Blind Deconvolution Algorithms for MIMO-FIR Systems Driven by Fourth-order Colored Signals

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Abstract

This paper deals with the blind deconvolution problem of MIMO-FIR channels driven by source signals which are temporally second-order uncorrelated but fourth-order correlated and spatially second- and fourth-order uncorrelated. Recently, we proposed a criterion for solving the blind deconvolution problem [8], [9]. However, we have not yet shown an algorithm to implement our proposed criterion. One of our objectives in the present paper is to propose an iterative algorithm for achieving the blind deconvolution by using this criterion. To achieve this objective, we consider an approach called the *non-deflation method*. We propose an iterative algorithm based on the method. Moreover, we present a *deflation* algorithm using this criterion for the blind separation of MIMO constant channels.

1. Introduction

Blind deconvolution is a problem of extracting source signals from their convolutive mixtures observed by sensors without knowing the source signals along with the transfer function between the sources and the sensors. The blind deconvolution has drawn an attention in diverse fields, for example, digital communications, speech processing, image processing, and array signal processing, etc.

The blind deconvolution problem has been studied by many researchers. For example, see [1]-[4] and references therein. Almost all the methods proposed to date have been developed under the assumption that the source signals are temporally independent and identically distributed (i.i.d.) and spatially independent (e.g., see [2]-[4]). However, there are some applications in which the condition of i.i.d. for the source signals is very strong. For example, in digital communications, the information bearing sequences are coded in order to combat noise corruption and channel distortion. These codes implicitly are not mutually independent among sequences and hence it is unlikely that they are i.i.d. signals. However, these code sequences are interleaved to avoid burst errors when the codes are transmitted [5]. These interleaved sequences are usually considered to be uncorrelated. To solve the blind deconvolution problem for such an application, therefore,

we should take account of an assumption that the source signals have a weaker condition than the i.i.d. condition, for example, the source signals are temporally second-order uncorrelated but high-order correlated.

In this paper, we deal with the blind deconvolution of MIMO-FIR channel systems driven by source signals which are temporally fourth-order colored signals (but temporally second-order white and spatially second- and fourth-order uncorrelated signals). The condition for the source signals is weaker than the i.i.d. condition. We previously proposed a criterion for blind deconvolution under this condition [8], [9]. However, we did not show any algorithms to implement the blind deconvolution using the proposed criterion.

One of our objectives in this paper is to propose an iterative algorithm to solve the blind deconvolution problem by using the proposed criterion. To this end, we consider the non-deflation approach for recovering the source signals simultaneously from their convolutive mixtures. Equalizers of MIMO-FIR channels are used to acquire the function of blind deconvolution. The proposed algorithm iteratively adjusts the parameters of equalizers so that the criterion takes a minimum. We carry out computer simulations using the proposed algorithm. The simulation results show that our algorithm can solve the blind deconvolution problem.

It is well-known that there is an approach called the deflation method for the blind separation of sources under the assumption that the source signals are i.i.d. and spatially independent. However, it is not clear whether the *deflation approach* can be applied to the MIMO-FIR channels in the case where the sources are fourth-order colored. It is shown in the present paper that this is the case for MIMO constant channels. Namely, we propose a deflation algorithm using this criterion for the blind separation of MIMO constant channels.

2. Problem formulation

We consider an MIMO-FIR system given by the following equation:

$$\mathbf{x}(t) = \sum_{k=0}^K \mathbf{H}^{(k)} \mathbf{s}(t-k), \quad (1)$$

where $\mathbf{x}(t)$ represents an m -column output vector called the *observed signal*, $\mathbf{s}(t)$ represents an n -column input vector called the *source signal*, $\{\mathbf{H}^{(k)}\}$ is an $m \times n$ matrix sequence representing the impulse response, and the number K denotes its order. Equation (1) can be written as

$$\mathbf{x}(t) = \mathbf{H}(z)\mathbf{s}(t), \quad (2)$$

where $\mathbf{H}(z)$ is the transfer function, which is defined by the z -transform of the impulse response $\{\mathbf{H}^{(k)}\}$, that is,

$$\mathbf{H}(z) = \sum_{k=0}^K \mathbf{H}^{(k)} z^k.$$

One of the objectives in this paper is to work out an iterative algorithm to solve the blind deconvolution problem. To this end, we consider the following FIR system called the *equalizer*.

$$\mathbf{y}(t) = \sum_{k=0}^L \mathbf{W}^{(k)} \mathbf{x}(t-k), \quad (3)$$

where $\mathbf{y}(t)$ is an n -column vector representing the output signal of the equalizer, $\{\mathbf{W}^{(k)}\}$ is an $n \times m$ matrix sequence, and the number L is the order of the equalizer. Equation (3) can be written as

$$\mathbf{y}(t) = \mathbf{W}(z)\mathbf{x}(t), \quad (4)$$

where $\mathbf{W}(z)$ is the transfer function of the equalizer defined by

$$\mathbf{W}(z) = \sum_{k=0}^L \mathbf{W}^{(k)} z^k.$$

Substituting (2) into (4), we have

$$\mathbf{y}(t) = \mathbf{W}(z)\mathbf{H}(z)\mathbf{s}(t) := \mathbf{G}(z)\mathbf{s}(t), \quad (5)$$

where

$$\mathbf{G}(z) := \mathbf{W}(z)\mathbf{H}(z) = \sum_{k=0}^{K+L} \mathbf{G}^{(k)} z^k. \quad (6)$$

Figure 1 shows the cascade system of an unknown system followed by an equalizer.

Figure 1 The cascade system of an unknown system and an equalizer.

We allow all the variables to be complex-valued. This is required for such an application using quadrature amplitude modulation (QAM) signals [5]. If $\mathbf{G}(z)$ in (6) becomes a matrix $\mathbf{G}_0(z)$ of the form defined by

$$\mathbf{G}_0(z) (= \mathbf{W}_0(z)\mathbf{H}(z)) = \Lambda(z)\mathbf{D}\mathbf{P}, \quad (7)$$

then the each component of $\mathbf{s}(t)$ is recovered by some component of $\mathbf{y}(t)$, except for three types of ambiguities, that is, the order ambiguity, the time-origin ambiguity, and the magnitude ambiguity, because $\Lambda(z)$ is a diagonal matrix with diagonal elements $\lambda_{ii}(z) = z^{l_i}$ ($i = 1, \dots, n$) (where l_i is

a nonnegative integer), $\mathbf{D}$ is a regular constant diagonal matrix, and $\mathbf{P}$ is a permutation matrix. Hence, the i -th component of $\mathbf{y}(t)$ is expressed as

$$y_i(t) = d_i s_{p_i}(t - l_i), \quad i = 1, \dots, n, \quad (8)$$

where $\{p_1, \dots, p_n\}$ is an arbitrary permutation of $\{1, \dots, n\}$. Therefore, we formulate the blind deconvolution problem as follows: Our objective is to find a matrix $\mathbf{W}_0(z)$ satisfying (7) without knowing the transfer function $\mathbf{H}(z)$ along with the source signals $s_i(t)$'s. [If the transfer function $\mathbf{G}(z)$ becomes $\mathbf{G}_0(z)$, $\mathbf{G}(z)$ is said to be *transparent*.]

To find a matrix $\mathbf{W}_0(z)$, we make the following assumptions.

- (A1) The transfer function $\mathbf{H}(z)$ in (2) is of full column rank for all nonzero complex $z \neq \infty$ (which implies that the unknown system of (2) has less inputs than outputs, that is, $m \geq n$).
- (A2) The input sequence $\{\mathbf{s}(t)\}$ is a zero-mean, nonzero-variance stationary vector process whose component processes $\{s_i(t)\}$ ($i = 1, \dots, n$) are temporally second-order white and spatially second- and fourth-order uncorrelated. No more than one of the components of $\{\mathbf{s}(t)\}$ is Gaussian, and all the others are non-Gaussian with

$$\sum_{\tau_1, \tau_2, \tau_3 \in Z} |\mathbf{C}s_i(\tau_1, \tau_2, \tau_3)|^2 \neq 0 \quad (< \infty) \quad (i = 1, \dots, n) \quad (9)$$

where Z denotes the set of all integers, and $\mathbf{C}v(\tau_1, \tau_2, \tau_3)$ is the fourth-order auto-cumulant of signal $v(t)$ defined by $\mathbf{C}v(\tau_1, \tau_2, \tau_3) := \text{cum}\{v(t), v(t+\tau_1)^*, v(t+\tau_2), v(t+\tau_3)^*\}$. Here the superscript $*$ denotes the complex conjugate.

- (A3) The transfer function $\mathbf{W}(z)$ in (4) is an MIMO-FIR filter.

Remark 1: The assumption (9) in (A2) is weaker than the assumption for the source signals in [9], that is, $\mathbf{C}s_i(0, 0, 0) \neq 0$.

Under these assumptions, we attempt to solve the blind deconvolution problem.

3. The non-deflation method

We proved the following theorem [9].

Theorem 1: Let $\mathbf{H}(z)$ and $\{\mathbf{s}(t)\}$ satisfy (A1) and (A2), respectively. Suppose that $\mathbf{G}(z)$ is a paraunitary matrix. Then, maximizing the following criterion function (10) makes $\mathbf{G}(z)$ transparent.

$$Q_1 := \sum_{i=1}^n K y_i \quad (10)$$

where, $K y_i := \sum_{\tau_1, \tau_2, \tau_3 \in Z} |\mathbf{C}y_i(\tau_1, \tau_2, \tau_3)|^2$

In [9], we did not propose any algorithms

implementing criterion (10). In this section, in order to propose the algorithms, we use the non-deflation approach whose objective is to extract simultaneously the inputs $\{s_i(t)\}$ ($i = 1, \dots, n$) of the unknown multichannel system from the observed signals $\{x_i(t)\}$ ($i = 1, \dots, m$). To apply the non-deflation approach using the criterion to the blind deconvolution problem, we consider the following function:

$$Q_A = -Q_1 + Q_2 + Q_3, \quad (11)$$

where

$$Q_2 = \frac{1}{2} \sum_{\tau=0}^{\infty} (E[y_i(t)y_i(t-\tau)^*] - \delta(\tau))^2, \quad (12)$$

$$Q_3 = \frac{1}{2} \sum_{i,j=1}^n \sum_{\tau=0}^{\infty} E[y_i(t)y_j(t-\tau)^*]^2. \quad (13)$$

Here $E[x]$ denotes the expectation of x , Q_2 is a penalty function which evaluates the auto-correlations of $y_i(t)$ ($i = 1, \dots, n$), $\delta(\tau)$ denotes the Kronecker delta, and Q_3 is a penalty function which evaluates the correlations between $y_i(t)$ and $y_j(t)$ ($i, j = 1, \dots, n; i \neq j$).

Remark 2: The transfer function $G(z)$ becomes a paraunitary matrix, if and only if functions Q_2 and Q_3 take minimum values (zeros).

From Theorem 1 and Remark 2, if Q_A takes a minimum, the blind deconvolution can be achieved. To minimize Q_A , taking into account the fact that the gradient of the criterion function Q_A with respect to the matrix W is given by

$$\nabla_W Q_A = -2 \frac{\partial Q_A}{\partial W^*},$$

see [11, p.890], we derive the following “online” stochastic gradient decent learning rule:

$$\Delta w_{ij}^{(k)} \propto \alpha_1 \frac{\partial Q_1}{\partial w_{ij}^{(k)*}} - \alpha_2 \frac{\partial Q_2}{\partial w_{ij}^{(k)*}} - \alpha_3 \frac{\partial Q_3}{\partial w_{ij}^{(k)*}}. \quad (14)$$

where $w_{ij}^{(k)}$ are the parameters of the element $w_{ij}(z) = \sum_{k=0}^L w_{ij}^{(k)} z^k$ of $W(z)$ and α_i ($i = 1, 2, 3$) are positive constants that regulate the step size. On the calculation of the derivative of a scalar function with respect to complex-valued parameters, see [11] (p. 890). Calculating the right-hand side of (14), we have

$$\begin{aligned} \Delta w_{ij}^{(k)} \propto & \alpha_1 \sum_{\tau_1, \tau_2, \tau_3 \in Z} [C y_i(\tau_1, \tau_2, \tau_3) \\ & \times \{E[y_i(t)y_i(t+\tau_2)y_i(t+\tau_3)^*x_j(t+k+\tau_1)^*] \\ & + E[y_i(t)y_i(t+\tau_1)^*y_i(t+\tau_2)x_j(t+k+\tau_3)^*] \\ & - E[y_i(t+\tau_2)y_i(t+\tau_3)^*]E[y_i(t)x_j(t+k+\tau_1)^*] \\ & - E[y_i(t)y_i(t+\tau_1)^*]E[y_i(t+\tau_2)x_j(t+k+\tau_3)^*] \\ & - E[y_i(t)y_i(t+\tau_2)](E[y_i(t+\tau_3)^*x_j(t+k+\tau_1)^*] \end{aligned}$$

$$\begin{aligned} & + E[y_i(t+\tau_1)^*x_j(t+k+\tau_3)^*]) \\ & - E[y_i(t+\tau_1)^*y_i(t+\tau_2)]E[y_i(t)x_j(t+k+\tau_3)^*] \\ & - E[y_i(t)y_i(t+\tau_3)^*]E[y_i(t+\tau_2)x_j(t+k+\tau_1)^*] \}] \\ & - \alpha_2 \{ (E[y_i(t)y_i(t)^*] - 1)E[y_i(t)x_j(t+k)^*] \\ & + \sum_{\tau=1}^{\infty} E[y_i(t)y_i(t+\tau)^*]E[y_i(t)x_j(t+k+\tau)^*] \} \\ & - \alpha_3 \{ \sum_{k=1}^n \sum_{\tau=0}^{\infty} E[y_k(t)y_i(t+\tau)^*]E[y_k(t)x_j(t+k+\tau)^*] \}, \quad (15) \end{aligned}$$

Here, let us summarize the non-deflation algorithm.

Step 1. Compute the outputs $y_i(t)$ ($i = 1, \dots, n$) of the equalizer using (3).

Step 2. Compute $C y_i(\tau_1, \tau_2, \tau_3)$, $E[y_i(t+\tau_2)y_i(t+\tau_3)^*]$, $E[y_i(t)y_i(t+\tau_1)^*]$, $E[y_i(t)y_i(t+\tau_2)]$, $E[y_i(t+\tau_1)^*y_i(t+\tau_2)]$, $E[y_i(t)y_i(t+\tau_3)^*]$, $E[y_i(t)y_i(t)^*]$, and $E[y_k(t)y_i(t+\tau)^*]$ in (15) by using the moving average, e.g., $E[y_i(t+\tau_1)^*y_i(t+\tau_2)]$ is estimated by

$$\begin{aligned} \phi_{ii}(\tau_1, \tau_2, t+1) &= \beta \phi_{ii}(\tau_1, \tau_2, t) \\ &+ (1-\beta)y_i(t+\tau_1)^*y_i(t+\tau_2) \quad (0 < \beta < 1), \end{aligned}$$

with an appropriate initial value $\phi_{ii}(\tau_1, \tau_2, 0)$.

Step 3. Modify $w_{ij}^{(k)}$ using the following equation:

$$\begin{aligned} \Delta w_{ij}^{(k)} \propto & \alpha_1 \sum_{\tau_1, \tau_2, \tau_3 \in Z} [\phi_{C y_i}(\tau_1, \tau_2, \tau_3) \\ & \times \{y_i(t)y_i(t+\tau_2)y_i(t+\tau_3)^*x_j(t+k+\tau_1)^* \\ & + y_i(t)y_i(t+\tau_1)^*y_i(t+\tau_2)x_j(t+k+\tau_3)^* \\ & - \phi_{ii}(\tau_2, \tau_3, t)y_i(t)x_j(t+k+\tau_1)^* - \phi_{ii}(0, \tau_1, t)y_i(t+\tau_2)x_j(t+k+\tau_1)^* \\ & - \phi_{ii}(0, \tau_2, t)(y_i(t+\tau_3)^*x_j(t+k+\tau_1)^* + y_i(t+\tau_1)^*x_j(t+k+\tau_3)^*) \\ & - \phi_{ii}(\tau_1, \tau_2, t)y_i(t)x_j(t+k+\tau_3)^* - \phi_{ii}(0, \tau_3, t)y_i(t+\tau_2)x_j(t+k+\tau_1)^* \}] \\ & - \alpha_2 \{ (\phi_{ii}(0, 0, t) - 1)y_i(t)x_j(t+k)^* \\ & + \sum_{\tau=1}^{\infty} \phi_{ii}(0, \tau, t)y_i(t)x_j(t+k+\tau)^* \} \\ & - \alpha_3 \{ \sum_{k=1}^n \sum_{\tau=0}^{\infty} \phi_{ki}(0, \tau, t)y_k(t)x_j(t+k+\tau)^* \}, \quad (16) \end{aligned}$$

where (16) is obtained by applying (15) to a stochastic gradient approach. Here $\phi_{C y_i}(\tau_1, \tau_2, \tau_3)$ computed by using the moving average is the estimation of $C y_i(\tau_1, \tau_2, \tau_3)$.

Step 4. Repeat the first three steps until the learning sufficiently converges.

4. The deflation method

In this section, we propose a deflation algorithm using the criterion function (10) for the blind separation of MIMO

constant channels. [Unfortunately we have not yet worked out any criterion for achieving the blind deconvolution of MIMO-FIR channels by using deflation approaches.] We consider, therefore, the following MIMO constant system:

$$\mathbf{x}(t) = \mathbf{H}^{(0)} \mathbf{s}(t), \quad (17)$$

where (17) is given by putting $K = 0$ in (1). The problem is to separate $\mathbf{s}(t)$ from $\mathbf{x}(t)$ without knowing $\mathbf{H}^{(0)}$ and observing $\mathbf{s}(t)$.

The deflation approach can recover $\{s_i(t)\}$ ($i = 1, \dots, n$) one by one from $\{x_i(t)\}$ ($i = 1, \dots, m$). In this approach, we first apply a maximization criterions to the outputs of the unknown multichannel system with m outputs and n inputs to extract only one source signal, and estimate the contribution of the extracted source signal to the output signals. Then, we remove this contribution from the output signals to define the outputs of a multichannel system with m outputs and $n-1$ inputs. We again apply the maximization criterion to the outputs of the unknown multichannel system with m outputs and $n-1$ inputs to extract the next source signal. We continue this process until we extract the last source signal.

To implement the idea of the deflation approach, a low vector $\mathbf{w} = [w_1, \dots, w_m]$ of the matrix $\mathbf{W}^{(0)}$ is used for representing the parameters of an equalizer. Therefore, the output of the equalizer is given by

$$\begin{aligned} y(t) &= \mathbf{w} \mathbf{x}(t) \\ &= \mathbf{g} \mathbf{s}(t), \end{aligned} \quad (18)$$

where $\mathbf{g}(z) = [g_1, \dots, g_n]$ is a low vector of the matrix $\mathbf{G}^{(0)}$. Criterion (10) is modified to the following criterion function.

$$Q_4 = \frac{1}{2} K_y \quad (19)$$

Here, we present the following theorem.

Theorem 2: Under $\mathbf{g} \mathbf{g}^{*T} = 1$, the deflation algorithm of maximizing criterion Q_4 gives a solution to the blind separation problem, that is,

$$\begin{aligned} g_i &= c & \text{for some } i = i_0 \\ &= 0 & \text{for } i \neq i_0 \end{aligned}$$

where c is a constant with unit magnitude $|c| = 1$ and $i_0 = \arg \max_{i \in \{1, 2, \dots, n\}} Ks_i$.

The outline of the proof of Theorem 2 is given by Appendix. Note that $\mathbf{g} \mathbf{g}^{*T} = 1$ implies that the variance of output signal $y(t)$ is unit and $\arg \max_{i \in \{1, 2, \dots, n\}} Ks_i$ is the value of number $i \in \{1, 2, \dots, n\}$ that maximizes Ks_i .

Based on Theorem 2, by applying the deflation approach to the blind separation problem, we propose the following minimization criterion:

$$Q_B = -Q_4 + Q_5, \quad (20)$$

where

$$Q_5 = \frac{1}{2} (E[y(t)y(t)^*] - 1)^2. \quad (21)$$

Here Q_5 is a penalty term added to satisfy the constraint $\mathbf{g} \mathbf{g}^{*T} = 1$.

Remark 3: The constraint $\mathbf{g} \mathbf{g}^{*T} = 1$ holds, if and only if function Q_5 takes a minimum (zero).

From Theorem 2 and Remark 3, one can see that if Q_B takes a minimum, the blind deconvolution can be achieved. In order to minimize Q_B , we use the same rule as (14);

$$\Delta w_i \propto \alpha_4 \frac{\partial Q_4}{\partial w_i^*} - \alpha_5 \frac{\partial Q_5}{\partial w_i^*}. \quad (22)$$

where α_i ($i=4,5$) are positive constants that regulate the step size. Calculating the right-hand side of (22), we have

$$\begin{aligned} \Delta w_i &\propto \alpha_4 \sum_{\tau_1, \tau_2, \tau_3 \in \mathbb{Z}} [C_y(\tau_1, \tau_2, \tau_3) \\ &\times \{ E[y(t)y(t+\tau_2)y(t+\tau_3)^* x_i(t+\tau_1)^*] \\ &\quad + E[y(t)y(t+\tau_1)^* y(t+\tau_2)x_i(t+\tau_3)^*] \\ &\quad - E[y(t+\tau_2)y(t+\tau_3)^*] E[y(t)x_i(t+\tau_1)^*] \\ &\quad - E[y(t)y(t+\tau_1)^*] E[y(t+\tau_2)x_i(t+\tau_3)^*] \\ &\quad - E[y(t)y(t+\tau_2)] (E[y(t+\tau_3)^* x_i(t+\tau_1)^*] \\ &\quad \quad + E[y(t+\tau_1)^* x_i(t+\tau_3)^*]) \\ &\quad - E[y(t+\tau_1)^* y(t+\tau_2)] E[y(t)x_i(t+\tau_3)^*] \\ &\quad - E[y(t)y(t+\tau_3)^*] E[y(t+\tau_2)x_i(t+\tau_1)^*] \}] \\ &- \alpha_5 \{ (E[y(t)y(t)^*] - 1) E[y(t)x_i(t)^*] \} \end{aligned} \quad (23)$$

Here, let us summarize an iterative procedure of extracting the source signals from the observed signals by using the deflation approach.

Step 1. Compute the output $y(t)$ of the equalizer, using (18).

Step 2. Compute $C_y(\tau_1, \tau_2, \tau_3)$, $E[y(t+\tau_2)y(t+\tau_3)^*]$, $E[y(t)y(t+\tau_1)^*]$, $E[y(t)y(t+\tau_2)]$, $E[y(t+\tau_1)^* y(t+\tau_2)]$, $E[y(t)y(t+\tau_3)^*]$, and $E[y(t)y(t)^*]$ in (23) by using the moving average. For example, $E[y(t+\tau_2)y(t+\tau_3)^*]$ is estimated by using the following rule:

$$\begin{aligned} \phi(\tau_2, \tau_3, t+1) &= \beta \phi(\tau_2, \tau_3, t) \\ &\quad + (1-\beta) y(t+\tau_2) y(t+\tau_3)^* \quad (0 < \beta < 1), \end{aligned}$$

with an appropriate initial value $\phi(\tau_2, \tau_3, 0)$.

Step 3. Modify w_i using the following equation:

$$\begin{aligned} \Delta w_i &\propto \alpha_4 \sum_{\tau_1, \tau_2, \tau_3 \in \mathbb{Z}} [|\phi_{Cy}(\tau_1, \tau_2, \tau_3)| \\ &\times \{ y(t)y(t+\tau_2)y(t+\tau_3)^* x_i(t+\tau_1)^* + \\ &\quad y(t)y(t+\tau_1)^* y(t+\tau_2)x_i(t+\tau_3)^* \\ &\quad - \phi(\tau_2, \tau_3, t) y(t)x_i(t+\tau_1)^* - \phi(0, \tau_1, t) y(t+\tau_2)x_i(t+\tau_1)^* \\ &\quad - \phi(0, \tau_2, t) (y(t+\tau_3)^* x_i(t+\tau_1)^* + y(t+\tau_1)^* x_i(t+\tau_3)^*) \} \end{aligned}$$

$$- \phi(\tau_1, \tau_2, t)y(t)x_i(t+\tau_3)^* - \phi(0, \tau_3, t)y(t+\tau_2)x_i(t+\tau_1)^*] \\ - \alpha_5 \{ (\phi(0, 0, t) - 1)y(t)x_i(t)^* \} \quad (24)$$

where (24) is obtained by applying (23) to a stochastic gradient approach. Here $\phi_{Cy}(\tau_1, \tau_2, \tau_3)$ computed by using the moving average is the estimation of $Cy(\tau_1, \tau_2, \tau_3)$.

Step 4. Repeat the first above three steps until the learning sufficiently converges. If the learning converges, go to the next step.

Step 5. Assume that the source signal $s_{p_i}(t)$ is recovered. Compute the scale of the input $s_{p_i}(t)$ by using the following equation:

$$v(t) = \sum_{j=1}^m w_{j0}x_j(t), \quad (25)$$

where w_{j0} 's are the parameters of the vector w_0 obtained by the first three steps.

Step 6. Estimate the scale of $h_{qp_i}^{(0)}$ by using the cross-correlation of the observed signals $x_q(t)$'s and $v(t)$'s as

$$\hat{h}_{qp_i}^{(0)} := E[x_q(t)v(t)^*], \quad (q = 1, 2, \dots, m). \quad (26)$$

where $h_{ij}^{(0)}$ is an element of the matrix $H^{(0)}$.

Step 7. Estimate the contribution of $s_{p_i}(t)$ to the observed signals $x_q(t)$ ($q = 1, 2, \dots, m$), that is, $h_{qp_i}s_{p_i}(t)$, using

$$\hat{x}_{qp_i}(t) = \hat{h}_{qp_i}^{(0)}v(t), \quad (q = 1, 2, \dots, m). \quad (27)$$

Step 8. Remove the above contribution using the following equation:

$$x_q^{(i)}(t) = x_q(t) - \hat{x}_{qp_i}(t), \quad (q = 1, 2, \dots, m). \quad (28)$$

where $x_q^{(i)}(t)$ ($q = 1, \dots, m$) are the outputs of a linear unknown multichannel system with m outputs and $n-1$ inputs.

Step 9. If the superscript (i) of $x_q^{(i)}(t)$ is less than n , then set $i = i + 1$ and $x_q(t) = x_q^{(i)}(t)$ ($q = 1, \dots, m$), and go to Step 1. If $i = n$, then stop here.

For the details of the above Step 5 to Step 9, see [10].

5. Computer Simulations

We carried out some computer simulations to demonstrate the validity of our proposed algorithms. This section describes two of them.

Example 1 (Non-deflation algorithm)

In this example, we show the computer simulation

result of the blind deconvolution obtained by using the non-deflation algorithm. We considered the following two-input and three-output FIR system.

$$H(z) = \begin{bmatrix} h_{11}(z) & h_{12}(z) \\ h_{21}(z) & h_{22}(z) \\ h_{31}(z) & h_{32}(z) \end{bmatrix} = \begin{bmatrix} 0.5 & z \\ z & 1 \\ 1 & 0.5z \end{bmatrix}.$$

The observed signal $x(t) = [x_1(t), x_2(t), x_3(t)]^T$ was calculated by (2). The source signals $\{s_i(t)\}$ ($i = 1, 2$) were generated by the following system:

$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} \frac{0.7+z}{1+0.7z} & 0 \\ 0 & \frac{0.3+z}{1+0.3z} \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}. \quad (29)$$

where $\{v_1(t)\}$ and $\{v_2(t)\}$ were non-Gaussian i.i.d. signals with zero mean and unit variance, but were independent with each other. Since the filters $(0.7+z)/(1+0.7z)$ and $(0.3+z)/(1+0.3z)$ in (29) were all-pass filters, the source signals $\{s_1(t)\}$ and $\{s_2(t)\}$ became temporally second-order white but temporally fourth-order colored signals.

The order of the equalizer in (3) was set to $L = 1$. We used the three-input and two-output FIR system as the equalizer. The values of τ_1 , τ_2 , and τ_3 in Q_1 were set to $[0, 2]$, $[0, 2]$, and $[0, 2]$, respectively. The values of the parameters τ in Q_2 and Q_3 were set to $[0, 2]$ and $[0, 2]$, respectively. The parameters α_1 , α_2 , and α_3 of the learning algorithm were set to 0.0000001, 0.001, 0.0005, respectively (see (16)). The initial values of w_{ij}^k ($i = 1, 2$; $j = 1, 2, 3$; $k = 0, 1$) were set to zero except for $w_{11}^0 = 1$ and $w_{23}^0 = 1$.

The following equalizer $W_0(z)$ was found after 100000 iterations using the procedure of Step 1 to Step 4 in the non-deflation approach:

$$W_0(z) = \begin{bmatrix} 1.2683 & -0.0117 & -0.5840 \\ -0.6569 & -0.0223 & 1.2945 \end{bmatrix} \\ + \begin{bmatrix} 0.0099 & 0.0018 & 0.0017 \\ -0.0329 & -0.0219 & 0.0096 \end{bmatrix} z$$

Therefore, the output signals $y_1(t)$ and $y_2(t)$, respectively, were

$$y_1(t) = 0.0502s_1(t) - 0.0051s_1(t-1) + 0.0018s_1(t-2) \\ - 0.0117s_2(t) + 0.9781s_2(t-1) + 0.0107s_2(t-2) \\ \approx s_2(t-1), \\ y_2(t) = 0.9661s_1(t) - 0.0291s_1(t-1) - 0.0219s_1(t-2) \\ - 0.0223s_2(t) - 0.0315s_2(t-1) - 0.0281s_2(t-2) \\ \approx s_1(t).$$

From this result, it can be seen that our proposed non-deflation algorithm can achieve the blind deconvolution.

Example 2 (Deflation algorithm)

In the example 2, we show the computer simulation result of the deflation algorithm. The channel matrix $H^{(0)}$ was set to

$$\mathbf{H}^{(0)} = \begin{bmatrix} h_{11}^{(0)} & h_{12}^{(0)} \\ h_{21}^{(0)} & h_{22}^{(0)} \\ h_{31}^{(0)} & h_{32}^{(0)} \end{bmatrix} = \begin{bmatrix} -0.6 & 0.1 \\ -0.3 & 0.2 \\ 0.8 & -1.7 \end{bmatrix}.$$

The observed signal $\mathbf{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$ was calculated by (17). The source signals $\{s_i(t)\}$ ($i = 1, 2$) were generated by the following system:

$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{0.3+z}{1+0.3z} \end{bmatrix} \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix}. \quad (30)$$

where the first component signal $\{\psi_1(t)\}$ was a Gaussian white signal and the second component signal $\{\psi_2(t)\}$ was a non-Gaussian, i.i.d. signal with zero mean and unit variance. The source signal $\{s_2(t)\}$ was temporally second-order white but temporally fourth-order colored signal.

The values of τ_1 , τ_2 , and τ_3 in Q_4 were set to $[0, 2]$, $[0, 2]$, and $[0, 2]$, respectively. The value of the parameter τ in Q_5 was set to $[0, 2]$. The parameters of the learning algorithm were chosen as $\alpha_4 = 0.000001$ and $\alpha_5 = 0.001$ (see (24)). The output of the equalizer is calculated by $y(t) = w_1x_1(t) + w_2x_2(t) + w_3x_3(t)$. The initial values of w_i ($i = 1, 2, 3$) were set to 0 except for $w_1 = 1$.

The following equalizer w_0 was found after 500000 iterations using the procedure of Step 1 to Step 4;

$$\mathbf{w}_0 = [-0.8359, 0.1479, -0.5881]$$

Therefore, the output of the equalizer was

$$y(t) = -0.0133s_1(t) + 1.0049s_2(t) \\ \approx s_2(t)$$

Since $Ks_2 > Ks_1 = 0$, one can see that the deflation algorithm can be used to extract the source signal which has the largest energy of the fourth-order auto-cumulant sequences among the source signals. Moreover, we confirmed that the source signal $\{s_1(t)\}$ was extracted by the procedure of Step 5 to Step 9.

6. Conclusions

We have proposed an iterative algorithm based on the non-deflation approach of the blind deconvolution for temporally second-order white and spatially second- and fourth-order uncorrelated signals. Moreover, we have presented an iterative algorithm based on the deflation approach for the blind separation of MIMO constant channels.

We have carried out computer simulations to demonstrate our proposed algorithms. The results have shown that the proposed algorithms could work well.

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Appendix: Outline of the proof of Theorem 2

We can derive the following inequalities:

$$K_y \leq \sum_{\tau_1, \tau_2, \tau_3 \in \mathbb{Z}} \left\{ \sum_{i=1}^n |g_i|^4 C_{s_i}(\tau_1, \tau_2, \tau_3) \right\}^2 \\ \leq K_0 \left(\sum_{i=1}^n |g_i|^4 \right)^2 \leq K_0 \left\{ \left(\sum_{i=1}^n |g_i|^2 \right)^2 \right\}^2 \quad (31)$$

where $K_0 = \max_{\{k=1, \dots, n\}} \sum_{\tau_1, \tau_2, \tau_3 \in \mathbb{Z}} |C_{s_k}(\tau_1, \tau_2, \tau_3)|^2$. Taking into account $\sum_{i=1}^n |g_i|^2 = 1$, we have $K_0 \left\{ \left(\sum_{i=1}^n |g_i|^2 \right)^2 \right\}^2 = K_0$.

The inequality of the third and fourth terms in (31) holds with equality if and only if

$$\sum_{i \neq k} |g_i|^2 |g_k|^2 = 0 \quad (32)$$

From $\sum_{i=1}^n |g_i|^2 = 1$ and (32), we have

$$|g_{i_0}| = 1 \quad \text{for some } i_0 \in I_0$$

$$|g_i| = 0 \quad \text{for any } i \neq i_0$$

Here, I_0 is the subset of natural numbers less than or equal to n defined by $I_0 = \{i \in \{1, \dots, n\} \mid Ks_i = K_0\}$. Therefore, we have $K_y = K_0$ when Q_B is maximized under $\mathbf{g}\mathbf{g}^{*T} = \mathbf{I}$ ($\sum_{i=1}^n |g_i|^2 = 1$).