An Introduction to a Framework to Evaluate the Transparency of a Firm

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Abstract

The aim of this research is to give a brief survey of framework to evaluate/quantize the transparency of a firm. We present a modification of Merton’s model for corporate debt where we assume that the process of the firm value is not observable but is strongly correlated with the sum of prices of stock and bonds which is observable and tradable. By a filtering argument, an explicit credit spread formula is obtained. Using the formula, "market implication" of transparency of a firm can be quantified. A numerical example is presented to illustrate the procedure.

1 Introduction

Thanks to the big crash in the summer 2007, we recognized that transparency of firm is quite important when we estimate credit risk of firm. In our model, we define the transparency of firm based on a modified Merton model. The Merton’s model [5] is the first but still standard "structural" model. There have been many structural models since then and also many credit risk models belonging to another class, called “reduced models” like Jarrow and Turnbull’s [3] or Duffie and Singleton’s [4]. The hybrid class was introduced firstly by Duffie-Lando [7], which is now recognized as a filtering model for credit risk. Our model is also a kind of hybrid model and can be called “filtering” model, but is totally different from Duffie-Lando’s approach.

We estimate the transparency not by using standard statistical techniques. In the modern finance/financial engineering, there is an alternative way to estimate a parameter, which is called “calibration” with a slight abuse of terminology. Typical example of the method is the one for so-called implied volatility; estimation of the volatility in the Black-Scholes model [2].

2 The setting and notation

Let \((\Omega, \mathcal{F}, P)\) be a probability space which Brownian Motion \(W_t\) can be defined, and \(\mathcal{F}_t\) be the natural filtration of \(W\). We assume Merton Model Economy [5]; i.e., firm value \(V := \{V_t\}_{0 \leq t \leq T}\) is assumed to be

\[ V_t = V_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma W_t), \]

where \(\mu \in \mathbb{R}\) and \(\sigma \geq 0\). However, in the market on the date of \((t_0, t_1, \cdots, t_k, \cdots, t_n)\), where

\[ 0 = t_0 \leq t_1 \leq t_2 \cdots \leq t_{n-1} \leq t_n = T, \]

we suppose that the firm value is only observable each \(t_k\) during the interval \((t_k, t_{k+1})\), the market "guess" market value of firm by which we mean the aggregation of the market suspects, and the value \(V' = \{V'_t\}_{0 \leq t \leq T}\) is assumed to be

\[ V'_t := \mathbb{E}[V_t | G_t], \]

where "\(G_t\)" is the market filtration which we assume

\[ G_t := \mathcal{F}_t' \lor \sigma(W_{t_1}, W_{t_2}, \cdots, W_{t_k}), \]

where \((t_k \leq t < t_{k+1})\), \(\mathcal{F}_t'\) is the natural filtration of \(W'\) and \(W'_t\) is standard 1-dimensional Brownian motion on \((\Omega, \mathcal{F}, P)\) that satisfies the following condition

\[ \langle W, W' \rangle_t = \rho t \quad (0 \leq \rho \leq 1). \]

For the proof see [1].

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Lemma 1 ([1]). For $t_k \leq t < t_{k+1}$, we have

$$V'_t = V_{t_k} \exp\left(\frac{\mu}{2}(t - t_k) + \sigma\rho(W'_t - W'_{t_k})\right).$$

At the announce times, the market value of firm coincides with the firm value. When $\rho = 1$, the market fully observe the firm value each the period $(t_k, t_{k+1})$.

We suppose that pay-off of corporate debt is $f(V_T)$, where

$$f : \mathbb{R} \to \mathbb{R}$$

is a bounded Borel function. We claim that the market value of corporate debt is given by the replication cost of

$$K^*_T := \mathbb{E}[f(V_T)|\mathcal{G}_T],$$

which is minimizer of

$$\inf\{||K_T - f(V_T)||_{L^2} : K_T \in L^2(\mathcal{G}_T)\}.$$

Since the market generated by $V'$ is complete, this means the market value $D^\rho_t$, for $t_{n-1} \leq t < T$, is given by

$$D^\rho_t = \mathbb{E}^Q[e^{-r(t_{n-1} - t)}K^*_t|\mathcal{G}_t],$$

where $r \geq 0$ is the risk free rate and $\mathbb{E}^Q$ denotes the expectation with respect to the equivalent martingale measure with respect to $V'$ i.e., the $\mathbb{P} \cdot \mathcal{F}'_t$ martingale $Z_t$ which assume that

$$Z_t := \exp(\theta W'_t - \frac{1}{2} \theta^2 t),$$

where

$$\theta := \left(-\frac{\mu - r}{\sigma}\right),$$

defines an equivalent probability martingale $Q$ by

$$\frac{dQ}{dP} = Z_t, t \in [0, T].$$

The probability $Q$ is usually called equivalent martingale measure, and is unique when $V'$ only is considered but not the case when $V$ is also involved.

When the company discloses information about its firm value, we should rearrange the strategy: an optimal hedging strategy at $t_{n-1}$ is given by

$$\kappa^*_{t_{n-1}} = \mathbb{E}[D^\rho_{t_{n-1}}|\mathcal{G}_{t_{n-1}}]$$

The market value of corporate debt $D^\rho_t$ for $t_{n-2} \leq t < t_{n-1}$ is given by

$$D^\rho_t = \mathbb{E}^Q[e^{-r(t_{n-1} - t)}\kappa^*_{t_{n-1}}|\mathcal{G}_t].$$

Inductively, the optimal strategy in the same spirit $t_{n-k}$ is given by

$$\kappa^*_{t_{n-k}} = \mathbb{E}[D^\rho_{t_{n-k}}|\mathcal{G}_{t_{n-k}}],$$

and the market value of corporate debt $D^\rho_t$ for $t_{n-(k+1)} \leq t < t_{n-k}$ is given by

$$D^\rho_t = \mathbb{E}^Q[e^{-r(t_{n-k} - t)}\kappa^*_{t_{n-k}}|\mathcal{G}_t].$$

Thus we have:

Lemma 2. For $t \geq 0$

$$D^\rho_t = e^{-r(T-t)}\mathbb{E}^Q[\kappa^*_T|\mathcal{G}_t].$$

For a detailed proof, see [1]. The optimal hedging strategy is rearranged each at $t_k$, so the hedging strategy is locally optimal. Lemma 2 show that the replication cost of the local strategy coincides with the replication cost $\mathbb{E}^Q[K^*_T|\mathcal{G}_t]$, that is, even if we rearranged the optimal strategy at each $t_k$, the strategy becomes the global optimal hedging strategy.

Now, let is,

$$K^{\sigma^2}_{n-k} := \frac{\sigma^2}{2}(T - t_{n-(k+1)})$$

$$+ \frac{(\sigma^2)^2}{2}(t_{n-k} - t_{n-(k+1)})$$

$$+ r(T - t_{n-k}),$$

$$t^\rho_{n-k} := T - t_{n-k+1} - \rho^2(t_{n-k} - t_{n-(k+1)}).$$
\[ K_{t_n-k}^\sigma := -\frac{\sigma^2}{2}(T-t_{n-(k+1)}^+)
\]
\[ + \frac{(\sigma \rho)^2}{2}(t-t_{n-(k+1)})
\]
\[ + r(T-t),
\]
and
\[ t^p_{t_n-k} := T - t_{n-k+1} - \rho^2(t - t_{n-(k+1)}).
\]

3 The result

Theorem 3 ([1]). We have the following explicit formula for the optimal strategy \( K_{t_n-k}^* \) and the market value of corporate debt \( D^p_t \):

\[ K_{t_n-k}^* = e^{-r(T-t_{n-k})} \int_{-\infty}^{\infty} f (V_{t_n-k} \exp(K_{t_n-k}^\sigma + \sigma z)) \exp(-\frac{z^2}{2\pi t_{n-k}^p}) \frac{dz}{\sqrt{2\pi t_{n-k}^p}}, \quad (k = 1, 2, \cdots) \]

and

\[ D^p_t = e^{-r(T-t)} \int_{-\infty}^{\infty} f (V'_{t_n-k} \exp(K_{t_n-k}^\sigma + \sigma z)) \exp(-\frac{z^2}{2\pi t_{n-k}^p}) \frac{dz}{\sqrt{2\pi t_{n-k}^p}}, \quad (t_{n-(k+1)} \leq t < t_{n-k}). \]

(3.1)

Moreover, we have a more explicit form at \( t_{n-k} \):

\[ D^p_{t_n-k} = e^{-r(T-t_{n-k})} \int_{-\infty}^{\infty} f (V_{t_{n-k}} \exp(-\frac{\sigma^2}{2}(T-t_{n-k})
\]
\[ + r(T-t_{n-k}) + \sigma z)) \exp(-\frac{z^2}{2(T-t_{n-k})}) \frac{dz}{\sqrt{2\pi(T-t_{n-k})}}. \]

(3.2)

4 Evaluation of transparency

We assume that the pay off \( f(V_T) \) is simply given by

\[ f(V_T) := \min(F, V_T) \]

to estimate explicitly transparency, where \( f \) is the face value of corporate debt. We also suppose that "market value" of the firm is the sum of prices of stocks and bonds, that is

\[ V'_t := \eta_1 S_t + \sum_{T} \eta^T D^T_t, \]

where \( \eta_1 \) is the number of issued stock and \( \eta^T \) is total value of the debt of maturity \( T \).

Then we obtain the following explicit formula thanks to Theorem 3:

\[ D^p_t = V'_t \Phi(\alpha)
\]
\[ - Fe^{-r(T-t)} \Phi(\alpha + \sigma \sqrt{t_{n-k}^p}) \]
\[ + Fe^{-r(T-t)}, \]

(4.1)

where

\[ \Phi(y) := \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}x^2} dx, \]
\[ \alpha := \frac{\log \frac{E_{V'_t} - \sigma^2 t_{n-k}^p}{2 \pi t_{n-k}^p} - r(T-t)}{\sigma \sqrt{t_{n-k}^p}}, \]
\[ t^p_{t_n-k} := T - t_{n-k} - \rho^2(t - t_{n-k}), \]

and

\[ D^p_{t_n-k} = V'_{t_n-k} \Phi(\beta)
\]
\[ - Fe^{-r(T-t)} \Phi(\beta + \sigma \sqrt{T-t_{n-k}}) \]
\[ + Fe^{-r(T-t)}, \]

(4.2)

where

\[ \beta := \frac{\log \frac{E_{V'_{t_n-k}} - (r + \frac{\sigma^2}{2}) \sqrt{T-t_{n-k}}}{2 \pi}}{\sigma \sqrt{T-t_{n-k}}}. \]

We understand there explicit formulas as an indication of transparency \( \rho \) which is not directly observable.

5 Illustrative example

Suppose that we are given the following situation;
1. A firm announces its accounting information 4 times a year.

2. The firm value at each announce time is available, let say, through the balance sheet.

3. The corporate bond price and the stock price of the firm i.e.: the current market value of the firm are quoted in the market.

4. Interest rates during the period are quoted in the market.

We shall quantify the transparency of a firm which is listed on the Tokyo Stock Exchange. Now we use fake data, since the purpose is to illustrate the procedure.

1. Let, the last announce time be \( t_k = 0 \), the present time \( t = 0.25 \) and the maturity of corporate debt \( T = 1.25 \).

2. The interest rate is \( r = 0.006 \).

3. At the firm disclosure time, the firm value is \( V_0 = 60 \).

4. The total face value of the corporate debt is \( F = 25.0 \).

At the announcement we can calculate the market evaluation of the volatility \( \sigma \) by (4.2) formula if we observed price of corporate debt.

Fig. 1: Announce times

Fig. 2: Volatility and Price of Debt

Fig. 2 describes the dependence between the prices of corporate debt \( D^o_i \) and the market evaluation of volatility \( \sigma \). The X-axis is for the prices of corporate debt and the Y-axis is for the market evaluation of volatility. Other parameters are as given above. For example, it we suppose that the corporate debt price at the announcement is \( D_k = 24.5 \), then, by the Fig.2, the market evaluation of volatility is \( \sigma^2 = 20.51\% \).

6. The current firm value, which is the sum of the prices of stock and bonds, is \( V_t = 60 \).

7. The current Corporate debt price is \( D_t = 24.6 \) just as the above example.

We can calculate the transparency by substituting these value for (4.1). Fig.3, describes the dependence between the prices of corporate debt \( D^o_i \) and the transparency \( \rho \). The X-axis is for the prices of corporate debt and the Y-axis is for the transparency. Other parameters are as given above. Then by the Fig.3, we notice that the transparency is \( \rho = 0.642 \).

6 Concluding Remark

In section 4, we assumed the pay-off is \( \min(F, V_T) \) and then we calculated transparency by using (4.1) and (4.2). A necessary tool is only Microsoft office Excel and we used the technic appearing in [9]. It is easy and quick.
to calculate transparency by using our framework. Furthermore we can say (3.1) and (3.2) are of simple form, because (3.2) coincides with the European option premium formula in the Black Scholes Model. In general, if the payoff function f is non-decreasing, $D^t_i$ is a non-decreasing function of V (see e.g. [8]). Therefore we can obtain the transparency in the similar way with section 4, that is, we can estimate the parameter $\sigma$ at each $t_k$ like the implied volatility technic and there when we assume a suitable assumption of f, we can calculate transparency $\rho$ on $(t_k, t_{k+1})$.

7 Issue for Future work

We assume the default is determined by the firm value, that is the firm is bankruptcy when the firm is just only debt default. That is usual in Japan. But in US or in Europe, they think in a different way for bankruptcy. They regard bankruptcy as one of the corporate strategies and the stockholders often determine the strategy. So they smash up the firm even when they can discharge a liability (see e.g. [7]). The difference should be recognized. The pay-off function of the corporate debt, or equivalently, the "recovery rate" should be chosen in a more careful way (see e.g. [6]). The result may depend on the choice, but we would look for robust evaluation that is indifferent to the choice.

The choice of the value process could also be more flexible. To generalize, we can choose the solution to stochastic differential equation which has the Markov property. In this case we need to rely on technic from Malliavin Calculus ([10])

References


