \leq -P_{1i}, \quad t \neq kh, \\ &\begin{bmatrix} X(kh^-) - \tilde{C}_{di}^T \tilde{C}_{di} & \tilde{A}_{di}^T \\ \tilde{A}_{di} & X^{-1}(kh) - \gamma^{-2} B_{di} B_{di}^T \end{bmatrix} \\ &\geq P_{3i}, \quad t = kh, \end{aligned}$$

where $\tilde{A}_{di} = A_{di} + B_{2i} F_j$ and $\tilde{C}_{di} = C_{di} + D F_i$, for $P_{1i} > 0$ and $P_{2i} > 0$. But these conditions are rather strong and we relax them in the following theorem.

Theorem 2.1 Suppose there exists a common bounded matrix $X(t) > 0$ that satisfies

$$\begin{aligned} &\dot{X}(t) + A_i^T X(t) + X(t) A_i + C_{1i}^T C_{1i} \\ &\quad + \gamma^{-2} X(t) B_i B_i^T X(t) \leq -P_{1i}, \quad \forall i, \\ &\begin{bmatrix} X(kh^-) - \tilde{C}_{di}^T \tilde{C}_{di} & \tilde{A}_{di}^T \\ \tilde{A}_{di} & X^{-1}(kh) - \gamma^{-2} B_{di} B_{di}^T \end{bmatrix} \\ &\geq P_{3i}, \quad \forall i, \\ &\begin{bmatrix} \Delta C_{dij}^T \Delta C_{dij} + \Delta F_{ij}^T \Delta F_{ij} & -\Delta F_{ij}^T \Delta B_{2ij}^T \\ -\Delta B_{2ij} \Delta F_{ij} & \gamma^{-2} \Delta B_{dij} \Delta B_{dij}^T \end{bmatrix} \\ &\quad + P_{3i} + P_{3j} \geq P_{3ij}, \quad i < j \end{aligned} \quad (11)$$

where P_{1i} , P_{2i} , P_{3i} and P_{3ij} are positive definite matrices, $\Delta B_{dij} = B_{di} - B_{dj}$, $\Delta B_{2ij} = B_{2i} - B_{2j}$, $\Delta C_{dij} = C_{di} - C_{dj}$, $\Delta F_{ij} = F_{di} - F_{dj}$. Then a controller in the form (9) is γ -suboptimal. Moreover, if $B_{2i} = B_2$, $i = 1, \dots, r$, then the condition (11) is always satisfied.

As a design method, we have the following theorem.

Theorem 2.2 Suppose there exists a common bounded matrix $X(t) > 0$ that satisfies

$$\begin{aligned} &\dot{X}(t) + A_i^T X(t) + X(t) A_i + C_{1i}^T C_{1i} \\ &\quad + \gamma^{-2} X(t) B_i B_i^T X(t) \leq -P_{1i}, \quad t \neq kh, \end{aligned} \quad (12)$$

$$X^{-1}(kh) - \gamma^{-2} B_{di} B_{di}^T \geq P_{2i}, \quad t = kh, \quad (13)$$

$$\begin{bmatrix} X(kh^-) - C_{di}^T C_{di} & A_{di}^T \\ A_{di} & \Delta_{22}(kh) \end{bmatrix} \quad (14)$$

$$\geq P_{3i}, \quad t = kh, \quad \forall i,$$

$$\begin{bmatrix} \Delta C_{dij}^T \Delta C_{dij} & 0 \\ 0 & \gamma^{-2} \Delta B_{dij} \Delta B_{dij}^T - \Delta B_{2ij} \Delta B_{2ij}^T \end{bmatrix} \\ + P_{3i} + P_{3j} \geq P_{3ij} \quad \forall i < j \quad (15)$$

where $\Delta_{22}(kh) = X^{-1}(kh) + B_{2i} B_{2i}^T - \gamma^{-2} B_{di} B_{di}^T$ and $P_{ki} > 0$, $k = 1, 2, 3$. Then the controller

$$u(k) = -B_2^T (X^{-1} + B_2 B_2^T - \gamma^{-2} B_d B_d^T)^{-1} A_d x(k)$$

is γ -suboptimal. Moreover, if $B_{2i} = B_2$, $i = 1, \dots, r$, then the condition (15) is always satisfied and the controller

$$u(k) = -B_2^T (X^{-1} + B_2 B_2^T - \gamma^{-2} B_d B_d^T)^{-1} A_d x(k)$$

is γ -suboptimal.

2.3 H_∞ Control with Output Feedback

Consider the system described by the following fuzzy rules:

$$\begin{aligned} \text{IF} & \quad \xi_1 \text{ is } M_{i1} \text{ and } \dots \text{ and } \xi_q \text{ is } M_{iq} \\ \text{THEN} & \quad \begin{aligned} \dot{x}(t) &= A_i x(t) + B_{1i} w(t), \quad t \neq kh, \\ x(kh^+) &= A_{di} x(kh) + B_{di} w(k) \\ &\quad + B_{2i} u(k), \quad t = kh, \\ z &= \begin{bmatrix} C_{1i} x(t) \\ C_{di} x(kh) + D_{12i} u(k) \end{bmatrix}, \\ y(k) &= C_{2i} x(kh) + D_{21i} w(k), \quad i = 1, \dots, r \end{aligned} \end{aligned} \quad (16)$$

where $y(k) \in \mathbb{R}^d$ is the observation and C_{2i} and D_{21i} are of appropriate dimensions. For simplicity, we assume

$$\begin{aligned} D_{12i}^T \begin{bmatrix} C_{di} & D_{12i} \end{bmatrix} &= \begin{bmatrix} 0 & I \end{bmatrix}, \\ D_{21i} \begin{bmatrix} B_{di}^T & D_{21i} \end{bmatrix} &= \begin{bmatrix} 0 & I \end{bmatrix}, \quad i = 1, \dots, r. \end{aligned} \quad (17)$$

The state equation, controlled output and observation are defined as follows;

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(\xi) (A_i x(t) + B_{1i} w(t)), \\ x(kh^+) &= \sum_{i=1}^r \lambda_i(\xi) (A_{di} x(kh) + B_{2i} u(k) + B_{di} w(k)), \\ z &= \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_{1i} x(t) \\ C_{di} x(kh) + D_{12i} u(k) \end{bmatrix}, \\ y(k) &= \sum_{i=1}^r \lambda_i(\xi) (C_{2i} x(kh) + D_{21i} w(k)). \end{aligned} \quad (18)$$

For each subsystem (16), we assume that the following rules concerning output feedback controllers are given.

$$\begin{aligned} \text{IF} & \quad \xi_1 \text{ is } M_{i1} \text{ and } \dots \text{ and } \xi_q \text{ is } M_{iq} \\ \text{THEN} & \quad \begin{aligned} \dot{\hat{x}}(t) &= \hat{A}_i \hat{x}(t), \quad t \neq kh, \\ \hat{x}(kh^+) &= \hat{A}_{di} \hat{x}(kh) + \hat{B}_i y(k), \quad t = kh, \\ u(k) &= \hat{C}_i \hat{x}(kh), \quad i = 1, \dots, r \end{aligned} \end{aligned}$$

where $\hat{x}(t) \in \mathbb{R}^n$ and all matrices are of compatible dimensions. Based on these rules, we take the following control:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^r \lambda_i(\xi) \hat{A}_i(t) \hat{x}(t), \quad t \neq kh, \\ \hat{x}(kh^+) &= \sum_{i=1}^r \lambda_i(\xi) (\hat{A}_{di} \hat{x}(kh) + \hat{B}_i y(k)), \\ t &= kh, \\ u(k) &= \sum_{i=1}^r \lambda_i(\xi) \hat{C}_i x(kh).\end{aligned}\quad (19)$$

We say that the feedback controller (19) is γ -suboptimal if the closed loop system (18) with (19) is stable and input-output stable with (2). For a given controller (19) we can apply Proposition 2.1 to the closed-loop system (18) with (19). Now we propose a method to design a γ -suboptimal controller based on the H_∞ controllers of subsystems. For usual linear discrete-time system we have the following result.

Theorem 2.3 Consider the following system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1 w(t), \quad t \neq kh, \\ x(kh^+) &= A_d x(kh) + B_2 u(k), \quad t = kh, \\ z &= \begin{bmatrix} z_c(t) \\ z_d(k) \end{bmatrix} = \begin{bmatrix} C_1 x(t) \\ C_d x(kh) + D_{12} u(k) \end{bmatrix}, \\ y(k) &= C_2 x(kh) + D_{21} w(k)\end{aligned}$$

with the conditions

$$D_{12}^T \begin{bmatrix} C_d & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}, \quad D_{21} D_{21}^T = I.$$

Then it is exponentially stable and (2) is satisfied if there exist bounded matrices $X(t) > 0$ and $Z(t) > 0$ which satisfy

$$\begin{aligned}&\dot{X}(t) + A^T X(t) + X(t) A + C_1^T C_1 \\&\quad + \gamma^{-2} X(t) B_1 B_1^T X(t) \leq -P_1 < 0, \\&-\dot{Z}(t) + (A + \gamma^{-2} B_1 B_1^T X(t)) Z(t) \\&\quad + Z(t) (A + \gamma^{-2} B_1 B_1^T X(t))^T + B_1 B_1^T \leq -P_2 < 0, \\&A_d^T X(kh) A_d - X(kh) + C_d^T C_d - A_d^T X(kh) B_2 \\&\quad \times (I + B_2^T X(kh) B_2)^{-1} B_2^T X(kh) A_d \leq -P_3 < 0, \\&Z^{-1}(kh) - \gamma^{-2} A_d^T X(kh) B_2 (I + B_2^T X(kh) B_2)^{-1} \\&\quad \times B_2^T X(kh) A_d \geq P_4 > 0, \\&A_d [Z^{-1}(kh) + C_2^T C_2 - \gamma^{-2} A_d^T X(kh) B_2 (I + \\&\quad B_2^T X(kh) B_2)^{-1} B_2^T X(kh) A_d]^{-1} A_d^T - Z(kh^+) \\&\quad \geq -P_5 < 0\end{aligned}$$

where $P_i > 0$, $i = 1, \dots, 5$. Then an output feedback controller

$$\begin{aligned}u(k) &= F(kh) \hat{x}(kh), \\ \dot{\hat{x}}(t) &= (A + \gamma^{-2} B_1 B_1^T X(t)) \hat{x}(t), \\ \hat{x}(kh^+) &= A_d \hat{x}(kh) + B_2 u(k) \\&\quad + A_d \Lambda_Z^{-1}(kh) C_2^T (y(k) - C_2 \hat{x}(kh))\end{aligned}$$

where

$$\begin{aligned}F(kh) &= -B_2^T (X^{-1}(kh) + B_2 B_2^T)^{-1} (kh) A_d, \\ \Lambda_Z(kh) &= Z^{-1}(kh) + C_2^T C_2 - \gamma^{-2} A_d^T X(kh) B_2 (I \\&\quad + B_2^T X(kh) B_2)^{-1} B_2^T X(kh) A_d\end{aligned}$$

is γ -suboptimal.

We now go back to the fuzzy system (18). As for the premise variables we consider three cases: (i) given functions, (ii) the observation, (iii) the state variables ([9]).

Case (i): ξ is given.

In this case any controller of the form (19) is feasible. Using Theorem 2.3, we can show the following.

Theorem 2.4 Suppose there exists a common bounded $X(t) > 0$ that satisfies (12)-(15), and that there exists a common bounded matrix $Z(t) > 0$ that satisfies

$$\begin{aligned}-\dot{Z}(t) + (A_i + \gamma^{-2} B_{1i} B_{1i}^T X(t)) Z(t) \\ + Z(t) (A_i + \gamma^{-2} B_{1i} B_{1i}^T X(t))^T + B_{1i} B_{1i}^T \\ \leq -P_{1i} < 0 \quad \forall i,\end{aligned}\quad (20)$$

$$\begin{aligned}0 < P_{1i} + P_{1j} + \Delta B_{1ij} \Delta B_{1ij}^T \\ + \gamma^{-2} \Delta B_{1ij} \Delta B_{1ij}^T X(t) Z(t) \\ + \gamma^{-2} Z(t) X(t) \Delta B_{1ij} \Delta B_{1ij}^T, \quad \forall i < j,\end{aligned}\quad (21)$$

$$\begin{aligned}\begin{bmatrix} Z^{-1}(kh) & A_{di}^T X(kh) B_{2i} \\ B_{2i}^T X(kh) A_{di} & \gamma^2 (I + B_{2i}^T X(kh) B_{2i}) \end{bmatrix} \\ \geq P_{2i} > 0 \quad \forall i, \\ P_{2i} + P_{2j} >\end{aligned}\quad (22)$$

$$\begin{aligned}\begin{bmatrix} 0 & \Delta A_{dij}^T X(kh) \Delta B_{2ij} \\ \Delta B_{2ij}^T X(kh) \Delta A_{dij} & \gamma^2 \Delta B_{2ij}^T X(kh) \Delta B_{2ij} \end{bmatrix} \\ \forall i < j,\end{aligned}\quad (23)$$

$$\begin{aligned}\begin{bmatrix} Z(kh^+) & A_{di} & 0 \\ A_{di}^T & Z^{-1}(kh) + C_{2i}^T C_{2i} & B_{2i}^T X(kh) A_{di} \\ 0 & A_{di}^T X(kh) B_{2i}^T & \Delta_{33i} \end{bmatrix} \\ \geq P_{3i} > 0 \quad \forall i, \\ P_{3i} + P_{3j} >\end{aligned}\quad (24)$$

$$\begin{aligned}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta C_{2ij}^T \Delta C_{2ij} & \Delta B_{2ij}^T X \Delta A_{dij} \\ 0 & \Delta A_{dij}^T X \Delta B_{2ij} & \gamma^2 \Delta B_{2ij}^T X \Delta B_{2ij} \end{bmatrix} \quad \forall i < j\end{aligned}\quad (25)$$

where $\Delta_{33i} = \gamma^2 (I + B_{2i}^T X(kh) B_{2i})$ and $P_i > 0$, $i = 1, \dots, 5$. Then an output feedback controller

$$\begin{aligned}u(k) &= \mathcal{F}(kh) \hat{x}(kh), \\ \dot{\hat{x}}(t) &= (A + \gamma^{-2} B_1 B_1^T X(t)) \hat{x}(t), \\ \hat{x}(kh^+) &= A_d \hat{x}(kh) + B_2 u(k) \\&\quad + A_d \Lambda_Z^{-1}(kh) C_2^T (y(k) - C_2 \hat{x}(kh))\end{aligned}\quad (26)$$

where

$$\begin{aligned}\mathcal{F}(kh) &= -B_2^T (X^{-1}(kh) + B_2 B_2^T)^{-1} A_d, \\ \Lambda_Z(kh) &= Z^{-1}(kh) + C_2^T C_2 - \gamma^{-2} A_d^T X(kh) B_2 (I \\&\quad + B_2^T X(kh) B_2)^{-1} B_2^T X(kh) A_d\end{aligned}$$

is γ -suboptimal.

Case (ii): $\xi = y$.

This is a natural case that the premise variable ξ coincides with the observation y of the underlying system. But in this case the fourth equation for y in (18) is not well-defined. Hence we assume that $C_{2i} = C_2$. Since y is a premise variable, we also assume $D_{21i} = 0$, $i = 1, \dots, r$. Then (16) leads to

$$\begin{aligned} \text{IF } & \xi_1 \text{ is } M_{i1} \text{ and } \dots \text{ and } \xi_q \text{ is } M_{iq} \\ \text{THEN } & \dot{x}(t) = A_i x(t) + B_{1i} w(t), \quad t \neq kh, \\ & x(kh^+) = A_{di} x(kh) + B_{di} w(k) + B_{2i} u(k), \\ & t = kh, \\ & z = \begin{bmatrix} C_{1i} x(t) \\ C_{di} x(kh) + D_{12i} u(k) \end{bmatrix}, \quad \forall i, \\ & y(k) = C_2 x(kh), \end{aligned}$$

and the fuzzy system (18) becomes

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(y) (A_i x(t) + B_{1i} w(t)), \\ x(kh^+) &= \sum_{i=1}^r \lambda_i(y) (A_{di} x(kh) + B_{di} w(k) \\ &\quad + B_{2i} u(k)), \\ z &= \sum_{i=1}^r \lambda_i(y) \begin{bmatrix} C_{1i} x(t) \\ C_{di} x(kh) + D_{12i} u(k) \end{bmatrix}, \\ y(k) &= C_2 x(kh). \end{aligned}$$

Then from Theorem 2.4 we have

Corollary 2.1 Suppose there exists a common bounded $X(t) > 0$ that satisfies (12)-(15) with $B_{di} = 0$, $i = 1, \dots, r$, and that there exists a common bounded matrix $Z(t) > 0$ that satisfies (20)-(24) and

$$\begin{bmatrix} Z(kh^+) & A_{di} & 0 \\ A_{di}^T & Z^{-1}(kh) + C_2^T C_2 & B_{2i}^T X(kh) A_{di} \\ 0 & A_{di}^T X(kh) B_{2i}^T & \Delta_{33i} \end{bmatrix} \geq R_{3i} > 0 \quad \forall i, \quad (27)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta B_{2ij}^T X \Delta A_{di} \\ 0 & \Delta A_{di}^T X \Delta B_{2ij} & \gamma^2 \Delta B_{2ij}^T X \Delta B_{2ij} \end{bmatrix} \geq R_{3i} + R_{3j} > 0 \quad \forall i < j. \quad (28)$$

Then an output feedback controller

$$\begin{aligned} u(k) &= \mathcal{F}(kh) \hat{x}(kh), \\ \dot{\hat{x}}(t) &= (A + \gamma^{-2} B_1 B_1^T X(t)) \hat{x}(t), \\ \hat{x}(kh^+) &= A_d \hat{x}(kh) + B_2 u(k) \\ &\quad + A_d \Lambda_Z^{-1}(kh) C_2^T (y(k) - C_2 \hat{x}(kh)) \end{aligned}$$

where

$$\Lambda_Z(kh) = Z^{-1}(kh) + C_2^T C_2 - \gamma^{-2} A_d^T X(kh) B_2 (I + B_2^T X(kh) B_2)^{-1} B_2^T X(kh) A_d$$

is γ -suboptimal.

Case (iii): $\xi = x$.

In this case we need the state x to generate the state equations in (18). However, the only observation available to us is y and hence the controller (26) with $\xi = x$ is no longer feasible. If we replace ξ by $\hat{x}$ in (26), then we obtain a feasible controller

$$\begin{aligned} u(k) &= \mathcal{F}(kh) \hat{x}(kh), \\ \dot{\hat{x}}(t) &= (A + \gamma^{-2} B_1 B_1^T X(t)) \hat{x}(t), \\ \hat{x}(kh^+) &= A_d \hat{x}(kh) + B_2 u(k) \\ &\quad + A_d \Lambda_Z^{-1}(kh) C_2^T (y(k) - C_2 \hat{x}(kh)) \end{aligned} \quad (29)$$

$$\Lambda_Z(kh) = Z^{-1}(kh) + C_2^T C_2 - \gamma^{-2} A_d^T X(kh) B_2 (I + B_2^T X(kh) B_2)^{-1} B_2^T X(kh) A_d.$$

This is a γ -suboptimal controller for the following approximation of (18) with $\xi = x$:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(\xi) (A_i x(t) + B_{1i} w(t)), \\ x(kh^+) &= \sum_{i=1}^r \lambda_i(\xi) (A_{di} x(kh) + B_{di} u(k)), \\ z &= \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_{1i} x(t) \\ C_{di} x(kh) + D_{12i} u(k) \end{bmatrix}, \\ y(k) &= \sum_{i=1}^r \lambda_i(\xi) (C_{2i} x(kh) + D_{21i} w(k)). \end{aligned} \quad (30)$$

In fact we can directly apply Theorem 2.4 to (29) and (30):

Corollary 2.2 Suppose there exists a common bounded $X(t) > 0$ that satisfies (12)-(15) with $B_{di} = 0$, $i = 1, \dots, r$, and that there exists a common bounded matrix $Z(t) > 0$ that satisfies (20)-(25). Then the output feedback controller (29) is γ -suboptimal.

3 Fuzzy Sampled-Data Systems

Consider the following sampled-data fuzzy system;

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \lambda_i(\xi) (A_i x(t) + B_{1i} w(t) + B_{2i} \tilde{u}(t)), \\ z &= \begin{bmatrix} z_c(t) \\ z_d(k) \end{bmatrix} = \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_{1i} x(t) \\ D_{12i} \tilde{u}(t) \end{bmatrix}, \\ y(k) &= \sum_{i=1}^r \lambda_i(\xi) (C_{2i} x(kh) + D_{21i} w(k)) \end{aligned} \quad (31)$$

where $\tilde{u} \in \mathbb{R}^m$ is the control realized through a zero-order hold, that is, $\tilde{u}(t) = u_k$, $kh < t \leq (k+1)h$. Since the control $\tilde{u}(t)$ is constant between two sampling instants, we can introduce the following state space representation:

$$\dot{\bar{x}} = 0, \quad \bar{x}(kh^+) = u_k, \quad kh < t \leq (k+1)h.$$

Then clearly $\tilde{u}(t) = \bar{x}(t)$. Define the new state variable $x_e(t) = [x^T \quad \bar{x}^T]^T$. Then the above sample-data system becomes the following jump system:

$$\begin{aligned}\dot{x}_e(t) &= \sum_{i=1}^r \lambda_i(\xi)(A_e x_e(t) + B_{e1} w(t)), \quad t \neq kh, \\ x_e(kh^+) &= \sum_{i=1}^r \lambda_i(\xi)(A_{ed} x_e(kh) + B_{e2} u(k)), \\ &\quad t = kh,\end{aligned}$$

$$\begin{aligned}z(t) &= \sum_{i=1}^r \lambda_i(\xi) \begin{bmatrix} C_{e1} x_e(t) \\ \sqrt{h} D_{12} u(t) \end{bmatrix}, \\ y(k) &= \sum_{i=1}^r \lambda_i(\xi)(C_{e2} x_e(kh) + D_{21} w(k))\end{aligned}$$

where

$$\begin{aligned}A_e &= \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix}, B_{e1} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, A_{ed} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \\ B_{e2} &= \begin{bmatrix} 0 \\ I \end{bmatrix}, C_{e1} = \begin{bmatrix} C_1 & 0 \end{bmatrix}, C_{e2} = \begin{bmatrix} C_2 & 0 \end{bmatrix},\end{aligned}$$

and h comes from

$$\begin{aligned}\int_0^\infty |D_{12} \tilde{u}(t)|^2 dt &= \sum_{k=0}^\infty \int_0^h |D_{12} u(k)|^2 dt \\ &= \sum_{k=0}^\infty |\sqrt{h} D_{12} u(k)|^2.\end{aligned}$$

We only consider the case where the premise variables are given functions. The following theorem follows from Theorem 2.4.

Theorem 3.1 Consider the fuzzy sampled-data system (31) with the conditions (17). Suppose that there exists a common bounded $X(t) = \begin{bmatrix} X_1(t) & X_{12}(t) \\ X_{12}^T(t) & X_2(t) \end{bmatrix} > 0$ that satisfies (12)-(15) with A, B_1, C_1, A_d replaced by $A_e, B_{e1}, C_{e1}, A_{ed}$, respectively, and that there exists a common bounded matrix $Z(t) > 0$ that satisfies (20)-(22) with $X(t)$ replaced by $X_1(t)$, respectively, and

$$\begin{aligned}\begin{bmatrix} Z^{-1}(kh) & X_{12}(kh) \\ X_{12}^T(kh) & \gamma^2(hI + X_2(kh)) \\ Z(kh^+) & I \\ I & Z^{-1}(kh) + C_{2i}^T C_{2i} \\ 0 & X_{12}^T(kh) \end{bmatrix} &\geq P_{2i} > 0 \quad \forall i, \\ &\geq P_{3i} > 0 \quad \forall i, \\ P_{3i} + P_{3j} &> \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta C_{2ij}^T \Delta C_{2ij} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \forall i < j\end{aligned}$$

where $P_i > 0$, $i = 2, 3$. Then an output feedback controller

$$\begin{aligned}u(k) &= F(kh) \hat{x}(kh), \\ \dot{\hat{x}}(t) &= (A + \gamma^{-2} B_1 B_1^T X_1(t)) \hat{x}(t) \\ &\quad + (B_2 + \gamma^{-2} B_1 B_1^T X_{12}(t)) \tilde{u}(t), \\ \hat{x}(kh^+) &= \hat{x}(kh) + \Lambda_Z^{-1}(kh) C_2^T (y(k) - C_2 \hat{x}(kh))\end{aligned}$$

where

$$\begin{aligned}F(kh) &= -(hI + X_2(kh))^{-1} X_{12}^T(kh), \\ \Lambda_Z(t) &= Z^{-1}(kh) + C_2^T C_2 \\ &\quad - \gamma^{-2} X_{12}(kh)(I + X_2(kh))^{-1} X_{12}^T(kh)\end{aligned}$$

is γ -suboptimal.

Remark 3.1 As for the premise variables, we have only considered the case where they are given functions. However, we can obtain γ -suboptimal controllers for the cases where the premise variables are the observation and the state variables.

4 Conclusion

We have considered the H_∞ control problems for Takagi-Sugeno fuzzy jump and sampled-data systems. We have extended the results for Takagi-Sugeno fuzzy continuous-time system to a fuzzy jump system. We have obtained sufficient conditions for controllers to be γ -suboptimal. The results for a fuzzy jump system have been applied to a sampled-data system.

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