Traffic Flow Control by Adjusting Split Time of Crossing Signal

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Abstract

This paper is concerned with proposing an optimal design of a traffic flow control by defining some concepts of waiting flow for the traffic crossing with multiple signals as well as single signal. The performance of the optimal control strategy is compared with the one of a fixed-type control strategy.

1. Introduction

In order to reduce traffic congestion, crowdness and stagnation around traffic crossings, there exist several different methods of controlling traffic signals. Traditionally, the performance for evaluating the efficiency of constructing a new traffic signal control system, no matter how the crossing is single or multiple, has been mostly judged on how many times the vehicle line flow delays or stops. First of all, the program methods which operate periodically to control traffic signals by the a-priori fixed rules are familiar. In literature [1], the optimal traffic light control problem for an intersection of two two-way streets is presented where the inflow quantity is considered to be constant.

The intelligent control method which runs in the real time according to the present traffic flow has not been studied so far. As for this problem, Yamane [2] have proposed the optimal setting method of the split time of traffic signals for single and multiple crossings under simplified assumptions and showed the effectiveness by computational simulations.

2. Optimal split time of traffic signal at single crossing

Suppose \( 	au \) means the present time. The outflows and the inflows at the crossing are defined as \( y_j(t) \) and \( v_j(t) (j = 1 \sim 4) \) respectively in four directions of the crossing. \( v_j(t) \) is assumed to be measurable.

Fig. 1 The traffic model of single crossing

Some fundamental definitions are written down briefly.

[Definition 1]

Traffic flows [3] such as inflow \( v(t) \in \mathbb{R}^4 \) and outflow \( y(t) \in \mathbb{R}^4 \) are defined as the number of total traffic vehicles to pass one section of the road in one way during the unit time.
A new idea of waiting quantity is introduced to describe traffic crossing capacity in Fig. 1 which constructs the traffic model from a more practical standpoint. At first, it is considered to describe a mathematical model of traffic control system shown in Fig. 1. Because the increase of inflow quantity \(v_j(t) - y_j(t)\) is equivalent to the change amount of the waiting flow quantity \(x_j(t)\), the traffic control system in continuous time of \(t\) at the traffic crossing can be obtained as

\[
\frac{dx}{dt} = v(t) - y(t) \quad (1)
\]

for \(x(t) \geq 0 \quad (2)

\[[\text{Definition 2}] \]

\(x(t) \in \mathbb{R}^4\) of Eq (1) is called as a waiting flow length.

Then the traffic control system equation of discrete time of \(t\) with sampling time \(h\) is written by

\[
x(t + 1) = Ax(t) + h(-y(t) + v(t)) \quad (3)
\]

where

\[
A = I
\]

The control signal of the crossing is denoted by \(u_j(t)(j = 1 \sim 4)\). When \(u_1(t) = u_3(t) = 1\), the first and the third direction signals of the crossing are green. Otherwise, when \(u_1(t) = u_3(t) = 0\), the second and the fourth direction signals of the crossing are green. The relation \(u_1(t) + u_2(t) = 1\) always holds between \(u_1\) and \(u_2\).

The following assumption statements for traffic flow control system are assumed.

\[[\text{Assumption 1}]\] The traffic streams straightly flow ahead.

\[[\text{Assumption 2}]\] The traffic control \(u(t) \in \mathbb{R}^4\) signal operates only when green and red.

For the waiting flows defined as \(x_j(t)(j = 1 \sim 4)\), it is assumed that there exist non-negative functions \(b_j\) respectively in four directions of the crossing such that

\[
\begin{align*}
  \text{if } x_j(t) \geq 0 & \quad h y_j = b_j x_j \\
  \text{where } b_j &= f_j(v, x)
\end{align*}
\]

that is

\[
\begin{align*}
  \text{if } x(t) \geq 0 & \quad h y(t) = -B(t)u(t) \\
  \text{otherwise } h y(t) &= 0
\end{align*}
\]

where

\[
B(t) = \begin{bmatrix}
  b_1(t) & 0 \\
  0 & b_2(t) \\
  b_3(t) & 0 \\
  0 & b_4(t)
\end{bmatrix}
\]

\(f \in \mathbb{R}^4\) is dependent on characteristics of each crossing flow state. For example,

\[
\begin{align*}
  \text{if } f < \mu \quad & \quad \text{then } f = h v + x \\
  \text{otherwise } f &= \mu
\end{align*}
\]

where \(\mu\) means a maximum flow capacity while limit outflows to pass capacity flow of crossing.

Substituting into Eq (3) yields

\[
x(t + 1) = Ax(t) + B(t)u(t) + hv(t) \quad (4)
\]

Suppose that \(z_1\), \(z_2\) and \(z_3\) mean the total volume of inflow, waiting flow length and outflow in sum of four directions. Multiplying \(c = (1, 1, 1, 1)\) to the traffic control system Eq (3) becomes the relation

\[
z_2(t + 1) = z_2(t) + z_1(t) - z_3(t) \quad (5)
\]

where

\[
\begin{align*}
  z_1(t) &= c^T h v(t) \\
  z_2(t) &= c^T x(t) \\
  z_3(t) &= c^T h y(t)
\end{align*}
\]

\(z_2(t + 1) - z_2(t)\) is equivalent to the increase of waiting quantity at each discrete time of \(t\).

Furthermore let \(p(k)\), \(q(k)\) and \(r(k)\) be the total volumes of inflow, waiting flow and outflow in the time interval from \(t = \tau\) to \(t = k\) to formulate the following statements.

\[
q(k) = p(k) - r(k) \quad (9)
\]
where

\[ p(k) = \sum_{t=\tau}^{k} z_1(t) \]  
(10)

\[ q(k) = \sum_{t=\tau}^{k} \{ z_2(t+1) - z_2(t) \} \]  
(11)

\[ r(k) = \sum_{t=\tau}^{k} z_3(t) \]  
(12)

Outflow quantity normalized by the total inflow one is defined as passage efficiency \( J_1(k)(k \geq \tau) \)

\[ J_1(k) = \frac{r(k)}{p(k)} = J_1\{u_1(\tau), u_1(\tau + 1), \ldots, u_1(k)\} \]  
(13)

The waiting efficiency \( J_2(k)(k > \tau) \) is also defined as

\[ J_2(k) = \frac{q(k)}{p(k)} = J_2\{u_1(\tau), u_1(\tau + 1), \ldots, u_1(k)\} \]  
(14)

Then it is clear from Eq(5) that

\[ J_1(k) + J_2(k) = 1 \]  
(15)

From this fact once \( J_1(k)(k \geq \tau) \) is maximized, while \( J_2(k)(k \geq \tau) \) is minimized. As for the waiting flow quantity

\[ q(k) = c^T\{x(k + 1) - x(\tau)\} \]  
(16)

holds, the minimization problem of \( J_2 \) is regarded as the output regulation one. The passage ratio \( J_1(k) \) can be easily computed as

\[ J_1(k) = 1 - \frac{c^T\{x(k + 1) - x(\tau)\}}{p(k)} \]  
(17)

Without direct calculation of \( r(k) \), two different cases corresponding to a sign of \( J_2 \) are classified as follows,

1) stable waiting flow \( x(k + 1) \geq x(\tau) \geq 0 \) such that \( 1 \geq J_1 \geq 0 \) for \( J_2 \geq 0 \). This is called unsaturated flow state.

2) unstable waiting flow \( x(k + 1) \leq x(\tau) \) such that \( J_1 \geq 1 \) for \( J_2 \leq 0 \). This corresponds to saturated flow state.

Let's consider the optimal signal setting with regard to traffic capacity of crossing. In order to determine an optimal condition of the split time the total outflow quantity against the total inflow quantity is considered.

Here, the problem of case 1) which maximizes the passage ratio \( J_1(k) \) for a given \( k \) becomes most realistic and plausible. It is so difficult to obtain optimal solutions to maximize \( J_1(k)(k > \tau) \) because the optimal control \( u_1(t) \) depends on \( v(t) \) in future. So when we consider of maximization problem where we restrict \( k(> \tau) \) to \( k = \tau \), the optimal control strategy is automatically obtained.

\[ \text{if} \ b_1(\tau) - b_2(\tau) + b_3(\tau) - b_4(\tau) > 0 \]

\[ \text{then} \ u_1(\tau) = 1 \text{ otherwise } u_1(\tau) = 0 \]  
(18)

From a special viewpoint we consider the traffic flow in steady state as \( i \to \infty \). When the waiting flow is stable, that is,

\[ \lim_{i \to \infty} x(t) = x(\tau) \]

then we have

\[ \lim_{i \to \infty} J_1(i) \to 1 \]  
(19)

Moreover, in the same manner, Yamane [2] have proposed an optimal setting method of split time for multiple traffic crossings. However, in constructing the mathematical model in References [2], the waiting quantity around the traffic crossing has not been considered there at all.

3. Optimal split time of traffic signal with multiple crossings

We consider about a description of a mathematical model of traffic control system with multiple crossings. It is shown in Fig.2 that the waiting flows are defined as \( x_j(t)(j = 1 \sim 8) \), the outflows are defined as \( y_j(t)(j = 1 \sim 8) \) and the inflows are defined as \( v_j(t)(j = 1 \sim 8) \) respectively for eight directions of A,B crossings. It is assumed that there exist nonnegative functions \( b_j \) respectively in
four directions of two crossing such that

\[
\text{if } x_j(t) \geq 0 \quad y_j = b_j u_j
\]

where  \( b_j = f_j(v, x) \)  \( (20) \)

\[
\begin{array}{c c c c c c c c c}
\uparrow & v_3 & \downarrow & \uparrow & v_7 & \downarrow & \uparrow & v_1 & \downarrow \\
v_4 & \uparrow & \downarrow & y_1, v_5 & \downarrow & y_5 & \downarrow & y_2 & \downarrow \\
A & \uparrow & \downarrow & y_2, v_6 & \uparrow & y_6 & \downarrow & y_3 & \downarrow \\
\uparrow & v_7 & \downarrow & \uparrow & v_1 & \downarrow & \uparrow & v_6 & \downarrow \\
\end{array}
\]

Fig. 2 The traffic model with two crossings

When the signal in the \( j \)-th direction shows green, we denote \( u_j(t) = 1 \). Suppose that the consuming time to pass each crossing of A, B requires \( t = 0 \), otherwise the time to pass the road between A and B consumes \( \rho \) sampling times.

The traffic control system equation is written in a similar way for single crossing.

\[
x(t + 1) = Ax(t) + B(t)u(t) + v(t) \quad (21)
\]

where

\[
x(t) \in R^8, v(t) \in R^8, y(t) \in R^8
\]

\[
u_A = (u_1, u_2)^T, u_2 = 1 - u_1
\]

\[
u_B = (u_3, u_4)^T, u_4 = 1 - u_3
\]

\[
c = (1, 1, 1, 1, 1, 1, 1, 1)^T
\]

\[
A = I
\]

\[
B(t) = \begin{bmatrix} B_1(t) & 0 \\ 0 & B_2(t) \end{bmatrix}
\]

\[
B_1(t) = \begin{bmatrix} b_1(t) & 0 \\ 0 & b_2(t) \end{bmatrix}
\]

\[
B_2(t) = \begin{bmatrix} b_6(t) & 0 \\ 0 & b_8(t) \end{bmatrix}
\]

At the same time, it is assumed that the following relations hold between \( v_2(t) \) and \( b_6(t), v_8(t) \) and \( b_4(t) \)

\[
v_2(t + \rho) = b_6(t)(1 - u_3(t)) \quad (22)
\]

\[
v_8(t + \rho) = b_4(t)(1 - u_1(t)) \quad (23)
\]

Multiplying c to the traffic control system Eq(21) leads to the following equation

\[
z_2(t + 1) = z_2(t) + z_1(t) - z_3(t) \quad (24)
\]

where

\[
z_1(t) = c^T h v(t) \quad (25)
\]

\[
z_2(t) = c^T x(t) \quad (26)
\]

\[
z_3(t) = c^T h y(t) \quad (27)
\]

We define \( p(k), q(k) \) and \( r(k) \) as the total volume of inflow, waiting flow and outflow of Eq (25)~(27) for two crossings.

To maximize \( J_1(\tau) \) at time \( \tau \), it is necessary to satisfy following conditions

\[
\text{if } b_1(\tau) - b_2(\tau) + b_3(\tau) - b_4(\tau) > 0 \quad \text{then } u_1(\tau) = 1
\]

\[
\text{otherwise } u_1(\tau) = 0 (28)
\]

\[
\text{and if } b_5(\tau) - b_6(\tau) + b_7(\tau) - b_8(\tau) > 0 \quad \text{then } u_3(\tau) = 1
\]

\[
\text{otherwise } u_3(\tau) = 0 (29)
\]

4. Simulation

Here the effectiveness of proposed method is illustrated for the optimal design of traffic control by some numerical examples for both of single and two crossings.

Here the effectiveness of proposed method is illustrated for the optimal design of traffic control by some numerical examples.

Suppose that the consuming time to pass the single crossing needs one sampling time \( (\rho = 1) \). The inflow \( v_j(t) (j = 1 \sim 4 \) or \( j = 1 \sim 8 \) with no correlation are given as random numbers \( \{ 0 \leq v_j(t) \leq m_1 \} \) and \( b_j \) is assumed to limit within \( 0 \leq b_j \leq m_2 \). Here, if \( x_j < 0 \) is satisfied, then \( b_j = 0 \) otherwise \( b_j = x_j - v_j \). When the traffic capacity ratio \( \varepsilon = \frac{m_2}{m_1} \) takes 0.5,0.75,1,2,3, the effectiveness of the technique are figured out by the numerical experiments.

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4.1 In case of single crossing

$J_1(t)$ is bounded within $0 < J_1 < 1$ and the efficiency improves as $J_1(t)$ approaches 1. Here we consider the four different kinds of the fixed-type control signal such that the $u_j(t)(j = 1 \sim 4)$ forms the regular which takes 0 and 1 alternately, and 0,1,1,1, and 1,0,0,0, and 0,0,1,1 periodically. They are denoted by $u^1$, $u^2$, $u^3$, and $u^4$ respectively. Simulation results are shown in Fig.3 that $u^1$ and $u^2$ are superior to $u^3$ and $u^4$. The simulation results of the vehicle passage efficiency of both the optimal and the fixed-type control methods for the single crossing are shown in Fig.4. The average value of the passage efficiency is 0.976 for the optimal strategy $u^{opt}$ and is 0.864 for fixed-type one $u^1$. The average by using the optimal strategy increased more 11.6 % than the fixed-type one. Also, the comparison of the passage efficiency between the optimal and the fixed-type methods is shown in Fig.5 for different traffic capacity ratio $\varepsilon$.

4.2 In case of two crossings

The simulation results of the passage efficiency of both the optimal and the fixed-type control methods for two crossings are shown in Fig.6. The average value of the passage efficiency is 0.948 for the optimal strategy $u^{opt}$ and is 0.716 for fixed-type one $u^1$. The optimal strategy increased more about 23 % in average than the fixed-type one . Also, the comparison of the vehicle passage efficiency between the optimal and the fixed-type methods in two crossings is shown in Fig.7 as traffic capacity ratio $\varepsilon$ varies.

5. Conclusion

In this study, the optimal setting method is proposed for traffic flow system model by introducing the new idea of the waiting flow, compared with the fixed-type strategy. The passage efficiency using optimal strategy improved 17.4% in average as shown in Figs.4,6. In Figs.5,7 when $\varepsilon > 1$, i.e. the value of outflow capacity becomes high, we can confirm that the passage efficiency substantially improved.

There remain unsolved problems as follows

1. If $v(t)$, $x(t)$ can be estimated in future time of $k > \tau$, the non-linear mathematical programing problem which maximizes $J_1(k)$ can be solved by using the genetic algorithm. Moreover, a new evaluation function $\delta_1 (1-J_1(k)) + \delta_2 J_2(k)$ ($\delta_1 > 0, \delta_2 > 0$) with weighting coefficients $\delta_1, \delta_2$ to both of the passage efficiency and the waiting efficiency is considered to formulate the problem generally.

2. The inflow level $\lambda_j$ where $0 \leq v_j(t) \leq \lambda_j (\lambda_j = m_1)$ effects an influence on $J_1$. To the extent generally, the smaller $m_1$ becomes, the higher $J_1$ approaches 1 and the efficiency becomes better.

3. The development of automatic algorithm for genetic control inputs based on present traffic flow lines.

4. Assumption 1 is to remove by additional amber signal lights.

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Fig. 3 Pass flow efficiency with fixed-type periodic control when $h = 1$ for signal crossing.
References


