An Accurate Pipe Length Measurement for Straight And Curved Pipes with Open Ends Using Stationary Waves

S. Tanaka and M. Okamoto

Faculty of Engineering, Yamaguchi University

2-16-1 Tokiwadai, Ube, Yamaguchi, JAPAN

tanaka@sens.eee.yamaguchi-u.ac.jp

Abstract

The authors previously proposed an on-line pipe length measurement for straight and curved pipes with closed ends using stationary waves which were generated in the pipes. The system adopted several modes of the stationary wave as state variables of a linear dynamic system and reduced the length measurement to a parameter estimation problem, and thereby realized a highly accurate pipe length measurement. This paper presents a new length measurement system for straight pipes with open ends. Experiments will show that a highly accurate pipe length measurement is realized without any trouble on the annoying open end compensation.

1 Introduction

In the field of measurement, we often encounter the situation where pipe lengths have to be indirectly measured. The examples are the length measurement of pipelines of gas, water and electric cables underground. For the measurement, pulse-echo method is well known [1]. The method is however difficult to use under noisy conditions. This is because the method requires a precise measurement of time-of-flight (TOF) to realize an accurate length measurement. In addition, the method needs an equipment for generating a sound wave to be incided into the pipe. This eventually leads to a high cost.

The authors therefore previously proposed a pipe length measurement system for straight and curved pipes with closed ends by making use of the stationary waves generated in the pipes spontaneously or made by hitting the pipes with a club [2, 3]. In the system, some of the stationary waves were adopted as the state variables of a linear dynamic system and Kalman filters and maximum likelihood method were applied to the system to estimate the unknown parameters L (pipe length) and V (observation noise variance).

Moreover, paying attention to the fact that high order modes of the stationary waves have a high resolution ability with respect to the measurement, the authors presented a selection criterion for the optimal modes of the stationary wave and realized a much higher accurate measurement system using the optimal modes. With the method, the high accuracy of the measurement error 1~2 mm was achieved for the straight and curved pipes about 10m long. The advantages of the system are firstly that a highly accurate on-line length measurement is achieved even under noisy environment and secondly that it is applicable without any input equipment (this yields the reduction of cost of the measurement system).

There are however practically lots of applications where length measurement is required for the straight and curved pipes with open ends. In order to measure the length of such a pipe, open end compensation must be taken into account [4]. But, the problem is that the compensation strongly depends on the environment around the open ends and furthermore it is not a priori accurately known.

The paper proposes a new measurement system which measures the length of the pipes with open ends. The idea we propose is that we get observation data with an acoustic sensor from the pipe in two different conditions. That is, first directly from the pipe, next from the pipe with one end covered with a cap. Using these two types of observation data, the pipe length and open end compensation are simultaneously measured using the similar method as our previously developed method [2]. Finally, experiments will show that an accurate pipe length measurement is realized together with the measurement of the open end compensation.

Although the proposed approach is effective to straight and curved pipes with one or two open ends, the discussion will be below focussed on straight pipes with two open ends for simplicity.

Fig.1 Stationary waves formed in the pipe with two open ends.

2 Measurement of pipe length and open end compensation using low order modes model

2.1 Linear dynamic model of the stationary wave of the pipe with two open ends

We consider a straight pipe of length L with two open ends as shown in Fig.1. Considering a short time interval, sound fluctuation of each mode of the stationary wave in the pipe is modeled as follows :

$$x_{Aj}(t) = \alpha_{Aj} \sin(\omega_{Aj}t + \phi_{Aj}) \quad (j = 1, 2, 3, \dots) \quad (1)$$

where ω_{Aj} represents the angular frequency of the j -th mode of the stationary wave and is given by $\omega_{Aj} = j \times 2\pi v / \{2(L + 2\delta)\}$, where v is the sound velocity in the pipe and δ represents an open end compensation. When the pipe is located considerably distant from the floor, δ is, as well-known, given by $\delta = cr/2$ where r is the radius of the pipe and $c = 0.613$ is the open end compensation coefficient [4]. Note, however, that the coefficient c is not a priori known in practical measurement environment.

Back to the stationary wave, similarly as our previous method[2], we approximate the sound fluctuation in the pipe $z_A(t)$ as a linear combination of the first three modes' components of the stationary wave as follows :

$$z_A(t) = \sum_{j \in \Omega_A} x_{Aj}(t) \quad (2)$$

where Ω_A is the set of mode numbers $\{1, 2, 3\}$. Each mode of the stationary wave, i.e., $x_{Aj}(t)$ satisfying the differential equation $dx_{Aj}^2(t)/dt^2 = -\omega_{Aj}^2 x_{Aj}(t)$, the following dynamic equation is obtained if the state vector $\mathbf{x}_A = (x_{A1}, \dot{x}_{A1}, x_{A2}, \dot{x}_{A2}, x_{A3}, \dot{x}_{A3})^T$ is defined.

$$\dot{\mathbf{x}}_A(t) = \mathbf{A}\mathbf{x}_A(t) + \mathbf{w}_A(t) \quad (3)$$

where

$$\left. \begin{aligned} \mathbf{A} &= \begin{bmatrix} A_1 & O & O \\ O & A_2 & O \\ O & O & A_3 \end{bmatrix} \\ A_j &= \begin{bmatrix} 0 & 1 \\ -\omega_{Aj}^2 & 0 \end{bmatrix} \quad (j = 1, 2, 3) \end{aligned} \right\} \quad (4)$$

where $\mathbf{w}_A(t) = (0, w_{A1}(t), 0, w_{A2}(t), 0, w_{A3}(t))^T$ represents a transition noise vector introduced to compensate for an attenuation of the stationary wave in the short time interval. The noises $w_{Aj}(t)$ ($1 \leq j \leq 3$) are assumed to be the independent white Gaussian ones

Fig.2 Stationary waves formed in the pipes with one open end.

with zero means and the same variance σ_A^2 .

Discretization of the dynamic system gives

$$\mathbf{x}_A(k+1) = \mathbf{F}_A \mathbf{x}_A(k) + \mathbf{w}_A(k) \quad (5)$$

where $\mathbf{x}_A(k)$ and $\mathbf{w}_A(k)$ are respectively the state vector and the transition noise vector at the sampling time $k\Delta T$ (ΔT : sampling period). $\mathbf{F}_A$ is the transition matrix defined as

$$\mathbf{F}_A = \Phi_A(\Delta T), \quad \Phi_A(t) = \mathcal{L}^{-1}(s\mathbf{I} - \mathbf{A})^{-1} \quad (6)$$

where $\mathbf{I}$ and $\mathcal{L}^{-1}[\cdot]$ mean respectively an identity matrix and Laplace inverse transform.

The observation equation is given by an acoustic sensor inside the pipe and is modeled as

$$y_{Ak} = \mathbf{h}_A \mathbf{x}_A(k) + v_{Ak} \quad (7)$$

where $\mathbf{h}_A = (1, 0, 1, 0, 1, 0)$. y_{Ak} and v_{Ak} are respectively the observation and the measurement noise at the sampling time $k\Delta T$. The measurement noise v_{Ak} includes not only the intrinsic sensor noise but also the other modes of the stationary wave than the first three ones. According to [2], the sum of various modes of the stationary wave can be regarded as a white Gaussian noise. Therefore, v_{Ak} can be assumed to be a white Gaussian noise with zero mean and variance V_A in the following discussions.

2.2 Linear dynamic model of the stationary wave of the pipe with one open end

We next consider the case where one end of the straight pipe is covered by a cap like Fig.2. Then, similarly as the case of the preceding two open ends, the sound fluctuation $z_B(t)$ in the pipe is modeled as

$$\begin{aligned} z_B(t) &= \sum_{j \in \Omega_B} x_{Bj}(t) \\ &= \sum_{j \in \Omega_B} \alpha_{Bj} \sin(\omega_{Bj}t + \phi_{Bj}) \end{aligned} \quad (8)$$

where Ω_B represents the set of mode numbers $\{1, 3, 5\}$ and ω_{Bj} is the angular frequency of the j -th mode of the stationary wave given as $\omega_{Bj} = j \times 2\pi v / \{4(L + \delta)\}$. δ is of course the open end compensation described before.

Defining now the state vector as $\mathbf{x}_B = (x_{B1}, \dot{x}_{B1}, x_{B3}, \dot{x}_{B3}, x_{B5}, \dot{x}_{B5})^T$ similarly as before, the dynamic and

observation equations concerning the first three modes of stationary wave are obtained as :

$$\begin{aligned} \mathbf{x}_{B(k+1)} &= F_B \mathbf{x}_{Bk} + \mathbf{w}_{Bk} \\ y_{Bk} &= \mathbf{h}_B \mathbf{x}_{Bk} + v_{Bk} \quad (k = 0, 1, 2, \dots) \end{aligned} \quad (9)$$

where $\mathbf{x}_{Bk}$ and $\mathbf{w}_{Bk}$ are respectively the state vector and the transition noise vector at the sampling time $k\Delta T$. Also, y_{Bk} and v_{Bk} are respectively observation and observation noise at the time and furthermore $\mathbf{h}_B = (1, 0, 1, 0, 1, 0)$. v_{Bk} can be here again assumed to be a white Gaussian noise with zero mean and variance V_B .

2.3 Combination of the two linear dynamic systems

Defining the state vector and the transition noise vector newly like $\mathbf{x}_k = (\mathbf{x}_{Ak}^T, \mathbf{x}_{Bk}^T)^T$ and $\mathbf{w}_k = (\mathbf{w}_{Ak}^T, \mathbf{w}_{Bk}^T)^T$, the unified dynamic and observation equations are obtained as :

$$\mathbf{x}_{k+1} = F \mathbf{x}_k + \mathbf{w}_k \quad (11)$$

$$\mathbf{y}_k = H \mathbf{x}_k + \mathbf{v}_k \quad (12)$$

where

$$F = \begin{bmatrix} F_A & O \\ O & F_B \end{bmatrix}, \quad H = \begin{bmatrix} \mathbf{h}_A & \mathbf{0}_{61} \\ \mathbf{0}_{61} & \mathbf{h}_B \end{bmatrix} \quad (13)$$

$\mathbf{y}_k$ and $\mathbf{v}_k$ are respectively the observation and the observation noise vectors defined as $\mathbf{y}_k = (y_{Ak}, y_{Bk})^T$ and $\mathbf{v} = (v_{Ak}, v_{Bk})^T$. Also, $\mathbf{0}_{61}$ represents the six dimensional zero row vector. Since v_{Ak} and v_{Bk} are originally defined as the sum of various modes corresponding to the different measurement conditions, they have a correlation. We thus define a non-zero covariance V_{AB} for them in the succeeding discussions.

2.4 On-Line measurement by low order modes model

The dynamic and observation equations (11) and (12) being linear and observation noise vector $\mathbf{v}_k$ having white Gaussian property, the state vector $\mathbf{x}_k$ is estimated by Kalman filter [5].

However, five parameters of pipe length L , open end compensation δ , observation noise covariance V_A , V_B and V_{AB} being all unknown, the Kalman filter cannot be directly applied. Therefore, the parameter vector $\boldsymbol{\theta} = (L, \delta, V_A, V_B, V_{AB})^T$ is defined and a value is assigned to the vector to estimate the state vector under the hypothesis $\boldsymbol{\theta}$ and the following likelihood function is calculated.

$$J(\boldsymbol{\theta}) = \prod_{k=1}^K p(\mathbf{y}_k / \boldsymbol{\theta}, Y^{k-1}) \quad (14)$$

where $p(\mathbf{y}_k / \boldsymbol{\theta}, Y^{k-1})$ represents the conditional probability density function of $\mathbf{y}_k$ under the condition that observation $Y^{k-1} \triangleq \{y_j; 0 \leq j \leq (k-1)\}$ and parameter vector $\boldsymbol{\theta}$ are given [2].

Consequently, pipe length measurement is achieved by

maximizing the criterion function (14) with respect to $\boldsymbol{\theta}$. In order to reduce the computational time, Powell method [6] is used because the method enables sequential one-dimensional searches in parameter space.

3 Highly accurate length measurement using optimal modes model

We first consider the selection of optimal modes in the usage of the measurement system. From the viewpoint of parameter discrimination (separability), Fisher's information matrix [7] is here introduced in order to evaluate the sensitivity of the parameters L and δ . Below, we discuss how the optimal modes of the stationary wave are selected.

3.1 Selection of the optimal modes

We first define the set of mode numbers of the stationary wave to be used for the two dynamic systems. That is, the set Ω_A for the dynamic system derived under the first measurement condition (two open ends), and the set Ω_B for that of another condition (i.e., one open end). Thus, using the angular frequencies $\{\omega_{Aj}; j \in \Omega_A\}$ and $\{\omega_{Bj}; j \in \Omega_B\}$, observations y_{Ak} , y_{Bk} are represented as follows :

$$y_{Ak} = \sum_{j \in \Omega_A} \{\alpha_{Aj} \sin(\omega_{Aj} k \Delta T + \phi_{Aj})\} + v_{Ak} \quad (15)$$

$$y_{Bk} = \sum_{j \in \Omega_B} \{\alpha_{Bj} \sin(\omega_{Bj} k \Delta T + \phi_{Bj})\} + v_{Bk} \quad (16)$$

where α_{Aj} represents the magnitude of the j -th mode of the stationary wave for the pipe with two open ends and α_{Bj} for the pipe with one open end. v_{Ak} and v_{Bk} are respectively observation noises for the pipe under corresponding measurement conditions. The observation noises being regarded as white Gaussian ones, if we define the observation vector as

$$\mathbf{y} = (y_{A1}, y_{B1}, y_{A2}, y_{B2}, \dots, y_{AM}, y_{BM})^T, \quad (17)$$

the probability density function of $\mathbf{y}$ is given by :

$$\begin{aligned} p(\mathbf{y}) &= \frac{1}{(2\pi)^M \det[V]^M} \\ &\times \exp \left[-\frac{1}{2} \sum_{k=1}^M (\mathbf{y}_k - \boldsymbol{\zeta}_k)^T V^{-1} (\mathbf{y}_k - \boldsymbol{\zeta}_k) \right] \end{aligned} \quad (18)$$

where M means the number of observations and

$$\mathbf{y}_k = (y_{Ak}, y_{Bk})^T \quad (19)$$

$$\begin{aligned} \boldsymbol{\zeta}_k &= \left(\sum_{j \in \Omega_A} \alpha_{Aj} \sin(\omega_{Aj} k \Delta T + \phi_{Aj}), \right. \\ &\quad \left. \sum_{j \in \Omega_B} \alpha_{Bj} \sin(\omega_{Bj} k \Delta T + \phi_{Bj}) \right)^T \end{aligned} \quad (20)$$

$$V = \begin{bmatrix} V_A & V_{AB} \\ V_{AB} & V_B \end{bmatrix} \quad (21)$$

Each element of the observation noise covariance matrix V being approximately given by replacing space aver-

age with time average, we get

$$\begin{aligned} V_A &= (1/T) \left[\int_0^T \sum_{j \notin \Omega_A} \alpha_{Aj}^2 \sin^2(\omega_{Aj}t + \phi_{Aj}) dt \right] + R \\ &= (1/2) \sum_{j \notin \Omega_A} (\alpha_{Aj})^2 + R \end{aligned} \quad (22)$$

$$\begin{aligned} V_B &= (1/T) \left[\int_0^T \sum_{j \notin \Omega_B} \alpha_{Bj}^2 \sin^2(\omega_{Bj}t + \phi_{Bj}) dt \right] + R \\ &= (1/2) \sum_{j \notin \Omega_B} (\alpha_{Bj})^2 + R \end{aligned} \quad (23)$$

$$\begin{aligned} V_{AB} &= (1/T) \left[\int_0^T \sum_{i \notin \Omega_A} \sum_{j \notin \Omega_B} \{ \alpha_{Aj} \sin(\omega_{Aj}t + \phi_{Aj}) \right. \\ &\quad \left. \times \alpha_{Bj} \sin(\omega_{Bj}t + \phi_{Bj}) \} dt \right] + R \\ &= (1/2) \sum_{\substack{i \notin \Omega_A, j \notin \Omega_B \\ \omega_{Ai} \approx \omega_{Bj}}} (\alpha_{Ai} \alpha_{Bj}) + R \end{aligned} \quad (24)$$

where $T = M\Delta T$. $\{\alpha_{Aj}\}$ and $\{\alpha_{Bj}\}$ in the equations (22) ~ (24) are respectively obtained by calculating the inner products of the observation signal and the sinusoidal functions with the angular frequencies $\{\omega_{Aj}\}$ and $\{\omega_{Bj}\}$. The values of the angular frequencies $\{\omega_{Aj}\}$ and $\{\omega_{Bj}\}$ are calculated using the measurement of the pipe length and open end compensation by the preceding our low order modes model.

In order to analyse the sensitivity of L and δ , we now define the vector $\beta = (L, \delta)^T$ and evaluate Fisher's information matrix [7] defined as

$$K(\beta) = \int \left[\frac{\partial}{\partial \beta} \ell n p(\mathbf{y}) \right] \left[\frac{\partial}{\partial \beta} \ell n p(\mathbf{y}) \right]^T p(\mathbf{y}) d\mathbf{y} \quad (25)$$

Substituting (18) into (25) gives

$$\begin{aligned} K(\beta) &= \int \sum_{k=1}^M \left[\left\{ \frac{\partial}{\partial \beta} (\mathbf{y}_k - \zeta_k) \right\}^T V^{-1} (\mathbf{y}_k - \zeta_k) \right] \\ &\quad \times \sum_{k=1}^M \left[(\mathbf{y}_k - \zeta_k)^T V^{-1} \frac{\partial}{\partial \beta} (\mathbf{y}_k - \zeta_k) \right] p(\mathbf{y}) d\mathbf{y} \\ &= \sum_{k=1}^M \left[\left\{ \frac{\partial \zeta_k}{\partial \beta} \right\}^T V^{-1} \frac{\partial \zeta_k}{\partial \beta} \right] \\ &= \begin{bmatrix} K_{11}(\beta) & K_{12}(\beta) \\ K_{21}(\beta) & K_{22}(\beta) \end{bmatrix} \end{aligned} \quad (26)$$

Obviously, $K_{12}(\theta) = K_{21}(\theta)$. Considering that the sampling period ΔT is sufficiently small, $K_{ij}(\theta)$ ($1 \leq i, j \leq 2$) are approximated by replacing the sum with respect to k with the integration on $[0, T]$ and become as follows :

$$\begin{aligned} K_{11}(\beta) &\simeq \sum_{i \in \Omega_A} \sum_{j \in \Omega_A} [i j \alpha_{Ai} \alpha_{Aj} \gamma(\omega_{Ai}, \omega_{Aj}) \\ &\quad \times \left\{ \frac{\partial \omega_{A1}}{\partial L} \right\}^2 \frac{V_B}{V_A V_B - V_{AB}^2}] \end{aligned}$$

$$\begin{aligned} &+ \sum_{i \in \Omega_A} \sum_{j \in \Omega_B} [i j \alpha_{Ai} \alpha_{Bj} \gamma(\omega_{Ai}, \omega_{Bj}) \\ &\quad \times \frac{\partial \omega_{A1}}{\partial L} \frac{\partial \omega_{B1}}{\partial L} \frac{V_{AB}}{V_A V_B - V_{AB}^2}] \\ &+ \sum_{i \in \Omega_B} \sum_{j \in \Omega_B} [i j \alpha_{Bi} \alpha_{Bj} \gamma(\omega_{Bi}, \omega_{Bj}) \\ &\quad \times \left\{ \frac{\partial \omega_{B1}}{\partial L} \right\}^2 \frac{V_A}{V_A V_B - V_{AB}^2}] \end{aligned} \quad (27)$$

$$\begin{aligned} K_{12}(\beta) &\simeq \sum_{i \in \Omega_A} \sum_{j \in \Omega_A} [i j \alpha_{Ai} \alpha_{Aj} \gamma(\omega_{Ai}, \omega_{Aj}) \\ &\quad \times \frac{\partial \omega_{A1}}{\partial L} \frac{\partial \omega_{A1}}{\partial \delta} \frac{V_B}{V_A V_B - V_{AB}^2}] \\ &+ \sum_{i \in \Omega_A} \sum_{j \in \Omega_B} [i j \alpha_{Ai} \alpha_{Bj} \gamma(\omega_{Ai}, \omega_{Bj}) \\ &\quad \times \frac{\partial \omega_{A1}}{\partial L} \frac{\partial \omega_{B1}}{\partial \delta} \frac{V_{AB}}{V_A V_B - V_{AB}^2}] \\ &+ \sum_{i \in \Omega_A} \sum_{j \in \Omega_B} [i j \alpha_{Ai} \alpha_{Bj} \gamma(\omega_{Ai}, \omega_{Bj}) \\ &\quad \times \frac{\partial \omega_{A1}}{\partial \delta} \frac{\partial \omega_{B1}}{\partial L} \frac{V_{AB}}{V_A V_B - V_{AB}^2}] \\ &+ \sum_{i \in \Omega_B} \sum_{j \in \Omega_B} [i j \alpha_{Bi} \alpha_{Bj} \gamma(\omega_{Bi}, \omega_{Bj}) \\ &\quad \times \frac{\partial \omega_{B1}}{\partial L} \frac{\partial \omega_{B1}}{\partial \delta} \frac{V_A}{V_A V_B - V_{AB}^2}] \end{aligned} \quad (28)$$

$$\begin{aligned} K_{22}(\beta) &\simeq \sum_{i \in \Omega_A} \sum_{j \in \Omega_A} [i j \alpha_{Ai} \alpha_{Aj} \gamma(\omega_{Ai}, \omega_{Aj}) \\ &\quad \times \left\{ \frac{\partial \omega_{A1}}{\partial \delta} \right\}^2 \frac{V_B}{V_A V_B - V_{AB}^2}] \\ &+ \sum_{i \in \Omega_A} \sum_{j \in \Omega_B} [i j \alpha_{Ai} \alpha_{Bj} \gamma(\omega_{Ai}, \omega_{Bj}) \\ &\quad \times \frac{\partial \omega_{A1}}{\partial \delta} \frac{\partial \omega_{B1}}{\partial \delta} \frac{V_{AB}}{V_A V_B - V_{AB}^2}] \\ &+ \sum_{i \in \Omega_B} \sum_{j \in \Omega_B} [i j \alpha_{Bi} \alpha_{Bj} \gamma(\omega_{Bi}, \omega_{Bj}) \\ &\quad \times \left\{ \frac{\partial \omega_{B1}}{\partial \delta} \right\}^2 \frac{V_A}{V_A V_B - V_{AB}^2}] \end{aligned} \quad (29)$$

where

$$\left. \begin{aligned} \frac{\partial \omega_{A1}}{\partial L} &= -\pi v / (L + 2\delta)^2 \\ \frac{\partial \omega_{A1}}{\partial \delta} &= -2\pi v / (L + 2\delta)^2 \\ \frac{\partial \omega_{B1}}{\partial L} &= -\pi v / \{2(L + \delta)^2\} \\ \frac{\partial \omega_{B1}}{\partial \delta} &= -\pi v / \{2(L + \delta)^2\} \end{aligned} \right\} \quad (30)$$

Moreover, $\gamma(\omega_1, \omega_2)$ is a function of the two angular frequencies ω_1 and ω_2 as follows :

$$\begin{aligned} \gamma(\omega_1, \omega_2) &= \int_0^T t^2 \cos(\omega_1 t + \phi_1) \cos(\omega_2 t + \phi_2) dt \\ &= \frac{T^2}{\omega_1 + \omega_2} \sin\{(\omega_1 + \omega_2)T + (\phi_1 + \phi_2)\} \\ &\quad + \frac{2T}{(\omega_1 + \omega_2)^2} \cos\{(\omega_1 + \omega_2)T + (\phi_1 + \phi_2)\} \end{aligned}$$

$$\begin{aligned}
& -\frac{2}{(\omega_1 + \omega_2)^3} \sin\{(\omega_1 + \omega_2)T + (\phi_1 + \phi_2)\} \\
& + \frac{2}{(\omega_1 + \omega_2)^3} \sin\{(\phi_1 + \phi_2)\} \\
& - \left[\frac{T^2}{\omega_1 - \omega_2} \sin\{(\omega_1 - \omega_2)T + (\phi_1 - \phi_2)\} \right. \\
& + \frac{2T}{(\omega_1 - \omega_2)^2} \cos\{(\omega_1 - \omega_2)T + (\phi_1 - \phi_2)\} \\
& - \frac{2}{(\omega_1 - \omega_2)^3} \sin\{(\omega_1 - \omega_2)T + (\phi_1 - \phi_2)\} \\
& \left. + \frac{2}{(\omega_1 - \omega_2)^3} \sin\{(\phi_1 - \phi_2)\} \right] \quad (31)
\end{aligned}$$

As a criterion function, we adopt here the determinant of the Fisher's matrix given as

$$\begin{aligned}
\det[K(\beta)] &= \frac{\pi v}{(L + 2\delta)^2} \frac{\pi v}{2(L + \delta)^2} \\
&\times \left[\sum_{i \in \Omega_A} \sum_{j \in \Omega_A} \frac{i \times j \times \alpha_{Ai} \times \alpha_{Aj} \times \gamma(\omega_{Ai}, \omega_{Aj}) V_B}{V_A V_B - V_{AB}^2} \right. \\
&\times \sum_{i \in \Omega_B} \sum_{j \in \Omega_B} \frac{i \times j \times \alpha_{Bi} \times \alpha_{Bj} \times \gamma(\omega_{Bi}, \omega_{Bj}) V_A}{V_A V_B - V_{AB}^2} \\
&\left. - \sum_{i \in \Omega_A} \sum_{j \in \Omega_B} \frac{i \times j \times \alpha_{Ai} \times \alpha_{Bj} \times \gamma(\omega_{Ai}, \omega_{Bj}) V_{AB}}{V_A V_B - V_{AB}^2} \right] \quad (32)
\end{aligned}$$

We see that the modes whose characteristic frequencies are different as much as possible and whose magnitude and mode number are both large can be the candidates of the optimal modes.

3.2 On-line measurement of the pipe length and open end compensation

In selecting the optimal modes, the values of pipe length L , open end compensation δ , observation noise covariances V_A , V_B , V_{AB} and $\{\alpha_{Aj}\}$, $\{\alpha_{Bj}\}$ are needed to calculate the criterion function. For this aim, we can use the measurements $\hat{L}$ and $\hat{\delta}$ obtained by the low order modes model. Using the measurements, angular frequencies of the stationary waves $\{\hat{\omega}_{Aj}\}$ and $\{\hat{\omega}_{Bj}\}$, the amplitudes of all modes of the stationary waves $\{\hat{\alpha}_{Aj}\}$ and $\{\hat{\alpha}_{Bj}\}$, and covariances $\hat{V}_A$, $\hat{V}_B$ and $\hat{V}_{AB}$ are easily obtained.

This means we can easily obtain the optimal modes using the measurements from the low order modes model. Using the optimal modes, much more accurate pipe length measurement can be realized with the same method as that mentioned before.

In the measurement with the optimal modes, we must only optimize the likelihood function with respect to L and δ .

Fig.3 Outline of the experiment.

Fig.4 Output signal obtained with an acoustic sensor.

4 Experiments

4.1 Behavior of the open end compensation

In order to see the behavior of the open end compensation coefficient, we first try to apply our previous measurement method for closed straight pipes to the straight pipe with two open ends. Figure 3 shows the outline of the experiment. The length and the diameter of the pipe are respectively 4,000mm and 90mm. An acoustic sensor is implemented in the pipe at one open end.

The height h of the pipe from the floor was changed from 0mm to 420mm by 26mm. The fluctuation of the sound pressure was generated by hitting the pipe with a wooden club. The acoustic sensor was a capacitor-type microphone and sampling period was taken as $\Delta T = 0.4\text{ms}$. The example of the sensor output is shown in Fig.4. The temperature inside the pipe was 31.0°C . The experiment was repeated twenty times. In the measurement, the data window adopted was 0.4s from the time of the hitting.

In the experiment, our previous method for closed straight pipes was applied and open end compensation $\hat{\delta}$ was derived as the difference $(\hat{L} - 4,000)/2$, where $\hat{L}$ is the measured pipe length. In each height, 20 times experiments were executed and the average of the compensation $\hat{\delta}$ was calculated. Fig.5 shows the variation of the average compensation with the height. Horizontal line in the figure shows the compensation for the ideal case where the pipe is considerably distant from the floor [4]. It is seen from the figure that the compensation considerably changes with the height of the pipe.

Therefore, we can see that if the coefficient 0.613 is

Fig.5 Variation of the compensation with the height.

always simply adopted in the measurement, measurement error will become large.

4.2 Results by the proposed method

The same pipe as in section 4.1 layed down directly at the floor and the types of observations are done following the experiments shown in Fig.6.

The same data window as the section 4.1 was adopted and the proposed method was applied. As a result, for an experiment, the measurements of pipe length and open end compensation with the low order modes model were respectively $\hat{L} = 4,004\text{mm}$ and $\hat{\delta} = 39\text{mm}$. The optimal modes selected based on the measurements were $\Omega_A^* = \{5, 13, 14\}$ and $\Omega_B^* = \{3, 19, 27\}$ and the final measurements of L^* and δ^* were obtained as $L^* = 4000\text{mm}$ and $\delta^* = 40\text{mm}$.

Table 1 shows the measurement results by proposed method under 20 times experiments. It is seen that mean of the pipe length measurement almost coincides with the true value of 4,000mm. From the measurement of the open end compensation 39.6mm, compensation coefficient is seen to be culculated as 0.88. This means that the coefficient takes a quite different value from the well-known value 0.613 in the experiment. In this way, the proposed system was found to be very effective even in the measurement of the open end compensation.

Finally, although the details are omitted, the same optimal modes are seen to be obtained using even other criteria than the determinant one, that is, min-max criterion (on eigenvalue) and trace minimum criterion are confirmed to give the same optimal modes. Adding to it, the computation time of this measurement system was only 1.3sec with the notebook computer of

Table 1. Measurement results for the length and the compensation value.

	L (mm)	δ (mm)
true	4000.0	
mean	4000.4	39.6
SD	1.3	0.4

SD : standard deviation

dell inspiron 7500.

5 Conclusion

A measurement system of both pipe length and open end compensation for straight and curved pipes was proposed using the stationary waves inside the pipe obtained in different two measurement conditions. Moreover, a criterion to select optimal modes of the stationary wave was presented to realize a much higher measurement accuracy using Fisher's information matrix.

Experiments showed that the proposed method gave a high accurate pipe length measurement without any trouble on the determination of the open end compensation.

References

- [1] T.Manabe,S.Tomita and T.Yabuta : Acoustic Inspection System for Underground Telecommunication Conduit, Proc. of IMTC'94
- [2] S.Tanaka and M.Okamoto : Dynamic Model Based Pipe Length Measurement Using Stationary Wave, Measurement, Vol.28, 249/259 (2000)
- [3] M.Nishimoto and S.Tanaka : Length Measurement for Pipes with Curves and Branches Using Stationary Wave, The Transactions of The Electrical Engineers of Japan, Vol.121-E, No.7, 402/411 (2001)
- [4] T.Igarashi : Acoustics and Vibrations, Kyoritsu Syuppan (1968)
- [5] S.Arimoto : Kalman Filter, Sangyo-Tosho (1977)
- [6] M.J.D.Powell : An Efficient Method for Finding the Minimum of a Function of Several Variables without Calculating Derivatives,The Computer Journal, Vol. 7, No. 2, 1964
- [7] S.Kullback : Information Theory and Statistics, John Wiley and Sons (1959)

Fig.6 Experiments by the proposed method.