Collision Detection and Estimation of Its Spot for a Flexible Beam Using Mode-function-ratio Method

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Abstract

Flexible manipulators in working space with environmental objects (walls, boxes, humans, other mobile machines, etc.) should avoid collision to achieve their mission. Unfortunately, if an unexpected obstacle collides with the manipulator, it is necessary to stop and replan the path for avoiding the obstacle when the collision is detected. However, in order to replan new path to avoid the obstacle, we have to know the flexible manipulator's spot where the obstacle collides. This paper presents a method to find a collision spot of a flexible beam based on the observation data measured by strain sensors pasted on it.

1 Introduction

Robotic manipulators will be used in a highly structured environment such as houses, hospitals, offices and so on. In such environment the manipulator should prevent collision with objects. The flexible manipulators have an advantage in safety operation, because such manipulators are light weighted and mechanically flexible. However, the collision of the flexible manipulator and the unexpected obstacle or other operated objects causes serious problems, e.g., undesirable vibration and damage of the obstacle and/or the manipulator. In order to avoid their damage due to the collision, its detection and the estimation of the spot where the obstacle collides will play a significant role for the flexible manipulators in operation. As for multi-link flexible manipulator, to obtain the information of the collision spot on the arm is important for replanning the path of the manipulator to avoid the obstacle.

In order to avoid such collision, there are two approaches to development of the collision detection method. One is based on the noncontact method using vision, ultrasonic, and so forth [1, 2]; the other is based on using tactile method or force/torque sensor to detect contact with objects [3]-[5]. The collision detection based on contact can be achieved comparatively low-cost because the noncontact collision detection requires the expensive devices. Contact or collision between the flexible manipulators and the environmental ob-

jects has been studied by several researchers. Kaneko, *et al.* [6, 7] developed an active antenna for mobile robots. They used the deformation of the flexible beam for measuring a position of the object. Moorehead and Wang [8] proposed a collision detection method using strain gauges to determine the intensity and position of externally applied force due to collision with an obstacle. The estimation of the contact position in their approach was achieved by the mechanical relation between the positions of two strain gauges and the bending moments measured by the sensors. The author [9, 10] proposed a collision detection algorithm for a flexible cantilevered beam subject to distributed random disturbance based on an innovation process. He also studied an estimation method of the collision force acting on a flexible beam [11] and a method of collision spot estimation using the Kalman filter for the augmented system composed by the second-order model of the collision input and the system model of the flexible beam [12].

In this paper, a method of on-line collision detection and estimation of the spot of the flexible structure where an unexpected obstacle collides is presented. The flexible structure is assumed to be a flexible cantilevered-beam subject to distributed random disturbance. The mathematical model of the beam is given by the Euler-Bernoulli type partial differential equation with the distributed random disturbance and the external input due to the collision with unexpected obstacle. The collision input is modeled by a spatially distributed function. The state of the structure and the size of collision input are estimated using Kalman filter. A mode-function-ratio is calculated using the ratio in the 1st and the 2nd mode of the estimated collision input, and a collision position is identified from the mode-function-ratio.

2 Beam Dynamics with Collision and Its System Model

Consider a flexible cantilevered beam with collision phenomenon and distributed random disturbance due to wind, mechanical vibration, etc. The mathematical model of the beam is described by the following Euler-Bernoulli type par-

tial differential equation:

$$\begin{aligned} \rho S \frac{\partial^2 u(t, x)}{\partial t^2} + c_D I \frac{\partial^5 u(t, x)}{\partial x^4 \partial t} + EI \frac{\partial^4 u(t, x)}{\partial x^4} \\ = g_1 \gamma(t, x) + g_2 s(t) \delta(x - x_c), \quad (1) \\ (0 < x < \ell), \end{aligned}$$

where ℓ denotes the length of the beam; $u(t, x)$ the transverse displacement from its equilibrium state of the beam at the time t and position x ; $\gamma(t, x)$ the distributed random disturbance modeled by white Gaussian noise; ρ the mass density; S the cross sectional area of the beam; E the Young's modulus; I the second moment of cross sectional area; c_D the coefficient of the Kelvin-Voigt damping; g_1 and g_2 are constants. The boundary and initial conditions of (1) are given by $u(t, 0) = \partial u(t, 0)/\partial x = \partial^2 u(t, \ell)/\partial x^2 = \partial^3 u(t, \ell)/\partial x^3 = 0$ and $u(0, x) = \dot{u}(0, x) = 0$.

The collision is assumed to be made at a single spot of the beam, i.e., $x = x_c$ which is an unknown parameter. $s(t)$ denotes the external force (collision input) caused by the collision with the unexpected obstacle. This function takes non-zero value while the obstacle collides with the beam. It can be expressed by

$$s(t) = \begin{cases} 0, & t < t_c \\ s_0(t), & t_c \leq t \leq t_e \\ 0, & t_e < t, \end{cases} \quad (2)$$

where $s_0(t)$ denotes the unknown function which gives the behavior of the collision force's strength in the time interval $t \in [t_c, t_e]$ (t_c, t_e : unknown parameters). The vibration of the beam is measured by P strain sensors and their time derivatives. The observation from the j -th sensor is given by

$$y_j(t) = a_j \frac{\partial^2 u(t, \xi_j)}{\partial x^2} + b_j \frac{\partial^3 u(t, \xi_j)}{\partial x^2 \partial t} + e_j \beta_j(t), \quad (3) \\ (j = 1, \dots, P),$$

where $0 \leq \xi_1 < \xi_2 < \dots < \xi_P < \ell$; $\{a_j, b_j, e_j\}$ are nonzero constants; $\{\beta_j(t)\}$ the observation noise modeled by the white Gaussian noise.

The finite-dimensional model of the beam can be obtained via the modal analysis procedure. The displacement $u(t, x)$ is approximated by a linear combination of mode functions

$$u(t, x) = \sum_{k=1}^N u_k(t) \phi_k(x), \quad (4)$$

where $\{u_k(t)\}_{k=1, \dots, N}$ denotes the modal displacement; N is the sufficient number of modes describing the mechanical behavior of the beam. The associated eigenvalues and mode functions, $\{\lambda_k\}$ and $\{\phi_k(x)\}$, are the solutions of the continuous eigenvalue problem given by

$$\mathcal{A} \phi_k(x) = \lambda_k \phi_k(x), \quad (k = 1, 2, \dots), \quad (5)$$

where the operator $\mathcal{A}$ is defined by $\mathcal{A} := (EI/\rho S) d^4/dx^4$ with its boundary conditions

$$\begin{aligned} \phi_k(0) = \frac{d\phi_k(0)}{dx} = \frac{d^2\phi_k(\ell)}{dx^2} = \frac{d^3\phi_k(\ell)}{dx^3} = 0. \quad (6) \\ (k = 1, 2, \dots) \end{aligned}$$

The mode function $\{\phi_k(x)\}$ can be shown to have orthogonality property

$$\int_0^\ell \phi_k(x) \phi_\nu(x) dx = \delta_{k\nu}, \quad (7)$$

where $\delta_{k\nu}$ denotes Kronecker delta.

The following set of modal representation model can be obtained from the mathematical model (1) using the orthogonality property (7) as well as the solution in (4):

$$\begin{aligned} \ddot{u}_k(t) + \frac{c_D}{E} \lambda_k \dot{u}_k(t) + \lambda_k u_k(t) = \frac{g_1}{\rho S} \gamma_k(t) + \frac{g_2}{\rho S} s_k(t; x_c), \\ (k = 1, 2, \dots, N), \end{aligned} \quad (8)$$

where $\{s_k(t; x_c)\}_{k=1, 2, \dots, N}$ represents the collision input in modal representation given by

$$\begin{aligned} s_k(t; x_c) = \int_0^\ell s(t) \delta(x - x_c) \phi_k(x) dx \\ = \phi_k(x_c) s(t) \end{aligned} \quad (9)$$

which implies that the behavior of $s_k(t; x_c)$ during the collision depends only on $s(t)$. The position of the collision affects the amplitude of each $s_k(t; x_c)$ as a coefficient.

In order to know when and where the unexpected obstacle collides, the information on the collision input is required. Unfortunately the distribution and the strength of the collision input can not be measured directly. Furthermore the actual mathematical model which describes the behavior of $s(t)$ during the collision is unknown. Instead of the actual model of the collision phenomenon, we consider a system model so that its solution tracks the behavior of the collision input. In this paper, the dynamic behavior of $s_0(t)$ is assumed to be expressed by

$$\ddot{s}_0(t) = \alpha \zeta(t), \quad (t_c \leq t \leq t_e), \quad (10)$$

where α is a constant, and $\zeta(t)$ is a white Gaussian noise process. Differentiating (9) twice with respect to t and substituting (10) into the resultant equation with the relation (2), we obtain

$$\begin{aligned} \ddot{s}_k(t) = \alpha \phi_k(x_c) \zeta(t) \\ = \alpha \zeta_k(t), \quad (t_c \leq t \leq t_e), \end{aligned} \quad (11)$$

where $\zeta_k(t) = \phi_k(x_c) \zeta(t)$.

Introducing a state vector $v(t) := [v_c^T(t), \sigma^T(t), \dot{\sigma}^T(t)]^T$, $v_N(t) := [u_1(t), u_2(t), \dots, u_N(t), \dot{u}_1(t), \dot{u}_2(t), \dots, \dot{u}_N(t)]^T$, $\sigma(t) := [s_1(t; x_c), s_2(t; x_c), \dots, s_N(t; x_c)]^T$, the system model can be written into the following augmented system:

$$dv(t) = Av(t)dt + Gdw(t) \quad (12)$$

$$dy(t) = Cv(t)dt + Ed\eta(t), \quad (13)$$

where $y(t) = [y_1(t), \dots, y_P(t)]^T$; $w(t)$ is the $2N$ -dimensional Brownian motion process corresponding to the distributed random disturbance $\gamma(t, x)$ and the random noise

$\zeta(t); \eta(t)$ is also Brownian motion process corresponding to the observation noise $\{\beta_j(t)\}_{j=1,\dots,P}$; $\mathcal{E}\{dw(t)dw^T(t)\} = W dt$ and $\mathcal{E}\{d\eta(t)d\eta^T(t)\} = V dt$. The system matrices are given by

$$A = \begin{bmatrix} A_N & G_2 & 0 \\ 0 & 0 & I_N \\ 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & 0 \\ 0 & 0 \\ 0 & \alpha I_N \end{bmatrix}$$

$$C = [C_N \quad 0 \quad 0], \quad E = \text{diag}\{e_1, \dots, e_P\}$$

$$A_N = \begin{bmatrix} 0 & I_N \\ -\Lambda & -(c_D/E)\Lambda \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0 \\ (g_1/\rho S)I_N \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 \\ (g_2/\rho S)I_N \end{bmatrix}$$

and $C_N = [\tilde{C}_c, \tilde{C}_e]$; $[\tilde{C}_c]_{ij} = a_i d^2 \phi_j(\xi_i)/dx^2$; $[\tilde{C}_e]_{ij} = b_i d^2 \phi_j(\xi_i)/dx^2$ ($i = 1, \dots, P, j = 1, \dots, N$), $\Lambda = \text{diag}\{\lambda_1, \dots, \lambda_N\}$ and I_* the unit matrix; α constant.

3 Estimation Algorithm of Collision Spot

3.1 Estimation of Collision Input

In order to find the collision spot x_c , the information on the collision input is required, because $\{s_k(t; x_c)\}$ consists of $\{\phi_k(x_c)\}$ and $s(t)$ as shown in previous section. However it is impossible to measure the collision input directly, because it appears as a disturbance in the mathematical model. In order to obtain $\{s_k(t; x_c)\}$, it is necessary to estimate it based on the observation data. In this paper we employ the Kalman filter for (12) and (13) to estimate the state of the beam and the collision input. The Kalman filter is given by

$$d\hat{v}(t) = A\hat{v}(t)dt + K\{dy(t) - C\hat{v}(t)dt\} \quad (14)$$

$$K = PC^T(EVE^T)^{-1} \quad (15)$$

$$AP + PA^T - PC^T(EVE^T)^{-1}CP + GVG^T = 0, \quad (16)$$

where $\hat{v}(t) := \mathcal{E}\{v(t)|Y_t\}$ ($Y_t = \{y(s)|0 \leq s \leq t\}$) denotes the state estimate of $v(t)$; $\mathcal{E}\{\cdot|Y_t\}$ the conditional expectation. The initial conditions of (14) is given by $\hat{v}(0) = \hat{v}_0$.

3.2 Idea of Collision Spot Estimation

The direct measurement of the collision spot is impossible because of suitable sensors not to be installed on the beam. Fortunately we already know that $\{s_k(t; x_c)\}$ is expressed by the strength of the collision force $s(t)$ and the value of the mode function at the collision spot $\phi_k(x_c)$. In this paper we pay attention to a ratio of the collision input corresponding to the first two modes as an indicator to find the collision spot.

Consider the ratio defined by $s_2(t; x_c)/s_1(t; x_c)$. Substituting (9) into this ratio, it is reduced into a function of x_c , i.e.,

$$\frac{s_2(t; x_c)}{s_1(t; x_c)} = \frac{s_0(t)\phi_2(x_c)}{s_0(t)\phi_1(x_c)} = \frac{\phi_2(x_c)}{\phi_1(x_c)} \equiv \theta(x_c), \quad (17)$$

$$(t_c \leq t \leq t_e).$$

This equation implies that the ratio $s_2(t; x_c)/s_1(t; x_c)$ is regarded as a time independent function only during the collision ($t_c \leq t \leq t_e$). That is to say, the ratio $s_2(t; x_c)/s_1(t; x_c)$ gives a ratio $\phi_2(x_c)/\phi_1(x_c)$. As a result, we obtain the following relation:

$$\theta(x_c) = \frac{s_2(t; x_c)}{s_1(t; x_c)}, \quad (t_c \leq t \leq t_e). \quad (18)$$

In this paper, $\theta(x_c)$ is named the mode-function-ratio (MFR).

The problem here is to calculate collision spot x_c from the MFR. The basic idea to find x_c is as follows. $\theta(x_c)$ is approximated based on the estimate of the collision input. Then inverting $\theta(x_c)$, the collision spot can be obtained from the value of $\theta(x_c)$.

However, it is difficult to obtain the inverse function of $\theta(x_c)$, analytically, because the MFR is quite complicated function which consists of the mode functions at the collision spot, $\{\phi_k(x_c)\}_{k=1,2}$. Then the MFR can be represented by the Taylor series as follows:

$$\theta(x_c) = \theta(x_0) + \sum_{k=1}^4 \frac{1}{k!} \theta^{(k)}(x_0)(x_c - x_0)^k + O((x_c - x_0)^5), \quad (19)$$

where x_0 denotes an arbitrary parameter ($0 \leq x_0 \leq \ell$). The series expansion of the inverse of (19) can be expressed by

$$x_c = x_0 + \sum_{k=1}^4 \alpha_k (\theta - \theta_0)^k + O((\theta - \theta_0)^5), \quad (20)$$

where $\theta_0 := \theta(x_0)$ and

$$\alpha_1 = b_1^{-1}, \quad \alpha_2 = -\frac{b_2}{b_1^3}, \quad \alpha_3 = -\frac{-2b_2^2 + b_3b_1}{b_1^5},$$

$$\alpha_4 = \frac{-b_4b_1^2 + 5b_3b_2b_1 - 5b_3^2}{b_1^7}$$

$$b_1 = \left. \frac{d\theta(x_c)}{dx_c} \right|_{x_c=x_0}, \quad b_2 = \left. \frac{1}{2} \frac{d^2\theta(x_c)}{dx_c^2} \right|_{x_c=x_0},$$

$$b_3 = \left. \frac{1}{6} \frac{d^3\theta(x_c)}{dx_c^3} \right|_{x_c=x_0}, \quad b_4 = \left. \frac{1}{24} \frac{d^4\theta(x_c)}{dx_c^4} \right|_{x_c=x_0}.$$

Although the computation of $\theta(x_c)$ requires the true collision input $\{s_k(t; x_c)\}_{k=1,2}$, the available information is the estimate of the collision input in actuality. Then, we consider that the approximation of the MFR is calculated based on $\{\hat{s}_k(t)\}_{k=1,2}$ in the time interval $[t, t + \Delta T]$, where ΔT is the short time interval for data collection. Using the least mean square method, the approximation of $\theta(x_c)$ is obtained. Since the relation among $\{s_k(t; x_c)\}_{k=1,2}$ and $\theta(x_c)$ is given by (18), the square error $\{\theta(x_c)\hat{s}_1(t) - \hat{s}_2(t)\}^2$ is

considered. Consequently, the approximation of the MFR is given by the value of $\hat{\theta}(t)$ so that the following criteria is minimized:

$$J(\hat{\theta}) = \int_{t-\Delta T}^t \left\{ \hat{\theta}(t) \hat{s}_1(\tau) - \hat{s}_2(\tau) \right\}^2 d\tau. \quad (21)$$

Solving the following equation with respect to $\hat{\theta}(t)$:

$$\frac{\partial J(\hat{\theta})}{\partial \hat{\theta}} = 0, \quad (22)$$

the approximation of the MFR is calculated by

$$\hat{\theta}(t) = \frac{h_2(t)}{h_1(t)}, \quad (t_c \leq t \leq t_e), \quad (23)$$

where

$$h_1(t) = \int_{t-\Delta T}^t \hat{s}_1^2(\tau) d\tau, \quad h_2(t) = \int_{t-\Delta T}^t \hat{s}_1(\tau) \hat{s}_2(\tau) d\tau.$$

By replacing θ in (20) by $\hat{\theta}(t)$, the estimate of collision spot can be obtained by

$$\hat{x}_c(\hat{\theta}(t)) := x_0 + \sum_{k=1}^4 \alpha_k \left\{ \hat{\theta}(t) - \theta_0 \right\}^k, \quad (24)$$

$$(t_c \leq t \leq t_e)$$

where the high order terms are neglected.

The detection of collision is achieved by calculating a value of a scalar function defined by $r(t) = \hat{\sigma}^T(t) \hat{\sigma}(t)$, where $\hat{\sigma}(t)$ denotes the estimate of $\sigma(t)$ given by the Kalman filter. When this function takes a value in excess of preassigned level, i.e., $r(t) > \varepsilon$, we regard that a collision occurs, where ε is constant.

4 Simulation Studies

4.1 Simulation Setup

A steel beam with its length $\ell = 1.0$ [m], the cross sectional area $S = 4.04 \times 10^{-2} \times 5.03 \times 10^{-4}$ [m²] and the second moment of cross sectional area $I = 4.28 \times 10^{-13}$ [m⁴] was assumed in the simulation studies. The other physical parameters of the flexible beam are the Young's modulus $E = 1.96 \times 10^5$ [MPa], the mass density $\rho = 7.86 \times 10^3$ [kg · m⁻³], the coefficient of Kelvin-Voigt damping $c_D = 4.82 \times 10^8$ [N · s · m⁻²]. In this numerical simulations, three strain sensors are located at $\xi_1 = 0.5$ [m], $\xi_2 = 0.75$ [m], $\xi_3 = 0.95$ [m]. The other parameters were set as $c_j = 10$, $e_j = 0.5$, ($j = 1, \dots, 3$), $g_1 = g_2 = 0.5$, $\alpha = 200$.

As for the numerical model of the beam, instead of the solution of the mathematical model (1), we employed the numerical solution of the modal representation model corresponding to the first eight modes $\{u_k(t)\}_{k=1, \dots, 8}$. The Kalman filter was designed for the first two modes, i.e., $N = 2$. The time division of the numerical simulation was set as $\Delta t = 5 \times 10^{-3}$ [s]. In the criterion (21), the time renewal

Fig. 1: Behavior of the collision input in modal representation $\{s_k(t; x_c)\}_{k=1,2}|_{x_c=0.5}$ for Case 1 ($x_c = 0.5$ [m]).

Fig. 2: Behavior of the estimates of the collision input $\{\hat{s}_k(t)\}_{k=1,2}$ for Case 1 ($x_c = 0.5$ [m]).

interval was set as $\Delta T = 0.25$ [s], and the arbitrarily preassigned spot x_0 was set as 0.5 [m]. The covariance matrices with respect to the system and the observation noises were set as $W = \text{diag}\{1, 1\}$ and $V = \text{diag}\{1, 1, 1\}$. The initial conditions of the beam was set as $\{u_k(0)\}_{k=1, \dots, 8} = 0$ and $\{\dot{u}_k(0)\}_{k=1, \dots, 8} = 0$. The initial conditions of the Kalman filter was given by $\hat{v}(0) = 0$.

In this simulation, the collision is assumed to occur during the time from $t_c = 10$ [s] to $t_e = 10.5$ [s]. The intensity of the collision input $s(t)$ was generated based on the following equation:

$$s(t) = \sin\left(\pi \frac{t-10}{0.5}\right) \{u_s(t-10) - u_s(t-10.5)\}, \quad (25)$$

where $u_s(t)$ represents the unit step function.

4.2 Simulation Results

In this section, two cases of the simulation results are provided.

Case 1 ($x_c = 0.5$ [m]): Figs.1 and 2 present the modal collision input and their estimates, $\{s_k(t; x_c)\}_{k=1,2}|_{x_c=0.5}$ and $\{\hat{s}_k(t)\}_{k=1,2}$, respectively. The estimation result of the collision spot is shown in Fig.3. The solid and dotted lines

Fig. 3: Estimate of the collision spot $\hat{x}_c$ for Case 1 ($x_c = 0.5$ [m]) given by the solid line. The dashed line represents the true position of the collision spot.

Fig. 4: Behavior of the modal collision input $\{s_k(t; x_c)\}_{k=1,2}|_{x_c=0.3}$ for Case 2 ($x_c = 0.3$ [m]).

depict the behavior of $\hat{x}_c$ computed by (24) and the true collision spot. The $\hat{x}_c$ has the meaning only while the collision has occurred. The numerical value of $\hat{x}_c$ indicated in the time interval when the collision has not occurred is meaningless. In this case, $\hat{x}_c$ during the collision indicates the collision spot well.

Case 2 ($x_c = 0.3$ [m]): Figs.4 and 5 depict the behavior of the true collision inputs and their estimates in modal representation, $\{s_k(t; x_c)\}_{k=1,2}|_{x_c=0.3}$ and $\{\hat{s}_k(t)\}_{k=1,2}$, respectively. The estimate of the collision spot $\hat{x}_c$ is shown in Fig.6.

5 Conclusion

In this paper, a method of the collision detection and estimation of its spot for the flexible cantilevered beam with distributed random disturbance has been studied. Using the Kalman filter for the augmented system consisting of

Fig. 5: Behavior of the estimates of the collision input $\{\hat{s}_k(t)\}_{k=1,2}$ for Case 2 ($x_c = 0.3$ [m]).

Fig. 6: Estimate of the collision spot $\hat{x}_c$ for Case 3 ($x_c = 0.3$ [m]) given by the solid line. The dashed line represents the true position of the collision spot.

the finite-dimensional model of the beam and the second-order model for tracking the behavior of the collision input in modal representation, the estimate of the collision input has been obtained. The collision spot estimation has been achieved by introducing the mode-function-ratio and its inverse.

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