An evaluation method of probability distribution on L_{eq} for non-stationary environmental noise based on its dB-level statistics

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Abstract

The fluctuation of L_{eq} over a long time period is statistically evaluated by means of its probability distribution form under the condition of the noise level observation on dB scale without observing a whole shape of its fluctuation pattern. First, by considering the non-linear relationship between a sound level and a sound intensity, the distribution parameter on the L_{eq} is estimated from the parameters on noise level fluctuation obtained by usual observation and the sampling number of averaging operation on the L_{eq} within a short time interval. The non-stationary property of the L_{eq} is also evaluated by the statistics of each parameters over a whole observation period. Next, to reflect the correlation effect among instantaneous sound levels, the joint moment statistics of sound intensity which reflect various correlation properties among sound levels during the averaging time period are estimated by using the joint moment statistics of sound level fluctuation. Finally, the proposed method has been applied to actual environmental noises and probability distribution form of the L_{eq} is theoretically established by using their statistics.

1 INTRODUCTION

A random city noise emitted from road traffic, a railway, and *etc.* shows a non-stationary property as a result of diversified fluctuation based on natural, physical and human factors with a lapse of time. Usually, the above environmental noise is evaluated in not only a sound level form but also an L_{eq} form whose averaging time interval is prolonged. Though, the L_{eq} assumed to be an invariant noise index within a local time, it very often changes temporally and becomes to show a multifarious stochastic property after a lapse of long time period. The L_{eq} is usually measured as the dB value of an arithmetic mean of sound energy values calculated by antilogarithms of discrete level observations. In previous studies [1]-[7], some theoretical estimation methods on L_{eq} value were proposed to remove the fundamental problems related to the anti-logarithm transformation to obtain the

sound intensity from the actual level observation. However, it seems that the effect of this averaging time (or sampling number) on the above fluctuation of L_{eq} was not still touched upon.

In this paper, as a new trial on evaluating the temporal fluctuation of L_{eq} over a long time period, an evaluation method on probability distribution of L_{eq} is theoretically derived under the condition of the noise level observation on [dB] scale without observing a whole shape of its instantaneous fluctuation pattern. As the first stage of consideration, an assumption is introduced that instantaneous sound levels to calculate an L_{eq} are independent each other. Then, by considering the non-linear relationship between a sound level, a sound intensity and the sampling number of averaging operation, the distribution parameter on the L_{eq} is estimated from the parameters on noise level fluctuation obtained by usual observation data within a short time interval. The non-stationary property of the L_{eq} is also evaluated by the statistics of each parameters over a whole observation time period. Next, to reflect the correlation effect among instantaneous sound levels, the joint moment statistics of sound intensity which reflect various type correlation properties among sound levels during the averaging time period are estimated by using the joint moment statistics of sound level fluctuation. Then, the probability distribution of the L_{eq} can be evaluated by employing the statistical Laguerre type expansion series expression [9]. Finally, the probability distribution form of the L_{eq} is theoretically established by using these statistics and this proposed method is experimentally confirmed by applying it to the actually observed L_{eq} data.

2 PREDICTION OF PROBABILITY DISTRIBUTION ON L_{eq}

For a stationary environmental noise, it is well-known that an L_{eq} is evaluated by using a finite number of instantaneous sound level data

$L_i[\text{dB}(A)]$ as follows:

$$L_{eq} \triangleq 10 \log_{10} E, \quad E \triangleq \frac{1}{N} \sum_{i=1}^N 10^{L_i/10}. \quad (1)$$

Here, the measuring time of L_{eq} given by the product of sampling number N and sampling period of L_i may influence the probability distribution form of L_{eq} itself.

2.1 Probability Distribution on L_{eq} within A Finite Time Period

Since the probability density function (abbr. p.d.f.) of L_i is given by the statistical Hermite type expansion series expression [9], the $M\theta$ th order moment statistics of instantaneous sound intensity x_i can be estimated as:

$$\langle x_i^{M\theta} \rangle = e^{\theta\mu + \frac{1}{2}\theta^2\sigma^2} \left\{ 1 + \sum_{n=3}^{\infty} \frac{A_n}{n!} \theta^n \right\} \quad (2)$$

with the distribution parameters of statistical Hermite type expansion series expression given by

$$\mu \triangleq \langle L_i \rangle, \quad \sigma^2 \triangleq \text{Var}[L_i]$$

and

$$A_n \triangleq \langle H_n((L_i - \mu)/\sigma) \rangle,$$

where $M \triangleq 10/\ln 10$ and $x_i = 10^{L_i/10}$. The moment generating function of E evaluated by N independent samples from a population having moment statistics $\langle x_i^{M\theta} \rangle$ becomes as:

$$\langle e^{\theta E} \rangle = \langle e^{\theta \sum x_i/N} \rangle = \prod_{i=1}^N \langle e^{\theta/N \cdot x_i} \rangle. \quad (3)$$

The j -th differential operation gives the j -th order moment statistics of E :

$$\begin{aligned} \langle E^j \rangle &= \left. \frac{d^j}{d\theta^j} \langle e^{\theta E} \rangle \right|_{\theta \rightarrow 0} \\ &= \left. \frac{d^j}{d\theta^j} \left[\sum_{i=1}^N \frac{N!}{(N-i)!i!} \left\{ \sum_{n=1}^{\infty} \frac{\langle x_i^n \rangle \theta^n}{n! N^n} \right\}^i \right] \right|_{\theta \rightarrow 0} \end{aligned} \quad (4)$$

On the other hand, when the p.d.f. of L_{eq} can be given in the unified form of the statistical Hermite

type expansion series expression, the moment generating function of L_{eq} becomes equal to the $M\theta$ th order moment statistics of E :

$$\begin{aligned} \langle e^{\theta L_{eq}} \rangle &= e^{\theta\mu_N + \frac{1}{2}\theta^2\sigma_N^2} \left\{ 1 + \sum_{n=3}^{\infty} \frac{A_{Nn}}{n!} \theta^n \right\} \\ &= \langle E^{M\theta} \rangle. \end{aligned} \quad (5)$$

Here μ_N , σ_N^2 and A_{Nn} are the parameters of the p.d.f. on L_{eq} and they are given by solving Eqs.(4) and (5) simultaneously. When the L_{eq} is almost Gaussian, Eqs.(4) and (5) have no higher terms for $n \geq 3$ and they become a simultaneous equation with two unknown parameters. Then, we have

$$P(L_{eq}) = \frac{1}{\sqrt{2\pi}\sigma_N} e^{-(L_{eq}-\mu_N)^2/2\sigma_N^2} \quad (6)$$

with

$$\mu_N = \mu + (\sigma^2 - \sigma_N^2)/2 \quad (7)$$

and

$$\sigma_N^2 = \ln \left\{ (N-1 + e^{\sigma^2})/N \right\}, \quad (8)$$

where μ_N and σ_N^2 are the parameters corresponding to a mean value and a variance of L_{eq} . Here, μ and σ^2 denote the mean value and the variance of sound level.

2.2 Probability Distribution on L_{eq} of Non-stationary Noise

The actual random noise having a temporal fluctuation of the stochastic property may show a non-stationary property over a long time interval. When one aims a probabilistic evaluation over a long time period, it is reasonable to positively assume the L_{eq} as Gaussian within a short time interval by taking the smoothing effect over a long time period into consideration in advance. That is, the fluctuation of L_{eq} within a short time interval can be described by Eq.(6) and its distribution parameters, μ_N and σ_N^2 , are estimated by Eq.(7). By evaluating the temporal fluctuation of them, the p.d.f. of L_{eq} can be reconstructed by using a prediction method on probability distribution of non-stationary noise intensity [8] as follows:

$$\begin{aligned} P(E) &= \sum_{k=0}^{\infty} M_k \sum_{j=0}^k a_{k-1,j} E^j \\ &\times \left(\frac{d}{dE} \right)^j \left[\frac{1}{\sqrt{2\pi}\sigma_N} e^{-(\ln E - \mu_{0N})^2/2\sigma_{0N}^2} \right] \end{aligned} \quad (9)$$

with

$$M_k = \sum_{n=0}^{[k/2]} (-1)^k \frac{(\sigma_N^2 - \sigma_{0N}^2)^n (\mu_N - \mu_{0N})^{k-2n}}{2^n N! (k-2n)!}, \quad (10)$$

where $a_{k,j} = (j+1)a_{k-1,j} + a_{k-1,j-1}$ and $a_{k,k} = a_{k,0} = 1$. Here, μ_{0N} and σ_{0N}^2 are newly introduced parameters under the criterion of stationarity.

2.3 Probability Distribution on L_{eq} of Correlated Noise Level

2.3.1 Estimation of moment statistics of averaged intensity E In the previous section, N samples of sound intensity have been treated as statistically independent ones. Considering the effect of banking of traffic flow and *etc.*, these samples must have some correlation each other. Now, let us consider the effect of this correlation effect into the estimation theory and derive the probabilistic evaluation of averaged sound intensity. When a noise level $\mathbf{X} (\triangleq (X_1, X_2, \dots, X_N))$ of continuous N samples over an averaging time period is observed as the natural logarithm of noise intensity $\varepsilon (\triangleq (\varepsilon_1, \dots, \varepsilon_N))$:

$$X = \ln \frac{\varepsilon}{\varepsilon_0}, \quad \varepsilon_0 \triangleq 10^{-12} [\text{watt/m}^2], \quad (11)$$

its joint moment generating function can be given as follows:

$$M_X(\theta) \triangleq \langle e^{\theta \mathbf{X}} \rangle = 1 + \sum_{n=1}^{\infty} \sum_{n_1 + \dots + n_N = n} \frac{\mu(n_1, \dots, n_N)}{n_1! n_2! \dots n_N!} \theta_1^{n_1} \dots \theta_N^{n_N}, \quad (12)$$

where $\mu(n_1, \dots, n_N)$ denotes the n -th order joint moment statistics of $\mathbf{X}$. On the other hand, by considering Eq.(11), the θ -th order joint moment statistics of intensity ε can be evaluated by the joint moment statistics of noise level $\mathbf{X}$ as follows:

$$\langle \varepsilon_1^{\theta_1} \dots \varepsilon_N^{\theta_N} \rangle = 1 + \sum_{n=1}^{\infty} \sum_{n_1 + \dots + n_N = n} \frac{\mu(n_1, \dots, n_N)}{n_1! \dots n_N!} \theta_1^{n_1} \dots \theta_N^{n_N}. \quad (13)$$

As the L_{eq} is an evaluation index on averaged sound intensity of N samples (see Eq.(1)), the probability distribution of L_{eq} can be evaluated

from the statistical characteristics of E . By considering Eq.(13) and the moment generating function of E :

$$m_E(\theta) \triangleq \langle e^{\theta E} \rangle = \langle e^{\frac{\theta}{N}(\varepsilon_1 + \dots + \varepsilon_N)} \rangle = 1 + \sum_{t=1}^{\infty} \sum_{t_1 + \dots + t_N = t} \frac{\langle \varepsilon_1^{t_1} \dots \varepsilon_N^{t_N} \rangle}{t_1! \dots t_N!} \left(\frac{\theta}{N} \right)^t. \quad (14)$$

As is well-known, we can have easily the ℓ -th order moment statistics of E as the ℓ -th differential of Eq.(14) as follows:

$$\begin{aligned} \langle E^\ell \rangle &= \left. \frac{d^\ell}{d\theta^\ell} m_E(\theta) \right|_{\theta \rightarrow 0} \\ &= \sum_{t_1 + \dots + t_N = \ell} \frac{\ell!}{t_1! \dots t_N!} \left(\frac{1}{N} \right)^\ell \left\{ 1 + \sum_{n=1}^{\infty} \right. \\ &\quad \times \sum_{n_1 + \dots + n_N = n} \frac{\mu(n_1, \dots, n_N)}{n_1! \dots n_N!} t_1^{n_1} \dots t_N^{n_N} \left. \right\}. \end{aligned} \quad (15)$$

Then, the moment statistics of E can be evaluated by the joint moment statistics of noise level obtained by the usual observation method.

2.3.2 Reconstruction of p.d.f. on L_{eq} The p.d.f. of L_{eq} can be estimated from the moment statistics of E by using the statistical Laguerre type expansion series expression [9].

$$\begin{aligned} P(E) &= \frac{E^{m-1} e^{-E/s}}{\Gamma(m) s^m} \\ &\times \left\{ 1 + \sum_{n=3}^{\infty} B_n \frac{n! \Gamma(m)}{\Gamma(n+m)} L_n^{(m-1)} \left(\frac{E}{s} \right) \right\} \end{aligned} \quad (16)$$

with

$$m \triangleq \frac{\langle E \rangle^2}{\langle E^2 \rangle - \langle E \rangle^2}, \quad s \triangleq \frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle}$$

and

$$B_n \triangleq \left\langle L_n^{(m-1)} \left(\frac{E}{s} \right) \right\rangle.$$

Figure 1: Actual situations of measuring traffic noise.

3 EXPERIMENTAL CONSIDERATION

3.1 Measurement of Road Traffic Noise

The theoretical methods have been confirmed experimentally too by applying them to the actual road traffic noise. Figure 1 shows the actual situation of noise measurement in two cases; Case A, light traffic situation at a walk in front of a College; Case B, heavy traffic situation at a road crossing. The parameters and the moment statistics on the noise level fluctuation has been observed every one minute by using a usual sound level meter with a time constant "FAST" and with A-weighting network. The L_{eq} data in one minute have been also observed simultaneously. The temporally fluctuating patterns of L_{eq} over long time period (Case A:90 minutes and Case B:60 minutes) are shown in Fig. 2.

Figure 2: Temporal fluctuation patterns of L_{eq} over whole observation time period.

3.2 Estimation Results of L_{eq} Distribution for An Un-correlative Noise

When the parameters, μ and σ , on level fluctuation can be recognized as constant, the moment statistics of averaged intensity within a defined time length (1 min) can be estimated by Eq. (7). Though the statistical independency has been assumed to derive Eq. (4), the actual noise level data have shown some mutual correlation (in Case A, about 1 sec; in Case B, about 10 sec). Then, to eliminate the effect of correlation time, we adopted the sample number N as the number of independent samples within 1 minute (that is, for Case A $N = 60$ and for Case B $N = 5$, in Eq. (4) and Eq. (7)). The theoretically estimated result on the probability distribution form of L_{eq} by using Eq.(6) for a stationary noise is shown in Fig. 3 under the assumption of Gaussian property. In this figure, as only two parameters, μ and σ^2 , have been used, the theoretical curve could not illustrate enough the experimentally sampled points.

3.3 Estimation Results of L_{eq} Distribution for A Non-stationary Noise

When the distribution parameters μ and σ^2 on level fluctuation shows non-stationary property, the fluctuation of L_{eq} (1min) can be evaluated by Eq. (9). The temporal fluctuation of parameters are evaluated by Eq. (10). The theoretically estimated result of probability distribution on L_{eq} over

Figure 3: Comparisons between experimentally sampled points and theoretically estimated curves for the cumulative probability distribution of L_{eq} (1 min.). Experimentally observed points are marked by $\bullet$ and theoretically estimated curves with use of Eq. (6) are shown as —.

Figure 4: Comparisons between experimentally sampled points and theoretically estimated curves for the cumulative probability distribution of L_{eq} (1 min.). Experimentally observed points are marked by $\bullet$ and theoretically estimated curves are shown with the degree of approximation n in Eq. (9) as: $\cdots$, $n = 0$; $---$, $n = 1$; —, $n = 6$ for Case A and 3 for Case B.

a long time period Eq.(9) is shown in Fig. 4 by reflecting the non-stationary property of parameters over a long time interval.

3.4 Estimation Results of Probability Distribution on L_{eq} Under the Correlation Effect of Noise Level

First, let us evaluate the joint moment statistics of noise levels over an observation time period of L_{eq} (1 min.). For sampling ratio of 1 sec, the numbers of instantaneous levels N to obtain an L_{eq} within 1 min. is 60 and the effect of correlation of noise levels over N samples have to be reflected into Eq. (15). However, by considering the fact that the joint moment statistics become to zero for a time lag larger than the correlative time length τ of noise level fluctuation, Eq. (15)

can be reduced as follows:

$$\begin{aligned} \langle E^\ell \rangle &= \left\langle \left\{ \frac{1}{N} (\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_N) \right\}^\ell \right\rangle \\ &= \frac{\ell!}{N^\ell} \sum_{\ell_1 + \ell_2 + \cdots + \ell_K = \ell} \prod_{j=1}^K \sum_{n_1 + n_2 + \cdots + n_\tau = \ell_j} \\ &\quad \times \frac{1}{n_1! n_2! \cdots n_\tau!} \left\{ 1 + \sum_{t=1}^{\infty} \sum_{t_1 + t_2 + \cdots + t_\tau = t} \right. \\ &\quad \times \left. \frac{\mu_X(t_1, t_2, \cdots, t_\tau)}{t_1! t_2! \cdots t_\tau!} n_1^{t_1} n_2^{t_2} \cdots n_\tau^{t_\tau} \right\}, \quad (17) \end{aligned}$$

where $K \triangleq [N/\tau]$ and $[]$ denotes the symbol of Gauss.

The theoretically estimated result by Eq.(16) is shown in Fig. 5 by using the joint moment

Figure 5: Comparisons between experimentally sampled points and theoretically estimated curves for the cumulative probability distribution of L_{eq} (1 min.). Experimentally observed points are marked by $\bullet$ and theoretically estimated curves with use of Eq. (16) are shown with the degree of approximation $n = 6$ as —. In Case B, as a comparison between un-correlated and correlated noise cases, theoretically estimated curve with use of Eq. (6) are shown as - - - - -

statistics of noise level which reflects the correlation among N samples within the averaging time of L_{eq} .

4 CONCLUSION

In this study, by considering that the L_{eq} assumed to be an invariant noise index within a local short time changes very often temporally with the lapse of time over a long time period and becomes to show a multifarious stochastic property, an evaluation method of L_{eq} fluctuation over a long time period has been proposed by means of its probability distribution form under the real situation of the noise level observation on [dB] scale without observing a whole shape of its instantaneous fluctuation pattern.

Then, an evaluating method of probability distribution on L_{eq} is proposed by reflecting the statistical characteristics of noise level observed by usual noise measurement. The probability distribution form of the L_{eq} has been theoretically established by using the usual noise level statistics and then this proposed method has been confirmed experimentally too by applying it to the actually observed L_{eq} data.

Acknowledgement

We express our cordial thanks to Prof. Kazutastu Hatakeyama and Mr. Shigeru Matsumoto for their helpful discussion and assistance.

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