Noninteracting PID Control using Precompensators with Series - Parallel Connections

Hee-Hyol Lee*

Fukuoka Inst. of Tech.

Won-Kyu Choi

CRESYS Co.,Ltd.

Michio Miyazaki

Kanto Gakuin Univ.

Kageo Akizuki

Waseda Univ.

* E-mail:lee@fit.ac.jp, 3-30-1 Wajirohigashi, Higashi-ku, Fukuoka 811-0295 JAPAN

Abstract

Controlled variables in multivariable systems interact each other in general, therefore, it is necessary to remove their interaction sufficiently for the design of control systems. A cross controller was used for removing the interaction thus far mainly, however, it was complicated and the controller was high-order. On the other hand, the precompensator by the inverse Nyquist array method was also used for noninteraction of the controlled variables. However it is difficult to remove the interaction of the controlled system, which has very strong interaction, when the precompensator is used only one. This paper proposes the series and/or parallel connections of the precompensators for noninteracting PID control.

1 Introduction

In multivariable control systems, controlled variables interact each other in general. In this case one compensation for a control loop exerts harmful influence upon other control loop through the interaction. If we could take away this interaction by some ways for a complete one to one corresponds between the manipulated variable and the controlled variable, then the design of control systems results in the individual compensation of the single loop control.

A cross controller was used for removing the interaction

thus far mainly, but it was complicated and the controller was high-order. On the other hand, the precompensator by the inverse Nyquist array method was also used for noninteraction of the controlled variables. However it is difficult to remove the interaction of the controlled system, which has very strong interaction, when the precompensator is used only one.

This paper proposes the series and/or parallel connections of the precompensators for noninteracting PID control, and the precompensators are applied to the control of the fluid temperature and liquid level interacting plant to confirm the effectiveness.

2 Noninteracting precompensators by pseudo diagonalization

Let the precompensator G_c be a square matrix of order m .

$$G_c = [h_{ij}] \quad (1)$$

Then the ij th element of $G(s) = G_p(s) G_c$ is represented by

$$\ell_{ij}(j\omega_0) = \sum_{k=1}^m g_{ik}(j\omega_0) h_{kj} \quad (2)$$

at $s = j\omega_0$, where $g_{ik}(s)$ is the ik th element of $G_p(s)$.

Here the elements of the p th row-vector $\mathbf{h}_p = (h_{1p}, h_{2p}, \dots, h_{mp})^T$ of G_c is selected as that the square-sum of the absolute value of the non-diagonal elements of the p th row-vector $\mathbf{h}_p$ of $G(j\omega_0)$,

$$\sum_{i=1, i \neq p}^m |\ell_{ip}(j\omega_0)|^2 \quad (3)$$

is minimized under condition of the constraint,

$$h_{1p}^2 + h_{2p}^2 + \dots + h_{mp}^2 = 1 \quad (4)$$

For evaluation of interaction, a diagonal dominance is used in general. Let $\tilde{G}(s)$ be the inverse matrix of a rational function matrix $G(s)$. Given the controlled system $G_p(s)$ and taking the pseudo diagonalization of the precompensator G_c , if $\tilde{G}(s) = (G_p(s) G_c)^{-1}$ becomes the diagonal dominance, then the open loop transfer function matrix $Q(s) = G_p(s) G_c K(s)$ is considered to be the diagonal dominance for a diagonal matrix $K(s)$. Therefore every loop of the main controller $K(s)$ can design independently neglecting effects of the non-diagonal elements of $\tilde{G}(s)$.

We can evaluate the diagonal dominance graphically. The diagonal dominance of the inverse matrix $\tilde{Q}(s) = \{\tilde{q}_{ij}(s)\}$ of the transfer function matrix $Q(s)$ on the imaginary axis is equivalent to that one side of the inequalities is valid for all ω ,

$$|\tilde{q}_{ii}(j\omega)| > d_i(j\omega) \equiv \sum_{j=1, j \neq i}^m |\tilde{q}_{ij}(j\omega)|, i = 1, 2, \dots, m \quad (5)$$

$$|\tilde{q}_{ii}(j\omega)| > d_i'(j\omega) \equiv \sum_{j=1, j \neq i}^m |\tilde{q}_{ji}(j\omega)|, i = 1, 2, \dots, m \quad (6)$$

The inequalities (5) and (6) mean that the circle with its center at $\tilde{q}_{ii}(j\omega)$ and with a radius of $d_i(j\omega)$ does not include the origin. The set of vector loci is the inverse Nyquist array, and $d_i(j\omega)$ and $d_i'(j\omega)$ are Gershgorin band, then the Gershgorin band shows a degree of the interaction for the controlled system.

3 Series-Parallel connections of precompensators

We are sometimes confronted with the difficult that noninteraction could not be achieved by one precompensator for the controlled system, which has very strong interaction. Here we consider the connections of precompensators: series, parallel and their combination.

Series connections:

Figure 1 : Series connections of precompensators

Parallel connections:

Figure 2 : Parallel connections of precompensators

For the series connections, the first precompensator G_{c1} is designed using a typical hunting frequency ω_{01} , and the diagonal dominance is evaluated by the inverse Nyquist array method. If the diagonal dominance does not succeed, the second precompensator G_{c2} is designed at ω_{02} , which is the frequency of strong interaction, for the new controlled system $G_p(s)' = G_p(s) G_{c1}$. The precompensators are connected one by one in this manner. On the hand, for the parallel connections, every precompensator G_{ci} , $i = 1, 2, \dots, n$, is designed using the hunting frequencies ω_{0i} , and then they are connected in parallel.

4 Noninteracting PID control using precompensators

4.1 Experimental plant

The fluid temperature and liquid level experimental plant used in this study is composed of a warm water tank, a cold water tank and a mixed tank. Cold water and warm water are supplied to the mixed tank by two pumps, and the fluid temperature and the liquid level of the mixed tank are measured by a thermocouple and a level meter, respectively. This mixed tank is a closed tank with a constant flow valve, and the controlled system has very strong interaction between the fluid temperature and liquid level controlled variables. The construction of the experimental plant and the step responses of the plant are illustrated in Fig.3 and Fig.4, respectively, and the transfer functions deduced from the step responses are as follow

$$G_T(s) = \frac{50277.498s^2 - 2320.5s + 35.7}{484466.55s^3 + 23768.33s^2 + 409s + 1}$$

Figure 3 : Construction of the experimental plant

Figure 4 : Step responses of the plant

$$G_{LT}(s) = \frac{76.893s^2 - 80.94s + 28.4}{460.275s^3 + 487.2075s^2 + 172.85s + 1}$$

$$G_{TL}(s) = \frac{1526.514s^2 - 458s + 45.8}{10132.32s^3 + 3037.33s^2 + 314s + 1}$$

$$G_L(s) = \frac{6376.32s^2 - 885.6s + 41}{40124.16s^3 + 5728.32s^2 + 279.6s + 1} \quad (7)$$

4.2 PID control by each single loop

For the controlled variables of the fluid temperature and liquid level, we design PID controllers for the each loop regarding other controlled variable as a disturbance.

Figure 5 is the result of the control, and it shows that the hunting occurs at two controlled variables with main period of about 4.3 [min].

Figure 5 : Result of the PID control

4.3 Series-Parallel connection of precompensators

4.3.1 Design of a precompensator

For the transfer function Eq.(7) of the fluid temperature and liquid level interacting plant, the precompensator G_{cl} is designed at the hunting frequency $\omega_{01} = 0.0245$ as

$$G_{cl} = \begin{bmatrix} 0.6845 & 0.9597 \\ -0.7290 & 0.2812 \end{bmatrix} \quad (8)$$

The inverse Nyquist diagrams and Gershgorin bands of the loop transfer function $G(s) = G_p(s) G_{c1}$ are illustrated in *Fig.6*, and the diagonal dominance does not succeed.

Figure 6 : Inverse Nyquist diagram by G_{c1}

4.3.2 Series connection of precompensators

When we use one precompensator, it is difficult to realize the diagonal dominance for the controlled system with very strong interaction. Accordingly, we connect the precompensators one by one. A new precompensator G_{c2} is connected in a series, and designed at ω_{02} , which is the frequency of strong interaction, regarding the transfer function $G_p(s) G_{c1}$ as a new controlled system $G_p(s)'$. If the diagonal dominance does not realize, then a further precompensator G_{c3} is connected in a series and designed in same manner. The block diagram of the precompensators connected in a series to the third stage is shown in *Fig.7*, and then the precompensator $G_{c1}G_{c2}G_{c3}$ is as

$$G_{c1} G_{c2} G_{c3} = \begin{bmatrix} 0.1858 & 0.2306 \\ 0.1973 & 0.1544 \end{bmatrix} \tag{9}$$

The inverse Nyquist diagram and Gershgorin bands are shown in *Fig.8*, and then the diagonal dominance is

realized at every frequency.

Figure 7 : Block diagram of series connection

Figure 8 : Inverse Nyquist diagram by series connection

4.3.3 Parallel connection of precompensators

A parallel connection is also possible. *Figure 9* is a block diagram of the parallel connection. The precompensator G_{c2} is designed regarding the transfer function $G_p(s) G_{c1}$ as a new controlled system $G_p(s)'$. Similarly the precompensator G_{c3} is designed regarding the transfer function $G_p(s)' G_{c2}$ as a new controlled system $G_p(s)''$. The precompensator $G_{c1}+G_{c2}+G_{c3}$ connected in parallel is

$$G_{c1} + G_{c2} + G_{c3} = \begin{bmatrix} 0.7407 & 2.6180 \\ -2.2479 & -0.8362 \end{bmatrix} \tag{10}$$

and the inverse Nyquist diagram and Gershgorin bands are shown in *Fig.10*, and then the diagonal dominance also realized.

Figure 9 : Block diagram of parallel connection

Figure 10 : Inverse Nyquist diagram by parallel connection

4.3.4 Parallel-Series connection of precompensators

A parallel-series connection of the precompensators is illustrated in *Fig.11* as an example. The precompensators G_{c11} and G_{c12} are designed using the hunting frequencies ω_{o1} and ω_{o2} of the result of PID control shown in *Fig.5*. If the diagonal dominance does not realize, then new precompensators G_{c21} and G_{c22} are connected. The transfer function is as

$$(G_{c11} + G_{c12})(G_{c21} + G_{c22}) = \begin{bmatrix} 1.4073 & 2.0880 \\ -2.0346 & -1.4258 \end{bmatrix} \quad (11)$$

and the inverse Nyquist diagram and Gershgorin bands are shown in *Fig.12*, then the diagonal dominance succeeds.

Figure 11 : Block diagram of parallel-Series connection

Figure 12 : Inverse Nyquist diagram by parallel-series connection

4.4 Control experiment

We apply the proposed precompensators with series and/or

parallel connection to noninteracting PID control of the fluid temperature and liquid level experimental plant shown in Fig.3. The precompensators are connected to the third stage in a series described in section 4.3.2, and the PI controller is designed for each loop of the fluid temperature and liquid level controlled variables, respectively. The experimental result is shown in Fig.13. The fluid temperature and liquid level controlled variables reach quickly the set points of the fluid temperature 50 [deg] and the liquid level 40 [cm], respectively, and furthermore the hunting does not occur in two controlled variable. As the result we can confirm the effectiveness of the proposed method.

Figure 13 : Noninteracting PI control by series connection of precompensators

5 Conclusion

Controlled variables in multivariable systems interact each other in general, therefore, it is necessary to remove their interaction sufficiently for the design of control systems. As one of the countermeasure, the precompensator by the inverse Nyquist array method was used for noninteraction of the controlled variables. However it was difficult to remove the interaction of the controlled system, which has very strong interaction, when the precompensator is used

only one. Accordingly, the series and/or parallel connections of the precompensators for noninteracting PID control were proposed in this paper, and the precompensators were applied to the control of the fluid temperature and liquid level interacting plant. As the result we could confirm the effectiveness of the proposed method.

References

- [1] H.H.Rosenbrock: *Design of Multivariable Control Systems using the Inverse Nyquist Array*, IEE Proc., **115** (1969)
- [2] P.L.Falb and W.A.Wolovich: *Decoupling in the Design and Synthesis of Multivariable Control Systems*, IEEE, **AC-12** (1967)
- [3] D.J.Hawkins: *Pseudo Diagonalization and the Inverse Nyquist Array Method*, IEE Proc., **119** (1972)
- [4] H.Kimura: *CAD of the Multivariable Control System*, J. of Soc. of Inst. and Conrl. Eng., Vol.15, No.12 (1976)
- [5] M.Ito et al.: *Theory of Design for Linear Control Systems*, SICE (1978)
- [6] H.Lee et al.: *Noninteracting PID Control of Fluid Temperature and Liquid Level Interacting System*, T.IEE Japan, Vol.119-C, No.8/9 (1999)