Passive Radar System Using a Reflection of Satellite

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Abstract

This paper introduces a new application of a passive radar system using a GPS satellite.

Radar is a system which uses reflection signals in order to detect and pursue targets. Passive radar makes use of reflection signals such as commercial broadcasts and communications signals to detect targets and to estimate parameters. However, passive radar systems are not always designed to detect and process the echoes from targets illuminated by transmitters. The system would lose the availability if the receivers couldn't detect these signals.

We are concerned with the estimation method for a target position by one echo from a target illuminated by a GPS satellite. When we could receive a reflection signal from a GPS satellite we can use constraint conditions which are the relative delay between a direct signal and reflection signals and the rate of change of phase differences. We can obtain the reflection position by solving simultaneous equations. The equations are presented as the equation relating the relative delay between a direct signal and reflection signals, and the equation relating differential of relative delay.

In this paper, we describe proposed method under the assumption that the target is a vertical plane. In addition, we apply proposed method to observation data and evaluate the availability of proposed method.

1 Introduction

Conventional radar systems comprise a collocated transmitter and receiver, which usually share a common antenna to transmit and receive.

Passive radar is a system which comprise a transmitter and a receiver which are separated by a considerable distance. A target location is found by an intersection point of more than three spheroid.

Passive radar systems encompass a class of radar systems that detect and track objects by processing reflections from non-cooperative signal sources in radio wave environment, such as commercial broadcasts and communications signals[1].

The receiving systems of passive radars are passive, and hence undetectable. This is attractive as they can reduce dramatically the costs of system hardware. The rapid growth in the number of RF emissions for TV

and radio broadcasts in addition to terrestrial and space based communications have resulted in development of wide range of signal types which are available for passive radar systems. Recently, there have been an upsurge of interest in passive radar systems[2]-[6].

In the previous work, we showed the possibility of passive radar system using GPS satellites[9]. GPS satellites radiate GPS signals which carry informations about the satellites' location and general system health (known as almanac and ephemeris data)[7]. The use of GPS satellites as transmitters enables to detect the target without the knowledge of transmitters' positions in advance. GPS satellite signals, however, have generally been found to be inadequate for passive radar use because radiated power is too low. We applied the method based on maximum likelihood estimation to estimate a relative delay between a direct and a reflection signal, and then we proposed the method for estimation of the reflection position using multiple GPS satellites by geometrical constraints.

However, passive radar system is not usually designed to detect and process the echoes from targets illuminated by multiple transmitters. For example, if a receiver receives only one echo from a target, the system cannot determine a target position by means of the previous method. Therefore, new method is required to be able to estimate the target position using fewer target echoes.

In this paper, we propose the estimation method to detect a target position by using a direct signal and a reflection signal which are radiated from a GPS satellite. A target position is determined by solving simultaneous equations. The simultaneous equations are based on the equation of relative delay between a direct signal and reflection signal, and that of differential of relative delay. We present the method and show an experimental result.

2 System model and parameters

First of all, we present the system model and parameters considered in this paper.

Fig. 1 shows the system model. A GPS satellite radiates C/A(Coarse/Aquisition) code signal to a receiver which is on the ground. The incident waves to the receiver and the reflector are plane waves. The

receiver can receive the signal illuminated from the reflector which assumes a vertical plane by geometrical constraints. Therefore, the receiver can obtain both the direct signal and the reflection signal.

Parameters are defined as follows,

- (x, y, z) : Transmitter position
- η : Azimuth of transmitter
- ζ : Elevation of transmitter
- d : Distance of receiver to reflector
- φ : Angle between receiver and reflector

3 Separation of received signals

We need to separate the received signals into a direct signal and a reflection signal before estimating the reflector position. In this section, we review the maximum likelihood estimation method[8].

The baseband signal model is defined as follows,

$$q(j) = \sum_{p=1}^P \alpha_p e^{i\theta_p} m(jT - \tau_p) \quad j = 1, \dots, N \quad (1)$$

where,

- T : Sampling rate
- N : signal length
- j : Index of sampling
- P : The number of received signals
- $m(t)$: Band-limited C/A code
- α_p : Amplitude of p-th signal
- θ_p : Initial phase of p-th signal
- τ_p : Code delay of p-th signal.

In addition,

$$\begin{aligned} \alpha &= (\alpha_1, \dots, \alpha_P)' \\ \theta &= (\theta_1, \dots, \theta_P)' \\ \tau &= (\tau_1, \dots, \tau_P)' \end{aligned}$$

"'" denotes transpose.

The received signal $r(j)$ is defined as

$$r(j) = q(j) + n(j).$$

$n(j)$ is the Gaussian noise whose mean value μ is 0 and variance is σ^2 .

We estimate parameters $\hat{\alpha}$, $\hat{\theta}$ and $\hat{\tau}$ which minimize the error sum of squares as follows,

$$(\hat{\alpha}, \hat{\theta}, \hat{\tau}) = \arg \min \left\{ \sum_j |r(j) - s(j)|^2 \right\} \quad (2)$$

By solving the above per iteration time in the case of $P = 2$, we obtain,

$$(\hat{\alpha}_{k,1}, \hat{\alpha}_{k,2}), (\hat{\theta}_{k,1}, \hat{\theta}_{k,2}), (\hat{\tau}_{k,1}, \hat{\tau}_{k,2}).$$

k is an index of iteration and numerals of subindex express the order of arrival.

Fig. 1: System model

Therefore, we can be calculated the estimated relative delay $\hat{L}(k)$ between the direct signal and reflection signal.

$$\hat{L}(k) = (\hat{\tau}_{k,2} - \hat{\tau}_{k,1}) \cdot c,$$

where, c is the light speed.

4 Estimation of the reflector position by the simultaneous equations

In this section, we formulate the simultaneous equations concerning relative delay, namely path difference, and the differential of relative delay in order to obtain d and φ .

4.1 The equation of relative delay

At first, we define the set of short interval S .

$$S = (S_1, \dots, S_\xi, \dots, S_q), \quad q = N/\|S\|.$$

$\|S\|$ means the number of k comprised in S .

For S , the average of $\hat{L}(k)$ is calculated as follows,

$$\bar{L}(S) = \frac{1}{\|S\|} \sum_{k \in S} \hat{L}(k). \quad (3)$$

$\bar{L}(S)$ is a good approximation of $\hat{L}(\bar{k})$. $\bar{k}$ is a median of $k \in S$.

Here, we obtain $\bar{L}(S)$ from a transmitter position. The transmitter position (x_ξ, y_ξ, z_ξ) is given by GPS signal. The angular of transmitter (η_ξ, ζ_ξ) can be calculated by (x_ξ, y_ξ, z_ξ) .

The relative delay between the direct and the reflection signal calculated by geometric constraints is formulated as

$$\bar{L}_\xi = 2d \cos(\eta_\xi + \varphi) \cos \zeta_\xi. \quad (4)$$

Fig. 2: $\hat{L}(k)$ (dotted line) and $\bar{L}_\xi$ ($\diamond$) for S (line)

Fig. 3: $\Delta\theta_k$ (dotted line) and linear approximation (line)

$\bar{L}_\xi$ is equal to $\bar{L}(S)$.

Fig. 2 shows the example of relative delay $\hat{L}(k)$ and $\bar{L}_\xi$ for a short interval S .

4.2 The equation of differential of relative delay

For secondary equation, we formulate the equation concerning the differential of relative delay when a transmitter moves for a short time. The differential of relative delay is calculated from values of the phase difference and the rate of change of angular of a transmitter.

The differential of eq. (4) is written as follows,

$$\frac{d\bar{L}_\xi}{dt} = 2d \left\{ -\sin(\eta_\xi + \varphi) \cos \zeta_\xi \frac{d\eta_\xi}{dt} - \cos(\eta_\xi + \varphi) \sin \zeta_\xi \frac{d\zeta_\xi}{dt} \right\}. \quad (5)$$

The left-hand side of eq. (5) denotes the rate of change of relative delay. The rate of change of relative delay can be calculated from phase difference $\Delta\theta_\xi$ between a direct and a reflection signal.

$$\Delta\theta_\xi = \hat{\theta}_{\xi,2} - \hat{\theta}_{\xi,1}. \quad (6)$$

$\Delta\theta_\xi$ can be expressed by linear approximation as shown in fig. 3.

$$\Delta\theta_\xi \approx \gamma t + \nu. \quad (7)$$

We substitute the left-hand side of eq. (5) by approximation of rate of change of relative delay,

$$\frac{d\hat{L}_\xi}{dt} = \frac{d\Delta\theta_\xi}{dt} \cdot \frac{c}{2\pi f} \approx \gamma \cdot \frac{c}{2\pi f}, \quad (8)$$

where, f is carrier frequency of a transmitter.

The right-hand side of eq. (5) contains the rate of change of angular of a transmitter, $d\eta_\xi/dt$ and $d\zeta_\xi/dt$.

The transmitter position is calculated based on navigation data including GPS signal. The rate of change for azimuth $d\eta_\xi/dt$ is written as

$$\frac{d\eta_\xi}{dt} = \frac{1}{\sin \eta_\xi \sqrt{(x_\xi^2 + y_\xi^2)^3}} \left((1 - x_\xi^2) \frac{dx_\xi}{dt} - x_\xi y_\xi \frac{dy_\xi}{dt} \right). \quad (9)$$

The rate of change for elevation $d\zeta_\xi/dt$ is written as

$$\frac{d\zeta_\xi}{dt} = \frac{1}{\cos \zeta_\xi \sqrt{(x_\xi^2 + y_\xi^2 + z_\xi^2)^3}} \cdot \left((1 - z_\xi^2) \frac{dz_\xi}{dt} - z_\xi y_\xi \frac{dy_\xi}{dt} - z_\xi x_\xi \frac{dx_\xi}{dt} \right). \quad (10)$$

The equation becomes independent from that of relative delay since the differential of relative delay is able to be formulated by means of the rate of change of relative delay and the rate of change of angular of transmitter.

4.3 The simultaneous equations and their solutions

As the results of previous sections, we formulate the simultaneous equations as follows.

$$\begin{cases} 2d \cos(\eta_\xi + \varphi) \cos \zeta_\xi = \bar{L}_\xi \\ 2d \left\{ -\sin(\eta_\xi + \varphi) \cos \zeta_\xi \frac{d\eta_\xi}{dt} - \cos(\eta_\xi + \varphi) \sin \zeta_\xi \frac{d\zeta_\xi}{dt} \right\} = \frac{d\hat{L}_\xi}{dt} \end{cases}. \quad (11)$$

d is eliminated from eq. (11).

$$\begin{aligned} & \left\{ -\cos \eta_\xi \cos \zeta_\xi \cdot \frac{d\hat{L}_\xi}{dt} + \bar{L}_\xi (-\cos \eta_\xi \sin \zeta_\xi \frac{d\zeta_\xi}{dt} - \sin \eta_\xi \cos \zeta_\xi \frac{d\eta_\xi}{dt}) \right\} \cos \varphi \\ &= \left\{ -\bar{L}_\xi (\sin \eta_\xi \sin \zeta_\xi \cdot \frac{d\zeta_\xi}{dt} - \cos \eta_\xi \cos \zeta_\xi \cdot \frac{d\eta_\xi}{dt}) \right. \\ & \quad \left. \pm \sin \eta_\xi \cos \zeta_\xi \cdot \frac{d\hat{L}_\xi}{dt} \right\} \sqrt{1 - \cos^2 \varphi}. \quad (12) \end{aligned}$$

Here,

$$C_1 = -\cos \eta_\xi \cos \zeta_\xi \cdot \frac{d\hat{L}_\xi}{dt} + \bar{L}_\xi \left(-\cos \eta_\xi \sin \zeta_\xi \frac{d\zeta_\xi}{dt} - \sin \eta_\xi \cos \zeta_\xi \frac{d\eta_\xi}{dt} \right). \quad (13)$$

$$C_2 = -\bar{L}_\xi \left(\sin \eta_\xi \sin \zeta_\xi \cdot \frac{d\zeta_\xi}{dt} - \cos \eta_\xi \cos \zeta_\xi \cdot \frac{d\eta_\xi}{dt} \right) \pm \sin \eta_\xi \cos \zeta_\xi \cdot \frac{d\hat{L}_\xi}{dt}. \quad (14)$$

The estimation solutions $\tilde{\varphi}$ and $\tilde{d}$ are calculated,

$$\begin{cases} \tilde{\varphi} = \pm \cos^{-1} \left(\pm \sqrt{\frac{C_1^2}{C_1^2 + C_2^2}} \right) \\ \tilde{d} = \frac{\bar{L}_\xi}{2 \cos(\eta_\xi + \tilde{\varphi}) \cos \zeta_\xi} \end{cases} \quad (15)$$

From the positional relation between a transmitter and a receiver, the sign of d can be determined, and the solutions of $\tilde{\varphi}$ are confined to

$$\tilde{\varphi} = \pm \cos^{-1} \left(\sqrt{\frac{C_1^2}{C_1^2 + C_2^2}} \right). \quad (16)$$

The optimum solution of $\tilde{\varphi}$ satisfies the following equation and that is unique.

$$f(\tilde{d}, \tilde{\varphi}) = \arg \min \left\{ \frac{d\hat{L}_\xi}{dt} - 2\tilde{d} \left\{ -\sin(\eta_\xi + \tilde{\varphi}) \cos \zeta_\xi \frac{d\eta_\xi}{dt} - \cos(\eta_\xi + \tilde{\varphi}) \sin \zeta_\xi \frac{d\zeta_\xi}{dt} \right\} \right\}. \quad (17)$$

5 Experimental results

In order to evaluate the performance of proposed method, we applied the method to field data.

5.1 experimentation environment

The experimentation environment is showed table 1, fig. 4 and fig. 5.

Table 1: Experimentation parameters

d	26.5m
φ	0 degree
Antenna type	circular polarized antenna
PRN	No.25
Carrier frequency	L1 (1.57542 GHz)
Sampling frequency	40MHz
Band width	12MHz
Iteration	200ms

Fig. 4 shows the receiver position and the reflector position. A receiver is located within the range of which the receiver can receive the reflection signal from the building on the west of it. The distance of receiver to the building is 26.5 meters and angle between receiver and reflector is 0 degree.

Fig. 5 is presented the GPS satellite constellation when we did the experimentation. Circles indicate elevation from 0 degree to 80 degrees. The receiver position is in the center of circles. We chose a satellite as transmitter, which PRN(Pseudo Random Noise) code is No.25. The satellite of PRN25 is a low elevation satellite in fig. 4, so the signal is reflection on the building.

5.2 experimental result

The experimental results are showed in fig. 6 through fig. 8.

Fig. 6 and fig. 7 show results estimated by using maximum likelihood estimation. Fig. 6 shows the relative delay between a direct signal and a reflection signal. The relative delay $\hat{L}_k$ is calculated by $\Delta\hat{\tau}$. The phase difference is presented in fig. 7. Observational artifact during times between 30 to 40 seconds indicates failure to detect signals because a direct signal and a reflection signal are reverse phase each other.

In fig. 6, values of $\hat{L}_k$ is maintained virtually constant. The value of relative delay gradually declines between 0 to 50 seconds because the transmitter moved away from the receiver. This indicates the more the transmitter goes away from a receiver, the more the incidence angle for the reflector grows.

This result shows that source separation is successful by using maximum likelihood estimation. On the other hand, it shows a change of them is not able to calculate precisely.

Fig. 7 shows that the phase difference is monotonically increasing. The change of phase is recognizable. This is the reason that carrier wavelength is very short. If $\Delta\theta_k$ is 10 degrees, the change of relative delay is approximately 5.5 millimeters. The use of the phase difference enables to estimate with a high degree of precision.

Fig. 8 shows the estimation result of the reflector position, $\tilde{d}$ and $\tilde{\varphi}$. The top of figure shows results of $\tilde{d}$, and the bottom of figure shows results of $\tilde{\varphi}$. The horizontal axis is the lapse of time. We plot data calculated as the average per 10 seconds.

$\tilde{d}$ is distributed between 26 meters and 29 meters. $\tilde{\varphi}$ is distributed during ± 5 degrees. The trend of change in the relative delay is similar that of the phase difference.

The best result appeared when the phase difference is getting near ± 90 degrees, which the direct signal and reflection signal is orthogonal. When the phase difference is close to 0 degree, the estimation result is inferior to the others because it is hard to distinguish between direct signal and reflection signal.

Fig. 4: Experimentation environment

Fig. 5: GPS satellites constellation

Fig. 6: The relative delay

Fig. 7: The phase difference(unwrapped)

Fig. 8: Estimation results

The variance of estimated result, however, is affected by length of smoothing time. The setting of length of smoothing time is decided empirically, so it is necessary to consider the length according to measurement circumstances.

As mentioned above, the result of the experiment shows that the proposed method is able to detect a target when the reflector assumed a vertical plane.

6 Conclusions

In this paper, we proposed the estimation method to detect a reflector position by using the simultaneous equations.

The equations consist of relative delay and differential of relative delay between a direct signal and a reflection signal. The equation relating relative delay is presented by estimated parameters calculated by means of maximum likelihood estimation. The equation relating differential of relative delay is presented by rate of change of relative delay and rate of change of a transmitter position.

We applied proposed method to observation data and obtained a reflector position by solving the simultaneous equations. From the experimental result, we conclude that proposed method is able to detect a target by using a direct signal and a reflection signal when the reflector is a vertical plane.

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