Calculation of the MEMM for geometric Lévy processes and its application to option pricing

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Abstract

In this paper, we shall present a useful method to obtain the MEMM (minimal entropy martingale measure) for the typical geometric Lévy processes such as compound Poisson, stable, VG(Variance Gamma), CGMY(Carr-Geman-Madan-Yor), NIG(Normal Inverse Gaussian), etc., and moreover we shall apply the method to value the European call option and Asian call option.

1 Introduction

It is well known that the MEMM (minimal entropy martingale measure) for geometric Lévy processes, proposed by Fujiwara and Miyahara [7], plays an important role in the mathematical finance. From the practical point of view, it is also important to get it concretely in order to evaluate the price of the derivative. While we can show the existence of the MEMM in many practical cases, it is difficult to calibrate it concretely because we have to solve rather complicated nonlinear integral equation (see (C2) or (C3) below). In this paper, we will propose some useful algorithms in terms of the numerical analysis to obtain the MEMM for the typical (geometric) Lévy processes such as compound Poisson, stable process, etc.. Moreover, we shall apply these results to the pricing of European call option and Asian call option.

2 MEMM

We review briefly the MEMM according to [7]. Suppose that the underlying asset process S_t is given by $S_t = S_0 \exp\{X_t\}$, where S_0 is an initial value and X_t is a Lévy process with the generating triplet $(\sigma^2, \nu(dx), b)$.

Definition An equivalent martingale measure P^* is called the MEMM of S_t if for any equivalent martingale measure Q ,

$$H(P^*|P) \leq H(Q|P),$$

where P is a given market measure and $H(Q|P)$ means the relative entropy of Q with respect to P , defined by the following formula:

$$H(Q|P) = \begin{cases} \int_{\Omega} \log \left[\frac{dQ}{dP} \right] dQ, & \text{when } Q \ll P, \\ \infty, & \text{otherwise} \end{cases}$$

Assume that there exists a constant θ^* such that

$$(C1) \quad \int_{\{|x| \leq 1\}} e^x e^{\theta^*(e^x - 1)} \nu(dx) < \infty,$$

$$(C2) \quad f(\theta^*) \equiv b + \left(\frac{1}{2} + \theta^*\right) \sigma^2 + \int ((e^x - 1) e^{\theta^*(e^x - 1)} - I_{\{|x| \leq 1\}} x) \nu(dx) = r,$$

where r is the return rate of the save asset. Then we have the following ([7]).

Theorem Assume that there exists a constant θ^* satisfying the conditions (C1) and (C2), then
(1) there exists the MEMM P^* ,

(2) X_t is again Lévy process under P^* with the generating triplet $(\sigma^{*2}, \nu^*(dx), b^*)$ such that

$$\begin{aligned} \sigma^{*2} &= \sigma^2, \\ \nu^*(dx) &= e^{\theta^*(e^x-1)} \nu(dx), \\ b^* &= b + \theta^* \sigma^2 + \int_{\{|x| \leq 1\}} x d(\nu^* - \nu). \end{aligned}$$

Especially, if the Lévy measure $\nu(dx)$ satisfies the following condition:

$$(3) \quad \int_{\{|x| \leq 1\}} x d\nu(dx) < \infty,$$

then the generating triplet is of the form $(\sigma, \nu(dx), b_0)$, and furthermore the condition (C2) turns into the following:

$$(C3) \quad f(\theta^*) \equiv b_0 + \left(\frac{1}{2} + \theta^*\right) \sigma^2 + \int_{R \setminus \{0\}} (e^x - 1) e^{\theta^*(e^x-1)} \nu(dx) = r.$$

It is easy to see that the function $f(\theta)$ is monotone non-decreasing and $f(0) \geq r$ due to the reasonable reason in the financial market, so that θ^* is nonpositive if it exists. Moreover, we can prove that there exist negative θ^* 's for many typical Lévy processes such as Brownian motion, compound Poisson, stable, VG, CGMY, NIG, etc.([7]).

3 Numerical analysis

Since (C2) and (C3) are nonlinear integral equations with respect to θ^* , it is difficult to solve them precisely. Instead, in the following, we are intend to get approximate solutions of θ^* satisfying (C2) or (C3) for typical $\nu(dx)$'s by using the numerical analysis. We will use here two well known methods, an iteration method and the Newton method([1]). Assume that $\sigma \neq 0$, then in order to solve approximately Eq.(C2) or Eq.(C3), we will apply an iteration method, i.e., define $\theta_n, n = 1, 2, \dots$, by the formula:

$$(4) \quad \theta_{n+1} = \frac{r - f(\theta_n) + \sigma^2 \theta_n}{\sigma^2}$$

In the case where $\sigma = 0$, the Newton method will be applied to solve the Eq.(C2), i.e., define $\theta_n, n = 1, 2, \dots$, by the formula:

$$(5) \quad \theta_{n+1} = \theta_n - \frac{f(\theta_n) - r}{f'(\theta_n)}.$$

Our methodology of evaluating θ^* has the following two stages. Firstly we transform $f(\theta)$ into suitable form to use the Monte Carlo method and secondly we apply an iteration method or the Newton one.

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4 Geometric Lévy Processes

4.1 Compound Poisson process

The Lévy measure of the compound Poisson is given by $\nu(dx) = \lambda F(dx)$, where the parameter λ represents the frequency of jumps and $F(dx)$ is the probability distribution of the jump width. In this case, $\nu(dx)$ satisfies the condition (3) so that from (C3), $f(\theta)$ is given by the following.

$$f(\theta) = b_0 + \left(\frac{1}{2} + \theta\right) \sigma^2 + \int \lambda (e^x - 1) e^{\theta(e^x-1)} F(dx)$$

Before solving the equation $f(\theta) = r$, note that the integral part of $f(\theta)$ is equal to the following:

$$\int \lambda (e^x - 1) e^{\theta(e^x-1)} F(dx) = \lambda E_F \left[(e^x - 1) e^{\theta(e^x-1)} \right],$$

where $E_F[\cdot]$ denotes the mathematical expectation with respect to $F(dx)$. It follows that we can obtain sufficiently many random samples having the distribution $F(dx)$ so as to compute the integral approximately. Assume that $\sigma^2 \neq 0$ and define θ_n by (4) as follows:

$$\theta_{n+1} = \frac{r - b_0 - \sigma^2/2 - \lambda E_F \left[(e^x - 1) e^{\theta_n(e^x-1)} \right]}{\sigma^2},$$

then we can get an approximate solution of Eq.(C3) by using the Monte Carlo method (cf.[5]).

Suppose $F(dx)$ is the normal distribution with mean μ and variance η^2 . We will give some simulation results in Table 1.

Table 1 Compound Poisson Process

r	b_0	σ^2	λ	μ	η^2	θ^*	$ g' $
0.040	1.150	0.616	0.211	0.25	2.250	-1.957	0.176
0.040	0.015	0.862	0.771	0	0.203	-0.475	0.155

It is obvious that θ^* is negative and $|g'| < 1$, where $g(\theta) = (r - f(\theta) + \sigma^2 \theta) / \sigma^2$

4.2 VG(Variance Gamma) process

The Lévy measure of the VG process is given by the following.

$$\nu(dx) = C(I_{\{x < 0\}} e^{-c_1|x|} + I_{\{x > 0\}} e^{-c_2x}) |x|^{-1} dx,$$

where C, c_1, c_2 are positive constants. In this case, the condition (3) is again satisfied so that $f(\theta)$ is represented as follows:

$$f(\theta) = b_0 + \left(\frac{1}{2} + \theta\right)\sigma^2 + C \int_0^\infty \frac{e^{-c_1 x}}{x} (e^{-x} - 1) e^{\theta(e^{-x}-1)} dx + C \int_0^\infty \frac{e^{-c_2 x}}{x} (e^x - 1) e^{\theta(e^x-1)} dx.$$

Then we obtain the following results ([7]):

1. if $c_2 \leq 1$, then the equation $f(\theta) = r$ has a unique negative solution,
2. if $c_2 > 1$ and $f(0) \geq r$, then the equation $f(\theta) = r$ has a unique nonpositive solution,
3. if $c_2 > 1$ and $f(0) < r$, then the equation $f(\theta) = r$ has no solution.

It is easy to see that the integral parts of $f(\theta)$ can be written as follows:

$$\begin{aligned} \int_0^\infty \frac{e^{-c_1 x}}{x} (e^{-x} - 1) e^{\theta(e^{-x}-1)} dx \\ = E_{c_1} \left[\frac{(e^{-x} - 1)}{c_1 x} e^{\theta(e^{-x}-1)} \right], \\ \int_0^\infty \frac{e^{-c_2 x}}{x} (e^x - 1) e^{\theta(e^x-1)} dx \\ = E_{c_2} \left[\frac{(e^x - 1)}{c_2 x} e^{\theta(e^x-1)} \right], \end{aligned}$$

where $E_{c_1}[\cdot]$ ($E_{c_2}[\cdot]$ resp.) denotes the mathematical expectation with respect to the exponential distribution with parameter c_1 (c_2 resp.). Assume that $\sigma^2 \neq 0$ and define θ_n by (4), so that

$$\begin{aligned} \theta_{n+1} = \frac{r - b_0 - \sigma^2/2 - C E_{c_1} \left[\frac{(e^{-x}-1)}{c_1 x} e^{\theta_n(e^{-x}-1)} \right]}{\sigma^2} \\ - \frac{C E_{c_2} \left[\frac{(e^x-1)}{c_2 x} e^{\theta_n(e^x-1)} \right]}{\sigma^2} \end{aligned}$$

Thus, in the case of 1 or 2, we can get an approximate solution of the equation $f(\theta) = r$ in the similar way to §4.1. We will give some simulation results in Table 2.

Table 2 VG process

r	b_0	σ^2	C	c_1	c_2	θ^*	$ g' $
0.040	1.120	0.448	3.400	2.478	0.826	-0.197	0.707
0.040	0.712	0.648	0.274	0.758	2.226	-0.960	0.316

4.3 Stable process

The Lévy measure $\nu(dx)$ of the stable process is given by the formula:

$$\nu(dx) = \begin{cases} c_1 \frac{1}{|x|^{1+\alpha}} dx & x < 0 \\ c_2 \frac{1}{x^{1+\alpha}} dx & x > 0 \end{cases},$$

where $0 < \alpha < 2$ and $c_1, c_2 \geq 0$, but at least either c_1 or c_2 is positive. It follows from (C2) that

$$\begin{aligned} f(\theta) = b + \left(\frac{1}{2} + \theta\right)\sigma^2 + \int_1^\infty \left[\frac{c_1(e^{-x} - 1)}{x^{1+\alpha}} e^{\theta(e^{-x}-1)} \right. \\ \left. + \frac{c_2(e^x - 1)}{x^{1+\alpha}} e^{\theta(e^x-1)} \right] dx + \int_0^1 \left[\frac{c_1}{x^{1+\alpha}} [(e^{-x} - 1) \right. \\ \left. e^{\theta(e^{-x}-1)} + x] + \frac{c_2}{x^{1+\alpha}} [(e^x - 1) e^{\theta(e^x-1)} - x] \right] dx. \end{aligned}$$

Since $f(\theta)$ is a monotone non-decreasing function of θ and

$$\lim_{\theta \rightarrow -\infty} f(\theta) = -\infty, \quad \lim_{\theta \rightarrow 0} f(\theta) = \infty,$$

there exists a unique negative solution of the equation $f(\theta) = r$ ([7]). Denote by $h_1(\theta)$ ($h_2(\theta)$ resp.) the first integral (the second one resp.) of the right side of $f(\theta)$. Since $h_1(\theta)$ is on the infinite interval $[1, \infty)$, we need some techniques to handle it. For this purpose, let $x = 1 + y^2$ ($y > 0$), then $h_1(\theta)$ is represented as follows:

$$\begin{aligned} h_1(\theta) = E_{[1, \infty)} \left[\frac{2y}{k} \left[c_1 (e^{-1-y^2} - 1) e^{\theta(e^{-1-y^2}-1)} \right. \right. \\ \left. \left. + c_2 (e^{1+y^2} - 1) e^{\theta(e^{1+y^2}-1)} \right] \right], \end{aligned}$$

where, $E_{[1, \infty)}[\cdot]$ denotes the mathematical expectation of the generalized Cauchy distribution with the density function $k(1 + y^2)^{-1-\alpha}$ in $[0, \infty)$, and $k = \Gamma(1 + \alpha)/(\sqrt{\pi}\Gamma(\alpha + 1/2))$ ($\Gamma(\cdot)$ is the Gamma function). Thus the formula (4) can be written as follows:

$$\theta_{n+1} = \frac{r - b - \sigma^2/2 - h_1(\theta_n) - h_2(\theta_n)}{\sigma^2},$$

from which we can get an approximate solution by means of the Monte Carlo method.

We will give the simulation results in Table 3.

Table 3 Stable process

r	b	σ^2	α	c_1	c_2	θ^*	$ g' $
0.040	0.592	0.344	1.226	0.020	0.119	-1.362	0.720
0.040	0.772	0.874	0.826	0.121	0.158	-0.779	0.868

4.4 CGMY(Carr-Geman-Madan-Yor) model

The Lévy measure of the CGMY model is given by the following.

$$\nu(dx) = C(I_{\{x<0\}}e^{-G|x|} + I_{\{x>0\}}e^{-M|x|})|x|^{-(1+Y)}dx,$$

where $C > 0, M \geq 0$ and $Y < 2$. When $Y \leq 0, G > 0$ and $M > 0$ are assumed. If $Y = 0$, then the CGMY model is the VG process, and furthermore if $G = M = 0$ and $0 < Y < 2$, then it is the symmetric stable process. From the condition (C2), $f(\theta)$ is represented as follows:

$$\begin{aligned} f(\theta) &= b + \left(\frac{1}{2} + \theta\right)\sigma^2 \\ &+ C \int_0^\infty \frac{e^{-Mx}}{x^{1+Y}} \left((e^x - 1)e^{\theta(e^x-1)} - \mathbf{1}_{x \in [0,1]}x \right) dx \\ &+ C \int_0^\infty \frac{e^{-Gx}}{x^{1+Y}} \left((e^{-x} - 1)e^{\theta(e^{-x}-1)} + \mathbf{1}_{x \in [0,1]}x \right) dx. \end{aligned}$$

It is shown ([7]) that the function $f(\theta)$ has the following properties:

1. if $M \leq 1$, then the equation $f(\theta) = r$ has a unique negative solution,
2. if $M > 1$ and $f(0) \geq r$, then the equation $f(\theta) = r$ has a unique nonpositive solution,
3. if $M > 1$ and $f(0) < r$, then the equation $f(\theta) = r$ has no solution.

By the same reason as the VG process in §4.2, we obtain the following formula.

$$\begin{aligned} f(\theta) &= b + \left(\frac{1}{2} + \theta\right)\sigma^2 \\ &+ CE_M \left[\frac{(e^x - 1)e^{\theta(e^x-1)} - \mathbf{1}_{x \in [0,1]}x}{Mx^{1+Y}} \right] \\ &+ CE_G \left[\frac{(e^{-x} - 1)e^{\theta(e^{-x}-1)} + \mathbf{1}_{x \in [0,1]}x}{Gx^{1+Y}} \right], \end{aligned}$$

where $E_G[\cdot]$ ($E_M[\cdot]$ resp.) means the mathematical expectation with respect to the exponential distribution with the parameter G (M resp.).

Thus the formula (4) can be written as follows:

$$\begin{aligned} \theta_{n+1} &= \frac{r - b - \sigma^2/2}{\sigma^2} \\ &- \frac{1}{\sigma^2} CE_M \left[\frac{(e^x - 1)e^{\theta_n(e^x-1)} - \mathbf{1}_{x \in [0,1]}x}{Mx^{1+Y}} \right] \\ &- \frac{1}{\sigma^2} CE_G \left[\frac{(e^{-x} - 1)e^{\theta_n(e^{-x}-1)} + \mathbf{1}_{x \in [0,1]}x}{Gx^{1+Y}} \right]. \end{aligned}$$

It is not difficult to get an approximate solution by using the Monte Carlo method. We will give some simulation results in Table 4.

Table 4 CGMY model

r	b	σ^2	C	M	G	Y	θ^*	$ g' $
0.040	0.400	0.449	0.33	0.741	1.256	1.225	-0.919	0.313
0.040	0.638	0.668	0.771	1.256	1.361	0.565	-1.252	0.102

4.5 NIG (Normal Inverse Gaussian) model

The Lévy measure of the NIG model is given by the following ([8]).

$$\nu(dx) = \delta\alpha(\pi|x|)^{-1} \exp(\beta x) K_1(\alpha|x|),$$

where $\alpha, \delta > 0, |\beta| \leq \alpha, x \in \mathbb{R}$. Moreover, $K_1(z)$ ($z > 0$) is the modified Bessel function of second kind of order 1 such that

$$K_1(z) = \sqrt{\frac{\pi}{2z}} e^{-z} (1 + O(\frac{1}{z})) \quad (z \rightarrow \infty),$$

$$K_1(z) = z^{-1} (1 + O(z^2 \log z)) \quad (z \rightarrow 0)$$

The function $f(\theta)$ in (C2) is written as follows:

$$\begin{aligned} f(\theta) &= b + \left(\frac{1}{2} + \theta\right)\sigma^2 + \int_0^\infty \frac{\delta\alpha e^{-(\alpha+\beta)x}}{\pi x e^{-\alpha x}} K_1(\alpha x) \\ &[(e^{-x} - 1)e^{\theta(e^{-x}-1)} + \mathbf{1}_{x \leq 1}x] dx + \int_0^\infty \frac{\delta\alpha e^{-(\alpha-\beta)x}}{\pi x e^{-\alpha x}} \\ &K_1(\alpha x) [(e^x - 1)e^{\theta(e^x-1)} - \mathbf{1}_{x \leq 1}x] dx. \end{aligned}$$

Then we can show the following.

1. if $\alpha - \beta < 1$, then equation $f(\theta) = r$ has a unique negative solution,
2. if $\alpha - \beta \geq 1$ and $f(0) \geq r$, then the equation $f(\theta) = r$ has a unique nonpositive solution,
3. if $\alpha - \beta \geq 1$ and $f(0) < r$, then the equation $f(\theta) = r$ has no solution.

Put

$$h_1 = E_{\alpha+\beta} \left[\frac{\delta \alpha e^{\alpha x} K_1(\alpha x)}{\pi x (\alpha + \beta)} [(e^{-x} - 1) e^{\theta(e^{-x} - 1)} + 1_{x \leq 1}] \right],$$

and

$$h_2 = E_{\alpha-\beta} \left[\frac{\delta \alpha e^{\alpha x} K_1(\alpha x)}{\pi x (\alpha - \beta)} [(e^x - 1) e^{\theta(e^x - 1)} - 1_{x \leq 1}] \right],$$

where, $E_{\alpha+\beta}[\cdot]$ ($E_{\alpha-\beta}[\cdot]$ resp.) means the mathematical expectation of the exponential distribution with the parameter $\alpha + \beta$ ($\alpha - \beta$ resp.). By the same reason as §4.2 or §4.4, in this case the formula (4) can be written as follows:

$$\theta_{n+1} = \frac{r - b - \sigma^2/2 - h_1 - h_2}{\sigma^2},$$

from which we can get an approximate solution by using the Monte Carlo method. We will give some simulation results in Table 5.

Table 5 NIG model

r	b	σ^2	δ	α	β	θ^*	$ g' $
0.040	0.677	0.661	0.446	1.441	0.772	-1.756	0.371
0.040	0.677	0.613	0.175	1.721	-0.271	-3.604	0.151

5 Option pricing

In this section, using the NIKKEI Average Index Weekly Data of 54 weeks from September 4, 2006 to September 9, 2007, we shall evaluate the prices of European call option and Asian call option based on our previous four asset models, i.e. (geometric) Brownian motion(BS), (geometric) compound Poisson process with the normal jump(NJ), (geometric) compound Poisson process with double exponential distribution(DEJ) (cf.[4]) and (geometric) stable process(Stable).

It is well known that the prices of European option $V_E(0)$ and Asian option $V_A(0)$ are obtained as the conditional expectation under the risk neutral probability ([7]) and they are given by the formulae:

$$(6) \quad V_E(0) = e^{-rT} E_{P^*} [\max(S(T) - K, 0)],$$

$$(7) \quad V_A(0) = e^{-rT} E_{P^*} [\max(\frac{1}{T} \int_0^T S(t) dt - K, 0)],$$

where $S(t)$, P^* , E_{P^*} , T and K mean the stock

price at time $t \in [0, T]$, the MEMM, the mathematical expectation with respect to the MEMM, the maturity of the option, the strike price, respectively, and moreover $\max(a, b) = a \vee b$.

Our methodology of valuation is the following. Firstly, we estimate the parameters of four geometric asset models from the data. For the compound Poisson processes, we use the estimation method of [6], on the other hand, we rely upon [3] for the stable process. Secondly, using these estimated parameters we can get the MEMM in the same manner as §4. Finally we evaluate the prices of European call option and Asian call option due to (6) and (7), respectively.

Figure 1 illustrates the graph of NIKKEI Average Index data from September 4, 2006 to September 9, 2007.

Figure 1 NIKKEI Average Index

In Table 6 are given the estimated θ^* 's of the MEMM of the three models by means of (4) or (5).

Table 6 MEMM result

Parameter	NJ	DEJ	Stable
θ^*	-0.2001	-0.3231	-0.4495

Here, we used an iteration method to calculate the θ^* for the compound Poisson processes under the hypothesis that $\sigma^2 \neq 0$, whereas for the stable process, the Newton method was applied under the hypothesis that $\sigma^2 = 0$. We should notice in Table 6 that $\theta^* < 0$ in every case, which assures the existence of the MEMM. By using these results and the formulae (6) and (7), the evaluated European and Asian option prices of four models for various maturities and strike prices are given in Table 7 and Table 8.

Table7 European Call Option

$T(\text{week})$	K	NJ	DEJ	BS	Stable
2	16000	32.978	24.455	133.053	0.945
6	13000	2048.923	1997.305	2885.474	2688.297
6	14500	627.783	570.611	1397.448	1152.518
6	16000	27.074	16.445	269.869	9.943
6	20000	0	0	0.0006	0.539

Table8 Asian Call Option

$T(\text{week})$	K	NJ	DEJ	BS	Stable
2	16000	0	0	0	1.294
3	14500	1182.493	1163.429	1390.361	1326.673
2	13000	2740.499	2727.081	2878.412	2855.282
6	12000	3469.372	3435.736	3885.736	3811.953
6	14500	965.521	936.799	1386.892	1279.224

Here, we assume that $r = 0.001$ and $S_0 = 15877.67$. It is easily seen in these tables that (i) the price of BS is higher than that of other models for each T in most cases except the extreme case, (ii) the prices of NJ and DEJ are nearly equal, (iii) those of BS and Stable are close in some cases, and (iv) the price of the Asian is "stable" compared with the European. It seems that the above features are closely related with the volatility smile of the BS model and the "fat-tailness" of the Lévy process with jumps.

6 Concluding Remarks

In this paper, we presented a very useful methodology to obtain concretely the MEMM of geometric Lévy processes such as compound Poisson and stable, and in addition, estimated them in terms of an iteration method. In every case (§4.1 ~ §4.5), the evaluated value of θ^* is negative and $|g'| < 1$ so that there exists a unique solution of Eq.(4) due to the general theory of iteration method, which also proves that our method is justified and effective. Moreover, we employed some techniques in order to calculate the integrals over infinite intervals with respect to the Lévy measure so as to apply the Monte Carlo method. In §5 we made use of the Newton method under the hypothesis that $\sigma^2 = 0$ for the stable process, because it has infinitely many but small jumps in any finite interval. For the compound Poisson processes we assumed that $\sigma^2 \neq 0$, and then applied an iteration method to obtain the MEMM.

In §5 of this paper, we evaluated the prices of European call option and Asian one based on four

asset models. It seems that the valuation results are very good fit to the empirical option prices of the Nikkei data.

The computation time of an iteration method is very short. For example, in the case of the stable process, we approximate the mathematical expectation with 20,000 samples, the computation time is about 6 seconds with CPU-Pentium4 2.66GZ computer. Moreover, it is a big advantage that the structure of an iteration method is simple and self-contained.

There are another methods of option valuation in terms of FFT(fast Fourier transform), Esscher transform, etc. (see [4],[9], e.g.).

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