Cycle Slips Detection in GPS Positioning
Based on Statistical Tests of Innovation Processes

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Abstract

In a precise GPS positioning, cycle slips deteriorate the accuracy of positioning. In this paper, in order to achieve accurate positioning, we propose a method to detect cycle slips in the GPS positioning by utilizing $\chi^2$ test and Likelihood Ratio test (LR test). The experimental results of positioning by using real receiver data are shown.

1 Introduction

In Carrier phase Differential Global Positioning System (CDGPS), the (integrated) carrier phase measurement is utilized [1]. The carrier phase measurement has to be provided by the receiver continuously, however the temporary loss of lock on the GPS signals is sometimes caused by the rapid changes in the receiver condition, etc.. The loss of lock is called "cycle slips." To keep the accurate positioning, we need to detect and reject the cycle slips efficiently.

In the present algorithm, the Kalman filter is efficiently utilized to continuously provide a precise positioning with detection of the cycle slips. It is known that under normal conditions, the innovation process from Kalman filter is a zero mean Gaussian white noise process [2].

We observe whether it has been changed from standard Gaussian or not by applying $\chi^2$ test and its mean has been changed from zero or not by applying Likelihood Ratio test to detect cycle slips. When cycle slips are detected, we apply the algorithm such as [8] to correct cycle slips for better accuracy.

2 DGPS and Carrier Phases

There are two types of GPS observations: pseudo-ranges and carrier phases. The pseudo-ranges are generally used for navigation. In high-precision positioning such as DGPS, the carrier phase is used. The observed phase is the difference between the phase of the carrier signal transmitted by the satellite and the one of the receiver oscillator. This difference in carrier phase can be regarded as the Doppler frequency shift that arises due to the relative motion between the satellite and the receiver. Then the Doppler frequency is integrated over the interval of the measurement period, therefore, it is called the integrated carrier phase measurement.

The integrated carrier phase measurement for satellite $p$ and receiver $k$ is given by Eq. (1). In static CDGPS, the measurements data $\varphi^p_k$ are given as follows [3] - [5]:

$$\varphi^p_k(t) = \varphi^p(t_0) + \frac{f}{c} \rho^p_k(t) - \varphi_k(t_0) - \frac{f}{c} f^p_k(t) + \frac{f}{c} T^p_k(t) + N^p_k + \epsilon^p_k(t),$$

where $t$ is the signal reception time measured by the receiver $k$, $t_0$ is the initial time based on the GPS time, $\varphi_k(t_0)$ is the phase of the receiver oscillator at $t_0$, $f$ is the carrier frequency (1575.42 or 1227.60 [MHz]), $c$ is the speed of light ($2.99792458 \times 10^8$[m/s]), $\epsilon^p_k$ is the observation noise, $f^p_k$ and $T^p_k$ are ionospheric and tropospheric propagation delays respectively, $N^p_k$ is the integer ambiguity, which is an unknown integer number associated with the ambiguity of carrier cycles at
the initial time and \( r_k^p \) denotes the distance between the satellite \( p \) and the receiver \( k \), as follows:

\[
r_k^p(t) = \sqrt{(x_k - x_p)^2 + (y_k - y_p)^2 + (z_k - z_p)^2},
\]

where \((x_k, y_k, z_k)\) and \((x_p, y_p, z_p)\) are the coordinates of the antenna at the receiver \( k \) and the satellite \( p \), respectively, in the WGS-84 frame [9, 5]. The WGS-84 (World Geodetic System 1984) is a right-handed orthogonal coordinate system where the origin is located at the earth's center of mass.

### 2.1 Double Differences

Fig. 1 shows a conceptual view of the double difference, where two receivers \( k \) and \( u \) are located at the known point and the unknown point respectively, and \( b \) represents the unknown baseline vector between them.

If two receivers \( k \) and \( u \) observe two satellites \( p \) and \( q \) simultaneously, four equations of the form Eq. (1) are obtained. For the receivers located sufficiently close to each other, ionospheric and tropospheric effects can be assumed to be the same, thus, \( I_k^p(t) = I_u^p(t) \) and \( T_k^p(t) = T_u^p(t) \) [9].

Then the double difference phase observation is

\[
\Delta_{ku}^{pq}(t) = \varphi_k^p(t) - \varphi_u^p(t)
\]

\[
- \{(\varphi_k^q(t) - \varphi_u^q(t))
\]

\[
- \{f \{r_k^p(t) - r_u^p(t)\} + \frac{c}{f} \{r_k^q(t) - r_u^q(t)\}
\]

\[
+ N_{ku}^{pq} + e_{ku}^{pq}(t)
\]

where \( N_{ku}^{pq} \) and \( e_{ku}^{pq} \) are the initial transmitted phase \( \varphi_k^p(t_0) \) and \( \varphi_u^p(t_0) \), and the initial receiver phase \( \varphi_k(t_0) \) and \( \varphi_u(t_0) \) have canceled.

### 2.2 System Model

In this paper, we consider the case that the two receivers \( k \) and \( u \) are static at the known point and the unknown point respectively, and the double difference data for all combinations of four satellites are available at both points. We denote \( p = 1, q = 2, 3, 4 \) then the measurement can be formed:

\[
\Delta(i) = [\Delta_{ku}^{12}(i), \Delta_{ku}^{13}(i), \Delta_{ku}^{14}(i)]^T,
\]

where \( t = i \delta_t \) (\( \delta_t \) is the data update interval of the receiver). In this positioning problem, we need to estimate three coordinates \( x = [x_u, y_u, z_u]^T \) for the unknown position and three values \( N = [N_{ku}^{12}, N_{ku}^{13}, N_{ku}^{14}]^T \) corresponding to the integer ambiguity in Eq. (8). Therefore the state vector \( \eta \) consists of 6 components, as follows:

\[
\eta(i) = [x_T(i) | N_T(i)]^T
\]

\[
= [x_u(i), y_u(i), z_u(i), N_{ku}^{12}(i), N_{ku}^{13}(i), N_{ku}^{14}(i)]^T.
\]
Since all components are constants due to static positioning, the state equation can be written as

\[ \eta(i + 1) = \eta(i) + B(i)w(i), \]

\[ i = 1, 2, \ldots, n, \]  

(10)

where \( w(i) \equiv [w_1(i), w_2(i), \ldots, w_n(i)]^T \) is the process noise, \( B(i) \) is the appropriate weighting matrix for the process noise, and \( n \) is the number of data.

The observation equation can be represented as follows:

\[ \Delta(i) = h(x(i)) + N(i) + D(i)v(i), \]

(11)

where \( v(i) \equiv [v_1(i), v_2(i), v_3(i)]^T \) is the measurement noise, and \( D(i) \) is the appropriate weighting matrix such that \( R(i) \equiv D(i)D^T(i) > 0 \) holds for any \( i \). The \( j \)-th component \( (j = 1, 2, 3) \) of \( h(x(i)) \) for satellites \( p, q \) is given by

\[ h_j(x(i)) = \frac{f}{c} \left\{ \rho_k^p(t) - \rho_k^q(t) \right\} = \frac{f}{c} \left\{ \sqrt{(x_k - x_p)^2 + (y_k - y_p)^2 + (z_k - z_p)^2} - \sqrt{(x_n - x_p)^2 + (y_n - y_p)^2 + (z_n - z_p)^2} \right\} \]

(12)

where

\[ (p, q) = \{(1,2),(1,3),(1,4)\}. \]

\[ y(i) \equiv \Delta(i) - h(\hat{x}(i|i - 1)) \]

\[ + H(i)\hat{x}(i|i - 1) \]

\[ = C(i)\eta(i) + D(i)v(i), \]  

(13)

where \( H(i) \) and \( C(i) \) are defined as follows:

\[ H(i) \equiv \left[ \frac{\partial h(x(i))}{\partial x(i)} \right]^T \]

(14)

\[ C(i) \equiv [H(i)]_I. \]  

(15)

In order to estimate the unknown value \( \eta(i) \), the Kalman filter is applied to the state Eq. (10) and the measurement Eq. (13). If the observation Eq. (11) is linear, by byproduct of the estimation process, the innovation process \( \nu(i) \) and its covariance matrix \( M(i) \) are obtained as follows:

\[ \nu(i) = y(i) - C(i)\eta(i|i - 1) \]  

(16)

\[ M(i) = E\{\nu(i)\nu^T(i)\} \]

\[ = C(i)P(i|i - 1)C^T(i) + R(i). \]  

(17)

And it is well known that under normal conditions, the innovation process \( \nu_j(i) \) is a zero mean Gaussian white noise process [2]. In the following cycle slip detection procedure, therefore, we assume that the process \( \nu_j(i) \) is approximately a zero mean Gaussian white noise process that the covariance matrix is given by Eq. (17).

3 Detecting Cycle Slips

3.1 Definition of Cycle Slips

When a GPS receiver is turned on, the fractional beat phase (i.e., the difference between satellite transmitted carrier and receiver’s generated replica signal) is observed and an integer counter is initialized. During tracking, the counter is incremented by one cycle whenever the fractional phase changes from \( 2\pi \) to \( 0 \). The initial integer number, called integer ambiguity \( n_i \), remains constant as long as no loss of the signal lock occurs. When signal lock loses, the integer counter is re-initialized which causes a jump in the instantaneous accumulated phase by an integer number of cycles. This jump is called cycle slips and is restricted to phase measurements.

Three sources for cycle slips can be distinguished. First, cycle slips are caused by obstructions of the satellite signal due to trees,
buildings, bridges, mountains, etc. This source is the most frequent one. The second source for cycle slips is a low SNR due to bad ionospheric conditions, multipath, high receiver dynamics, or low satellite elevation. The third source is a failure in the receiver software which leads to incorrect signal processing [6]. As seen in Fig. 2, it is difficult to detect and correct cycle slips by watching observation data only due to time varying of observation data.

3.2 Applying Chi-Square test

If a cycle slip occurs at $t = r$, then the covariance matrix of the innovation process changes from $M$, e.g.

$$ E\{\nu_1(i)^2\} = \begin{cases} M_{11}(i) & (1 \leq i \leq r - 1) \\ M_{11}(i) & (r \leq i), \end{cases} $$

(18)

where $\nu_1(i)$ is the first element of $\nu(i)$ in (16), $M_{11}(i)$ and $M_{11}(i)$ are the (1, 1) element of $M(i)$ and $M(i)$ in (17) respectively. Therefore, we formulate two hypotheses such that

$H_{X,0}$: (the change does not occur)

$H_{X,1}$: (the change occurs).

The decision is, therefore, based on $\chi^2$ test. The hypotheses $H_{X,1}$ is accepted when a cycle slip occurs. To implement $\chi^2$ test, we let $L$ as follows:

$$ L = \begin{bmatrix} L_{12}^{13} \\ L_{12}^{13} \\ L_{14}^{14} \\ L_{14}^{14} \end{bmatrix} = \begin{bmatrix} \nu_1 M_{11}^{-1} \nu_1 \\ \nu_2 M_{22}^{-1} \nu_2 \\ \nu_3 M_{33}^{-1} \nu_3 \end{bmatrix}. $$

(19)

$L_{12}^{13}$, $(j = 2, 3, 4)$ is $\chi^2$ distributed with 1 degree of freedom. To detect cycle slips, $\chi^2$ test is implemented for each element of $L_i$ at every recursion of the Kalman filter. If $H_{X,1}$ is accepted, we implement Likelihood Ratio test such as [7] to avoid error of the $\chi^2$ test.

3.3 Applying Likelihood Ratio test

It is known that under normal conditions, the mean of innovation process from Kalman filter is zero [2]. If a cycle slip occurs at time $k = r$, then the mean of the innovation process changes from 0 to unknown value $\mu$, e.g.

$$ E\{\nu(k)\} = \begin{cases} 0 & (1 \leq k \leq r - 1) \\ \mu & (r \leq k \leq i). \end{cases} $$

(20)

Therefore, we formulate two hypotheses such that

$H_{X,0}$: (the change does not occur)

$H_{X,1}$: (the change occurs at time $r$).

The Likelihood Ratio between these two hypotheses is

$$ \prod_{k=r}^{i} \frac{p_1(\nu(k))}{p_0(\nu(k))}. $$

(21)

Its logarithm is thus

$$ \Lambda_i(r, \mu) = \frac{\mu}{\sigma^2} \sum_{k=r}^{i} \left( \nu(k) - \frac{\mu}{2} \right), $$

(22)

where $\sigma^2$ is the variance of $\nu$. Replacing the unknown changed time $r$ by its Maximum Likelihood Estimate (MLE) under $H_{X,1}$, namely

$$ \hat{r}(i) = \arg\max_{1 \leq r \leq i} \Lambda_i(r, \hat{\mu}), $$

(23)

where $\hat{\mu}$ is estimated jump magnitude by its MLE under $H_{X,1}$ as follows:

$$ \hat{\mu}(i) = \frac{1}{i - r + 1} \sum_{k=r}^{i} \nu(k). $$

(24)

Then we get the following change detector,
\[ g(i) = \max_{1 \leq r \leq i} \Lambda_i(r, \hat{\mu}) \]

\[ = \max_{1 \leq r \leq i} \frac{1}{2\sigma^2(i - r + 1)} \left\{ \sum_{k=\tau}^{i} \nu(k) \right\}^2. \]  

(25)

If \( g(i) \) is greater than threshold \( \lambda \), hypothesis \( H_{T,1} \) is accepted with changing time \( \hat{\tau} \) otherwise \( H_{T,0} \) is accepted. This test is implemented when hypothesis \( H_{X,1} \) is accepted, so if \( H_{T,0} \) is accepted, it shows that \( \chi^2 \) test caused type I error, i.e. the test should not have accepted \( H_{X,1} \). On the other hand, if \( H_{T,1} \) is accepted and estimated changing time \( \hat{\tau} \) is earlier than detected time \( i \), it shows that although cycle slips have been detected, \( \chi^2 \) test caused type II error, i.e. the test should have accepted \( H_{X,1} \) at time \( \hat{\tau} \). If \( H_{T,1} \) is accepted with estimated changing time \( \hat{\tau} = i \), cycle slips have been detected correctly.

3.4 Correcting Cycle Slips

When a cycle slip is detected, the related ambiguity value of \( n \) is re-initialized by the simulation algorithm such as [8] to correct cycle slips. The new ambiguity is set as follows:

\[ N_j(i) = \Delta_j(i) - c_j(\hat{\chi}(i-1)). \]  

(26)

Although this re-initializing will not be implemented when \( \chi^2 \) test causes type I error, it will be done at detected time \( i \) when type II error is caused.

4 Experimental Results

The algorithms presented in this paper were examined by using real receiver data. We obtain GPS observation data that were collected by the Geographical Survey Institute (GSI) in Japan. For the experiment, two receivers (Ashtech Z-Xll3) located at (i) Koumyou elementary school (Takarazuka City, Hyogo, Japan), and (ii) Yamate elementary school (Ashiya City, Hyogo, Japan) were selected, and their coordinates surveyed with some surveying techniques have been announced by GSI. From the announced coordinates, the baseline length is 7590.2035 [m]. And we assume that the coordinates of the receiver position (i) is known and (ii) is unknown. A total of 250 epochs data were collected in a 30 seconds interval on August 17, 1995, from 3:00 to 5:05 (UTC). In the experiment, we use three levels of significance such as \( \alpha = 0.025, 0.05, \) and 0.1. We also compare the results between the estimation by using \( \chi^2 \) and LR tests and the one such as [8], (by using \( \chi^2 \) test only). Table 1 shows the number of detected cycle slips and errors which \( \chi^2 \) tests caused. From Table 1, the lower level of significance \( \alpha \) causes the more cycle slips corrections with Type I errors, and the higher \( \alpha \) causes the fewer cycle slips corrections and the more type II errors.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.025</th>
<th>0.050</th>
<th>0.100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detected by ( \chi^2 )</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Corrected</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Type I errors</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Type II errors</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Results of the \( \chi^2 \) and LR tests

Fig. 3: Estimated baseline length

Table 2: 3-dimensional and baseline length estimated errors

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.025</th>
<th>0.050</th>
<th>0.100</th>
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<tbody>
<tr>
<td>3-dimensional [m]</td>
<td>0.120</td>
<td>0.105</td>
<td>0.104</td>
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<tr>
<td>baseline length [m]</td>
<td>0.066</td>
<td>0.048</td>
<td>0.036</td>
</tr>
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</table>

Fig. 3 shows the result of estimating baseline vector from (a) to (b). Table 2 shows the error from nominal value. 3-dimensional and baseline length errors are shown. From Fig. 3 and Table 2, the result with higher \( \alpha \)
Table 3: Comparison of the errors between three algorithms

<table>
<thead>
<tr>
<th></th>
<th>χ² and L.R</th>
<th>χ²</th>
<th>no tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-diagonal [m]</td>
<td>0.104</td>
<td>0.118</td>
<td>0.115</td>
</tr>
<tr>
<td>baseline length [m]</td>
<td>0.036</td>
<td>0.049</td>
<td>0.049</td>
</tr>
</tbody>
</table>

shows the lower error due to more corrections of cycle slips. Fig. 4 and Table 3 shows the comparison between (a) the algorithm in this paper (with χ² and LR tests), (b) the one such as [8] (with χ² test only) and (c) the one with no tests. This result shows that the result of (a) have better positioning accuracy than the one of (b) and (c) due to correcting cycle slips with fewer testing errors.

5 Conclusion

In this paper, the algorithm with cycle slips detection and correction by using χ² and LR tests has been proposed. The experimental results show that the algorithm in this paper can detect and correct cycle slips efficiently, and it provides better positioning accuracy. In the future study, the algorithm will be examined in kinematic environment.

References