A study of crossover operator in real coded genetic algorithm

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Abstract

In this paper, we propose a new crossover operator for real coded genetic algorithm (real-coded GA). Real-coded GA is expected to be powerful optimization technique for the nonlinear functions. A designing of crossover operator is important for a GA with good performance. At this point, for real-coded GA, many crossover operators are proposed. As a major crossover, the BLX- α proposed by Eshelman et al., the unimodal normal distribution crossover (UNDX) proposed by Ono et al.. However, the conventional crossover operators have merits and demerits of the accuracy and the processing speed. Then, in this paper, we propose the center distribution crossover (CDX) as a new crossover operator which improves the above problems. The performance of the proposed method is evaluated with the standard test functions. The performance of the proposed method are compared with the conventional methods.

Keywords: Genetic Algorithm, Real-Coded GA, Crossover operator, Function Optimization

1 Introduction

Genetic Algorithm (GA) is a stochastic search and an optimization algorithm. The GA is based on the principles of biological evolution [1-3, 5]. The GA searches for a solution using the selection operator, crossover operator and mutation operator in multiple searching points. The GA is applied to many optimization problems and the effectiveness are reported [4].

Recently, in this GA, real-coded GA attract the attention as an optimization methods for the

nonlinear functions and the effectiveness are reported [6]. For real-coded GA, there have been proposed many crossover operators so far. As a major crossover operator, the BLX- α proposed by Eshelman et al. [7], the unimodal normal crossover (UNDX) proposed by Ono et al. [8, 9].

However, the above crossover operators have merits and demerits of the accuracy and the processing speed. Then, in this paper, we propose the center distribution crossover (CDX) as a new crossover operator which improves the above problems. We also show the efficiency of the proposed method by applying to the standard test functions [13].

2 Real-coded genetic algorithm

In this section, we review the UNDX and the MGG model as a basic crossover operator and generation alternation model for real-coded GA.

2.1 UNDX

The UNDX proposed by Ono et al. [8, 9] shows good performance in the optimization problems. The UNDX generates children obeying a normal distribution around the parents as shown in Fig1. The normal distribution is centered at the middle point of parents x^1 and x^2 . The standard deviation is proportional to the distance between parents x^1 and x^2 in the direction of the line connecting the parents. Moreover, the standard deviation in the verticality directions are proportional to the distance of the parent x^3 and the line connecting the parents x^1 and x^2 .

The mathematical descriptions of the UNDX show the following : where x^c is child, and x^p is the center of parents x^1 and x^2 , and D is distance from parent x^3 to the line connecting the parents

Fig 1: Unimodal Normal Distribution Crossover, UNDX

Fig 2: Minimal Generation Gap, MGG

x^1 and x^2 , and ξ and η_i are normal distribution random number, and e_i is the unit vector.

$$x^c = x^p + \xi d + D \sum_{i=1}^{n-1} \eta_i e_i \quad (1)$$

$$x^p = (x^1 + x^2)/2 \quad (2)$$

$$d = x^1 - x^2 \quad (3)$$

$$\xi \sim N(0, \sigma_\xi^2), \eta_i \sim N(0, \sigma_\eta^2)$$

2.2 MGG

The MGG model proposed by Satoh et al. [11, 12] is an effective one of generation alternation models. In the MGG model, we randomly select parents from population, and we generate children by

crossover. From the parents and their children, we select an elite individual and a random one using the roulette selection based on the fitness value. The original parents are replaced by the selected children. In this model, the original parents are two individuals, and replacing individuals are also two. Fig2 shows the outline of the MGG model.

3 Proposed method

3.1 Center Distribution Crossover, CDX

We show the outline of the CDX as the proposed method. In the CDX, $n + 1$ parents are selected from the population and m children are generated around the center of parents.

The mathematical descriptions of CDX show the following : where C is child, and G is the mean of parents, and D is average distance from G to each parents, and η_i is normal distribution random number (average is 0, variance is σ_η^2), and e_i is the unit vector, and x_i is parental vector.

$$C = G + D \sum_{i=1}^{n+1} \eta_i e_i \quad (4)$$

$$G = \frac{1}{n+1} \sum_{i=1}^{n+1} x_i \quad (5)$$

$$D = \frac{1}{n+1} \sum_{i=1}^{n+1} |G - x_i| \quad (6)$$

$$\eta_i \sim N(0, \sigma_\eta^2)$$

In the conventional method, depending on how we select parents, it is easy to bias a searching area. On the other, the CDX effectively avoids a bias about a searching area. From the above point, the CDX effectively avoids a stagnation of a searching solution.

3.2 The configuration of GA with the CDX

We show the configuration of the real-coded GA with the proposed method. We use the MGG model as a generation alternation model. However, the proposed method uses many parents than the UNDX. We adjust the MGG model as below. In addition, Fig3 shows the outline.

1. We randomly select $n + 1$ parents from populations.

Fig 3: The configuration of GA with the CDX

2. We generate m children by crossover.
3. We randomly select 2 individuals from parents.
4. From the above 2 parents and m children, we select an elite individual and a random one using the roulette selection, and the 2 parents are replaced by the selected individuals.

4 Simulation and Results

In order to confirm the effectiveness of the proposed method, we compare the proposed method with the UNDX. In the first subsection, we show the test functions. In the second subsection, we show the experimental setting. In the last subsection, we show the simulation results and discuss the simulation results.

4.1 Test functions

In this paper, the performance of the proposed method is examined with the standard test functions [13]. The optimization problems used here are the minimization of the Rosenbrock function ($F1$), the Rastrigin function ($F2$), the Griewank function ($F3$), the Ridge function ($F4$). Table1 shows the test functions and its domains.

The Rosenbrock function is the unimodal function and it has dependence on its variables (non-separable function). The Rastrigin function is the multimodal function and it has independence on its variables (separable function). The Griewank function is the multimodal function and it has dependence on its variables (nonseparable function). The Ridge function is unimodal function and it has

Table 1: Test functions and its domains

$$F_1(x_i) = \sum_{i=1}^n [100(x_1^2 - x_i)^2 + (x_i - 1)^2]$$

$$-2.048 \leq x_i < 2.048$$

$$F_2(x_i) = 10n + \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i))$$

$$-5.12 \leq x_i < 5.12$$

$$F_3(x_i) = 1 + \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \left(\cos\left(\frac{x_i}{\sqrt{i}}\right) \right)$$

$$-512 \leq x_i < 512$$

$$F_4(x_i) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$$

$$-64 \leq x_i < 64$$

dependence on its variables (nonseparable function).

The unimodal function has only one global optimum. On the other hand, the multimodal function has many local optima, but only one global optimum.

4.2 Experimental setting

In this paper, we compare the performance of the proposed method with the UNDX. An experimental setting is shown in Table2. On the unimodal functions ($F1$, $F4$), number of individual is 300. On the multimodal functions ($F2$, $F3$), number of individual is 600. On all functions, number of variable is 20, number of applying crossover (UNDX) is 200, number of generation is 30000, parameters of UNDX σ_ξ is $1/2$ and σ_η is $0.35/\sqrt{n}$. The initial population is generated randomly following the uniform distribution inside the region described in Table1. All experiments were carried out by changing the seed for random number.

4.3 Simulation results

Table3 shows the average evaluations where the global optimum is discovered and the number of convergence times. These results are the average of 20 trials. Fig4, 5, 6, 7 show the histories of the

Table 2: Experimental Setting

Number of individuals	300($F1, F4$) / 600($F2, F3$)
Number of variables	20
Number of max generations	30000
Crossover	UNDX / CDX
σ_ξ	1/2 / -
σ_η	$0.35/\sqrt{n}$ / $1/\sqrt{n}$
Number of crossover time	200

Table 3: The average evaluations where the global optimum is discovered and the number of convergence times

	Crossover operator	Average evaluations	Convergence times
Rosenbrock function ($F1$)	UNDX	6000000	0/20
	Proposed method	3289200	20/20
Rastrigin function ($F2$)	UNDX	5215020	20/20
	Proposed method	3913960	20/20
Griewank function ($F3$)	UNDX	4936230	20/20
	Proposed method	3106440	20/20
Ridge function ($F4$)	UNDX	4951070	20/20
	Proposed method	1512030	20/20

best function value of the proposed method. Fig8, 9, 10, 11 show the histories of the best function value of the UNDX. In these figures, the y axis shows the function value, and the x axis shows the generation. The y axis is also log-scale.

On the results (Table3, Fig4, Fig8) of the test function $F1$, this function is unimodal function but it is nonseparable function, it is difficult for GA to find the optimal solution. However, the proposed method can find the optimal solution in all trials. In addition, the proposed method has the average evaluations where the global optimum is discovered less than the UNDX.

Secondly, on the results (Table3, Fig5, Fig9) of the test function $F2$, this function is separable function but it is multimodal function and it has many local optima around global optimum. The proposed method also has the average evaluations where the global optimum is discovered less than the UNDX. In addition, the proposed method can stably search solution.

Then, on the results (Table3, Fig6, Fig10) of the test function $F3$, this function is multimodal function and it is nonseparable function, it is difficult for GA to find optimal solution. The pro-

posed method has the average evaluations where the global optimum is discovered less than the UNDX.

Finally, on the results (Table3, Fig7, Fig11) of the test function $F4$, this function is unimodal function but it is nonseparable function. The proposed method has the average evaluations where the global optimum is discovered less than the UNDX.

We summarize all experimental results. The proposed method can find optimal solution in all trials. And, the proposed method has the average evaluations where the global optimum is discovered less than the UNDX. And, the proposed method can stably search solution. Especially, in the nonseparable functions ($F1, F3, F4$), the proposed method shows the good performance.

5 Conclusion

In this paper, we proposed the center distribution crossover (CDX) as a new crossover operator for real-coded GA. The performance of the CDX was evaluated with standard test functions. The performance of the proposed method was compared with the UNDX. Simulation results suggested that

the CDX has high accuracy and processing speed. Especially, in the nonseparable functions ($F1$, $F3$, $F4$), the proposed method shows the good performance.

In the near future, we try to compare the proposed method with another crossover operators, apply to another optimization problems.

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Fig 4: History of best function value in $F1$

Fig 8: History of best function value in $F1$

Fig 5: History of best function value in $F2$

Fig 9: History of best function value in $F2$

Fig 6: History of best function value in $F3$

Fig 10: History of best function value in $F3$

Fig 7: History of best function value in $F4$

Fig 11: History of best function value in $F4$