Unit Root of Cumulative Call Rate and Cointegration Analysis of Economic System

Yoji MORITA and Shigeyoshi MIYAGAWA

Faculty of Economics, Kyoto Gakuen University

Sogabe-cho, Kameoka, Kyoto 621-8555 JAPAN

morita-y@kyotogakuen.ac.jp

Abstract

In this paper, a unit root of cumulative call rate is investigated with Monte Carlo experiments. Call rate is shown to be $I(0)$ with a linear trend in [1970,1998]. Vector Error Correction model consists of only $I(1)$ variables in a frame work of Johansen's cointegration analysis, and stationary variables like a call rate are often added to the RHS of the resultant VEC model without considering cointegration. However, if cumulative call rate is $I(1)$, this variable should be contained in cointegration analysis. Since call rate has a linear trend, a unit root of cumulative call rate should be checked around a quadratic trend. MacKinnon's critical values for unit root are extended to a quadratic trend case with Monte Carlo simulation experiments. It is shown that cumulative call rate has a unit root with a quadratic trend. Cointegration properties are investigated among cumulative call rate, GDP, money supply and bank loan and VEC model is constructed.

1 Introduction and Problem Statements

There are a lot of works on unit root and cointegration analysis from empirical and statistical point of view. If a nonstationary variable is contained in the endogeneous and/or exogeneous variables of the system, there is a possibility of spurious regression reported by Granger and Newbold[1]. The importance of stationarity property is emphasized and cointegration analysis is introduced when transforming $I(1)$ variable into $I(0)$ variable by a difference operation.

Let us consider the following example. Let income and money be denoted by $y(t)$ and $m(t)$ respectively. Assume that each variable is $I(1)$ and that $y(t)$ and $m(t)$ are modelled by linear equations:

$$y(t) = \mu_y + a_1 y(t-1) + a_2 y(t-2) + b_1 m(t-1) + b_2 m(t-2) + \varepsilon_y(t). \quad (1)$$

$$m(t) = \mu_m + c_1 y(t-1) + c_2 y(t-2) + d_1 m(t-1) + d_2 m(t-2) + \varepsilon_m(t). \quad (2)$$

Defining $x(t) = (y(t), m(t))'$, Eqs.(1) and (2) are rewritten by a vector form:

$$x(t) = \mu + \Pi_1 x(t-1) + \Pi_2 x(t-2) + \varepsilon(t). \quad (3)$$

Using the difference operator $\Delta x(t) = x(t) - x(t-1)$, Eq.(3) can be transformed :

$$\Delta x(t) = \Gamma \Delta x(t-1) + \Pi x(t-2) + \mu + \varepsilon(t), \quad (4)$$

where

$$\Gamma = -(I - \Pi_1), \quad (5)$$

and where

$$\Pi = -(I - \Pi_1 - \Pi_2). \quad (6)$$

Since $x(t)$ is nonstationary and $\Pi x(t-2)$ is stationary, the matrix Π can be classified by two cases:

(case1) $\text{rank}\Pi = 0$, i.e., Π is the null matrix.

(case2) $\text{rank}\Pi = 1$, which implies that there are 2×1 matrices α and β such that $\Pi = \alpha\beta'$ and that $\beta'x(t-2)$ is stationary even though $x(t)$ itself is non-stationary. β is called the cointegration vectors. Our interest is concerned with (case2) and Eq.(4) in this case is called VECM (vector error correction model). Johansen[3] gives us the method of rank estimation along with hypothesis test. Hence, if there is no cointegration, an ordinary AR model for $(\Delta y(t), \Delta m(t))$ is available, while, in the existence of cointegration, error correction terms should be added to the above AR model. Now, let us consider another situation such that a nonstationary process $y(t)$ is combined with a stationary process $r(t)$. Johansen's method is not directly applicable to this case since all the variables should be nonstationary in his frame work. Therefore many papers deal this case in such a manner that a set of stationary variables $(\Delta y(t), r(t))$ is used to construct AR model. However, letting $R(t) \equiv r(1) + r(2) + \dots + r(t)$, that is, $\Delta R(t) \equiv r(t)$, and assuming that $R(t)$ is nonstationary, AR model applied to $(\Delta y(t), \Delta R(t))$ is not correct if we do not investigate a cointegration property between $y(t)$ and $R(t)$.

2 The Empirical Analysis

[**Data Description**] The data employed here is Japanese data, quarterly observed from 1970:1 through 1998:4 and original(i.e.,seasonally unadjusted). They are obtained from Nikkei Needs Database. Denoting the GDP deflator by $p(t)$, main variables used here are real GDP,real money,real loan,call rate and cumulative call rate to be listed as:

$$y(t) = \ln(\text{real GDP}(t)) \quad (7)$$

$$m(t) = \ln(M_2 + CD/p(t)) \quad (8)$$

$$l(t) = \ln(\text{loan}/p(t)) \quad (9)$$

$$r(t) = \text{call rate}(t) \quad (10)$$

$$R(t) = \sum_{i=1}^t r(i). \quad (11)$$

[**Unit Root Test**] Dicky-Fuller test is applied for check of stationarity. This test requires to calculate t-ratio of the parameter λ_0 in the following equation:

$$x(t) - x(t-1) = a + bt + \lambda_0 x(t-1) + \sum_{i=1}^{k_0} \lambda_i \Delta x(t-i) + \Psi D(t), \quad (12)$$

where $x(t)$ is a scalar variable to be tested, the delay parameter k_0 is decided by *AIC* criterion and where $D(t)$ is a seasonal dummy variable with elements $D_2(t)$, $D_3(t)$ and $D_4(t)$. If the null hypothesis $\lambda_0 = 0$ is rejected, then the $x(t)$ process does not have a unit root and becomes stationary. Table 1 shows results of the test *with trend* ($b \neq 0$) and *without trend* ($b = 0$).

Table 1 Unit Root Tests

var.	lags	with trend	lags	without trend
y	5	-1.05187	5	-2.18168
m	2	-2.90397	2	-2.01160
l	1	-0.91695	1	-1.82782
r	2	-4.06200 ***	2	-2.63382*

where ***, ** and * denote respectively rejection of the hypothesis at 1 %, 5 % and 10 % significance level. Critical values tabulated by MacKinnon's table[2] are -4.04, -3.45 and -3.15 for 1 %, 5 % and 10 % in *with trend*, while the values are -3.49, -2.89 and -2.58 for 1 %, 5 % and 10 % significance levels respectively *without trend*. The lag order is decided by AIC. We conclude that $y(t)$, $m(t)$ and $l(t)$ are nonstationary and $r(t)$ is stationary *with trend*. Cumulative call rate $R(t)$ is somewhat difficult to judge the unit root. Since $\Delta r(t)$ has a drift, that is, $E\{\Delta r(t)\} = -0.0713$, the unit root test of $r(t)$ should be done around a linear time trend. The result of ADF test implies that the null hypothesis of unit root is rejected with 1 % significance level.

Figure 1: behavior of $\Delta r(t)$

Figure 2: histogram and statistics of $\Delta r(t)$

Figure 3: behavior of call rate $r(t)$

Figure 4: behavior of cumulative call rate $R(t)$

In the next step, the unit root of cumulative call rate $R(t)$ should be examined. Since $\Delta R(t) = r(t)$ has a linear trend, $R(t)$ itself may fluctuate around a quadratic trend.

Letting L be a lagged operator, we consider the $z(t)$ -process with a quadratic trend.

$$(1 - \phi_1 L)(1 - \rho L)\{z(t) - (\mu + \beta_1 t + \beta_2 t^2)\} = u(t), \quad (13)$$

where $u(t)$ is a series of *i.i.d.* $N(0, 1)$ variables. The L.H.S. of the above equation becomes

$$\begin{aligned} L.H.S. &= z(t) - (\phi_1 + \rho)z(t-1) + \phi_1 \rho z(t-2) \\ &\quad - \mu' - \beta'_1 t - \beta'_2 t^2 \\ &= \Delta z(t) + (1 - \phi_1)(1 - \rho)z(t-1) \\ &\quad - \phi_1 \rho \Delta z(t-1) - \mu' - \beta'_1 t - \beta'_2 t^2, \end{aligned} \quad (14)$$

where

$$\begin{aligned} \mu' &= (1 - \phi_1)(1 - \rho)\mu + \{(\phi_1 + \rho) - 2\phi_1 \rho\}\beta_1 \\ &\quad + \{(\phi_1 + \rho) - 4\phi_1 \rho\}\beta_2, \end{aligned} \quad (15)$$

$$\beta'_1 = (1 - \phi_1)(1 - \rho)\beta_1 + 2\{(\phi_1 + \rho) - 2\phi_1 \rho\}\beta_2, \quad (16)$$

$$\beta'_2 = (1 - \phi_1)(1 - \rho)\beta_2. \quad (17)$$

Finally, Eq.(13) is rewritten by

$$\Delta z(t) = \mu' + \beta'_1 t + \beta'_2 t^2 + \alpha_0 z(t-1) + \alpha_1 \Delta z(t-1) + u(t), \quad (18)$$

where

$$\alpha_0 = -(1 - \phi_1)(1 - \rho), \text{ and } \alpha_1 = \phi_1 \rho. \quad (19)$$

Inserting the relation $\rho = 1$ into Eq.(18), the null hypothesis of unit root is stated with $\beta'_2 = 0$ and $\alpha_0 = 0$:

$$\Delta z(t) = \mu' + \beta'_1 t + \phi_1 \Delta z(t-1) + u(t), \quad (20)$$

where

$$\mu' = (1 - \phi_1)\beta_1 + (1 - 3\phi_1)\beta_2, \quad (21)$$

$$\beta'_1 = 2(1 - \phi_1)\beta_2. \quad (22)$$

The alternative hypothesis is given by Eq.(18) with $\beta'_2 \neq 0$ and $\alpha_0 \neq 0$. The unit root should be judged by comparing t-statistics of the estimated α_0 with critical values. These critical values are given by Monte Carlo experiments.

[**Monte Carlo Experiments**] Calculation of critical values for unit root is carried out.

(1)step-1: Eq.(20) in the null hypothesis is simulated with the sample size $T = 113$.

(2)step-2: Based on the data given in step-1, the parameter α_0 in Eq.(18) and its t-statistics are calculated.

(3)step-3: Each simulation experiment in step-1 and

step-2 consists of 25,000 replications for a single value of T which is determined by the sample size of data period 1970:1 to 1998:4.

(4)step-4: The 25,000 values of t-statistics of estimated α_0 are shown in the cumulative distribution function(denoted by cdf).

Figures 5 and 6 depict the cdf and its expanded map respectively. The cumulative call rate $R(t)$ in the period 1970:1 to 1998:4 is identified in Eq.(18) and t-statistics of α_0 becomes to be -3.38. This value gives us the p-value 0.14, which can not reject the null hypothesis of unit root with 10 % significance level.

Figure 5: Empirical CDF of t-statistics of estimated α_0

Figure 6: Expanded map of Empirical CDF

[**Cointegration Property(Johansen Test[3])**] Johansen gives us the method of rank estimation along with hypothesis test. Table 2 is results of Johansen test.

Table 2 Cointegration Tests (Johansen) on (y, m, l, R)

<i>Eigen values</i>	<i>L.Ratio</i>	<i>critical (5 %)</i>	<i>critical (1 %)</i>	<i>hypothesized No. of CE</i>
0.34	74.76	54.64	61.24	$r = 0 **$
0.16	28.12	34.55	40.49	$r \leq 1$
0.058	8.85	18.17	23.46	$r \leq 2$
0.019	2.17	3.74	6.4	$r \leq 3$

where $**$ ($*$) denotes rejection of hypothesis at 1 % (5 %) significance level and where lagged differences are $n = 4$ by AIC.

From Johansen test, $x(t) = (y(t), m(t), l(t), R(t))$ has a cointegrating vector, that is, $r = 1$. The cointegrating vector is calculated as:

$$\begin{aligned}
 y(t) &= c_1 + c_2 t + c_3 m(t) + c_4 l(t) + c_5 R(t) \\
 &= -3.0266 - 0.01697t \\
 &\quad + 2.4596m(t) - 1.0539l(t) + 0.00156R(t), \\
 &\quad (1.31074) \quad (0.65907) \quad (0.00076) \quad (23)
 \end{aligned}$$

where t means time trend, the number in parentheses are the asymptotic standard errors and where Eq.(23) contains seasonal dummy variables on the RHS of their equations.

The error correcting term $z_1(t)$ can be defined as the residual of the above relation and is described by

$$z_1(t) = y(t) - c_1 - c_2 t - c_3 m(t) - c_4 l(t) - c_5 R(t), \quad (24)$$

In order to support this result, Johansen tests are carried out in case of several combinations of variables to be shown in Table 3. See Appendix.

[**Basic System Model**] Our aim is to research how monetary policy (*call rate* $r(t)$) transmits to output $\Delta y(t)$ through channels of $\Delta m(t)$, $\Delta l(t)$ and $z_1(t)$. Since cointegration in Johansen test assumes a set of variables with $I(1)$ and call rate is, on the contrary, stationary in its level (i.e., $I(0)$), an introduction of $I(1)$ variable $R(t)$ plays an important role to the analysis of monetary transmission mechanism. The VEC model is described by

$$\begin{aligned}
 \Delta y(t) &= \sum_{i=1}^k a_{1i} \Delta y(t-i) + \sum_{i=1}^k a_{2i} \Delta m(t-i) \\
 &\quad + \sum_{i=1}^k a_{3i} \Delta l(t-i) + \sum_{i=1}^k a_{4i} \Delta R(t-i) \\
 &\quad + a_5 z_1(t-1) + \mu_y + a_6 t \\
 &\quad + \sum_{j=2}^4 s_{yj} D_j(t) + \sigma_y \varepsilon_y(t) \quad (25)
 \end{aligned}$$

$$\begin{aligned}
 \Delta m(t) &= \sum_{i=1}^k b_{1i} \Delta y(t-i) + \sum_{i=1}^k b_{2i} \Delta m(t-i) \\
 &\quad + \sum_{i=1}^k b_{3i} \Delta l(t-i) + \sum_{i=1}^k b_{4i} \Delta R(t-i) \\
 &\quad + b_5 z_1(t-1) + \mu_m + b_6 t \\
 &\quad + \sum_{j=2}^4 s_{mj} D_j(t) + \sigma_m \varepsilon_m(t) \quad (26)
 \end{aligned}$$

$$\begin{aligned}
 \Delta l(t) &= \sum_{i=1}^k c_{1i} \Delta y(t-i) + \sum_{i=1}^k c_{2i} \Delta m(t-i) \\
 &\quad + \sum_{i=1}^k c_{3i} \Delta l(t-i) + \sum_{i=1}^k c_{4i} \Delta R(t-i) \\
 &\quad + c_5 z_1(t-1) + \mu_l + c_6 t \\
 &\quad + \sum_{j=2}^4 s_{lj} D_j(t) + \sigma_l \varepsilon_l(t) \quad (27)
 \end{aligned}$$

$$\begin{aligned}
 \Delta R(t) &= \sum_{i=1}^k d_{1i} \Delta y(t-i) + \sum_{i=1}^k d_{2i} \Delta m(t-i) \\
 &\quad + \sum_{i=1}^k d_{3i} \Delta l(t-i) + \sum_{i=1}^k d_{4i} \Delta R(t-i) \\
 &\quad + d_5 z_1(t-1) + \mu_{call} + d_6 t \\
 &\quad + \sum_{j=2}^4 s_{cj} D_j(t) + \sigma_{call} \varepsilon_{call}(t) \quad (28)
 \end{aligned}$$

where $\varepsilon_y(t)$, $\dots$, $\varepsilon_{call}(t)$ are sequences of i.i.d. random variables with standard deviations σ_y , $\dots$, σ_{call} respectively and where t denotes a time trend. From the relation $\Delta R(t) \equiv r(t)$, it should be noted that the model constructed above represents an economic system driven by call rate. The delay order k was decided by AIC to be $k = 4$. Basic behaviors of $\Delta y(t)$, $\Delta m(t)$, $\Delta l(t)$ and $r(t)$ are depicted in Fig.7 in a form of impulse responses subjected to the shock of $r(t)$.

The monetary transmission problem can be discussed by the basic model derived above, see Morita et al.[4].

3 Conclusions

A new variable $R(t)$ which is cumulative call rate was introduced and an economic system model driven by

Figure 7: Impulse Responses in Basic System Model

call rate was reasonably constructed associated with cointegration analysis. A combination of nonstationary variables and a stationary variable (call rate) was theoretically analyzed within Johansen's cointegration framework. The VEC model was calculated and the resultant impulse responses were shown.

References

- [1] Granger, C.W.J. and Newbold, P. Forecasting Economic Time Series, (2nd ed.), *Academic Press*, 1986.
- [2] Mackinnon, J.G. Critical Values for Cointegration Tests, in *R.F. Engle and C.W.J. Granger ed., Long-Run Economic Relationships: Oxford University Press*, 1991, pp.267-276.
- [3] Johansen, S. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, Vol.59, 1991, pp.1551-1580.
- [4] Morita, Y., Miyagawa, S. and Nakazaki, H. On the Role of Call Rate in Cointegrating Analysis of Monetary Transmission Mechanism, *The Kyoto-gakuen University Review*, March, 2001, pp.41-54.

Appendix Table 3 Summary of Cointegration Tests (Johansen)

<i>variables</i>	$n = 2$	$n = 3$
(y, m, l, R)	$r = 0^{**} \quad r \leq 1 \quad r \leq 2 \quad r \leq 3^*$	$r = 0^{**} \quad r \leq 1 \quad r \leq 2 \quad r \leq 3$
(y, m, R)	$r = 0^* \quad r \leq 1 \quad r \leq 2$	$r = 0^{**} \quad r \leq 1 \quad r \leq 2$
(y, l, R)	$r = 0^{**} \quad r \leq 1^* \quad r \leq 2^*$	$r = 0^{**} \quad r \leq 1 \quad r \leq 2^*$
(y, R)	$r = 0^{**} \quad r \leq 1^*$	$r = 0^{**} \quad r \leq 1^*$
(m, R)	$r = 0 \quad r \leq 1$	$r = 0^* \quad r \leq 1$
(l, R)	$r = 0 \quad r \leq 1^*$	$r = 0^* \quad r \leq 1^*$
(y, m)	$r = 0 \quad r \leq 1^*$	$r = 0^{**} \quad r \leq 1$
(y, l)	$r = 0 \quad r \leq 1$	$r = 0 \quad r \leq 1$
(y, m, l)	$r = 0 \quad r \leq 1 \quad r \leq 2$	$r = 0^* \quad r \leq 1 \quad r \leq 2$
<i>variables</i>	$n = 4$	$n = 5$
(y, m, l, R)	$r = 0^{**} \quad r \leq 1 \quad r \leq 2 \quad r \leq 3$	$r = 0^* \quad r \leq 1 \quad r \leq 2 \quad r \leq 3$
(y, m, R)	$r = 0 \quad r \leq 1 \quad r \leq 2$	$r = 0 \quad r \leq 1 \quad r \leq 2$
(y, l, R)	$r = 0^* \quad r \leq 1 \quad r \leq 2^*$	$r = 0^* \quad r \leq 1 \quad r \leq 2^*$
(y, R)	$r = 0^* \quad r \leq 1^*$	$r = 0^* \quad r \leq 1^*$
(m, R)	$r = 0^* \quad r \leq 1$	$r = 0 \quad r \leq 1$
(l, R)	$r = 0 \quad r \leq 1^*$	$r = 0 \quad r \leq 1^*$
(y, m)	$r = 0 \quad r \leq 1$	$r = 0 \quad r \leq 1$
(y, l)	$r = 0 \quad r \leq 1$	$r = 0 \quad r \leq 1$
(y, m, l)	$r = 0 \quad r \leq 1 \quad r \leq 2$	$r = 0 \quad r \leq 1 \quad r \leq 2$
<i>variables</i>	$n = 6$	$n = 7$
(y, m, l, R)	$r = 0^* \quad r \leq 1 \quad r \leq 2 \quad r \leq 3$	$r = 0^{**} \quad r \leq 1 \quad r \leq 2 \quad r \leq 3$
(y, m, R)	$r = 0 \quad r \leq 1 \quad r \leq 2$	$r = 0 \quad r \leq 1 \quad r \leq 2^*$
(y, l, R)	$r = 0 \quad r \leq 1 \quad r \leq 2^*$	$r = 0^* \quad r \leq 1 \quad r \leq 2^{**}$
(y, R)	$r = 0 \quad r \leq 1^{**}$	$r = 0 \quad r \leq 1^*$
(m, R)	$r = 0 \quad r \leq 1$	$r = 0 \quad r \leq 1^*$
(l, R)	$r = 0 \quad r \leq 1^*$	$r = 0 \quad r \leq 1^*$
(y, m)	$r = 0 \quad r \leq 1$	$r = 0 \quad r \leq 1$
(y, l)	$r = 0 \quad r \leq 1$	$r = 0^* \quad r \leq 1$
(y, m, l)	$r = 0 \quad r \leq 1 \quad r \leq 2$	$r = 0 \quad r \leq 1 \quad r \leq 2$
<i>variables</i>	$n = 8$	$n = 9$
(y, m, l, R)	$r = 0^{**} \quad r \leq 1^* \quad r \leq 2 \quad r \leq 3$	$r = 0^{**} \quad r \leq 1^* \quad r \leq 2 \quad r \leq 3$
(y, m, R)	$r = 0 \quad r \leq 1 \quad r \leq 2^{**}$	$r = 0 \quad r \leq 1^* \quad r \leq 2^*$
(y, l, R)	$r = 0^* \quad r \leq 1 \quad r \leq 2^*$	$r = 0^{**} \quad r \leq 1 \quad r \leq 2^*$
(y, R)	$r = 0 \quad r \leq 1^*$	$r = 0^* \quad r \leq 1^{**}$
(m, R)	$r = 0 \quad r \leq 1^{**}$	$r = 0^* \quad r \leq 1^{**}$
(l, R)	$r = 0 \quad r \leq 1^*$	$r = 0 \quad r \leq 1^*$
(y, m)	$r = 0 \quad r \leq 1$	$r = 0 \quad r \leq 1$
(y, l)	$r = 0^* \quad r \leq 1^*$	$r = 0^* \quad r \leq 1^*$
(y, m, l)	$r = 0 \quad r \leq 1 \quad r \leq 2$	$r = 0 \quad r \leq 1 \quad r \leq 2$