Identification of Unknown Exogenous Input to Stochastic Linear Systems via Pseudomeasurement Approach

AKIRA OHSUMI,* TAKURO KIMURA,** and MICHIO KONO**

* Graduate School of Engineering,

** Interdisciplinary Graduate School of Agriculture and Engineering,
University of Miyazaki, Kibana, Gakuen, Miyazaki 889-2192, Japan

E-mail: {ohsumi, kouno}@cs.miyazaki-u.ac.jp

Abstract

The pseudomeasurement approach which was developed by the authors is applied to identify the exogenous input whose form is quite unknown.

1. Introduction

The idea of pseudomeasurement was introduced in system identification and its usefulness has been demonstrated by simulation studies for identifying unknown stepwise and impulsive exogenous inputs by the authors [1]–[5]. The *pseudomeasurement* used in system identification is the additional information about the unknown parameter vector contained in the dynamical system model, and this is augmented with the original observation data. The unknown parameter vector is estimated by constructing the augmented state vector and using the Kalman filter or the state observer according to the problem situation. The identification problems of the magnitude and the discharge time of stepwise or impulsive exogenous input are investigated in [2], [4], [5] for stochastic systems and in [3] for deterministic systems.

In this paper we consider a class of linear stochastic systems subjected to some unknown exogenous input. Different from such exogenous inputs considered in [2]–[5], we assume that the form of exogenous input vector is quite unknown except its dimension.

In the practical systems, a complete knowledge of the system input is unavailable. Examples of these include aircraft tracking systems where aircraft control inputs are unavailable as inputs to an observer-based tracker [6] or environmental systems that is subjected to sudden pollutive load by unlawful discharge or unforeseen accident [7]. This

paper extends the method proposed in [2]–[5] to the case of completely unknown exogenous input.

2. Identification Problem

Consider an identification problem for the stochastic system which is described by Itô stochastic differential equations:

$$dx(t) = Ax(t) dt + B\xi(t) dt + Cu(t) dt + G dw(t), \quad x(0) = x_0 \quad (1)$$

$$dy(t) = Hx(t) dt + R dv(t), \quad (2)$$

where $x(t) \in R^n$, $y(t) \in R^m$ ($m \leq n$), $u(t) \in R^\ell$; $w(t) \in R^{d_1}$ and $v(t) \in R^{d_2}$ are mutually independent Wiener processes with covariance matrices Q and I (unit matrix); and $\xi(t) \in R^q$ is the unknown exogeneous input to be identified.

Our problem is to identify the unknown exogenous input vector $\xi(t)$ as well as the system state $x(t)$ from a couple of input and output data $\{u(t), y(t)\}$. All matrices such as A , B and so on are time-invariant. The pair (A, H) is assumed to be observable.

In the event of being no a priori information available about the exogenous input $\xi(t)$, it may be reasonable to assume it as a polynomial model of order p

$$\xi(t) = \sum_{k=0}^p b_k \phi_k(t) \quad (3)$$

in which $\{\phi_k(t)\}$ is a set of (scalar) basis functions of time t , while $\{b_i\}$ is a set of unknown constant vectors, $b_k \in R^q$. The model (3) can be expressed as

$$\xi(t) = \Phi(t) \theta, \quad (4)$$

where $\theta = [b_0^T, b_1^T, \dots, b_p^T]^T \in R^{(p+1)q}$ (T denotes transpose), $\Phi(t) = [\phi_0(t)I_q, \phi_1(t)I_q, \dots, \phi_p(t)I_q] \in R^{q \times (p+1)q}$ (I_q : unit matrix of dimension q) or using the Kronecker product,

$$\Phi(t) = \phi^T(t) \otimes I_q, \quad (5)$$

(the symbol $\otimes$ is the Kronecker product), and $\phi(t) = [\phi_0(t), \phi_1(t), \dots, \phi_p(t)]^T$.

The conventional approach to identify the unknown constant vector θ can be stated as follows. For the stochastic system (1), it is a custom to use the expression for θ as $\theta(t)$ and to write as

$$\dot{\theta}(t) = G_\theta \gamma_\theta(t), \quad \theta(0) = \theta_0$$

or in the stochastic differential form,

$$d\theta(t) = G_\theta dw_\theta(t), \quad \theta(0) = \theta_0, \quad (6)$$

where $\gamma_\theta(t)$ is the $(p+1)q$ -vector zero-mean Gaussian random process and $w_\theta(t)$ is the corresponding Wiener process, viz., $w_\theta(t) = \int_0^t \gamma_\theta(\tau) d\tau$, with covariance matrix Q_θ . The initial value θ_0 is pre-assigned and may be set as $\theta_0 = 0$.

Then, augmenting (6) with the system dynamics (1), we have for the augmented state vector $z(t) = [x^T(t), \theta^T(t)]^T \in R^{n+(p+1)q}$:

$$\begin{aligned} dz(t) &= A_0(t)z(t) dt + C_0 u(t) dt \\ &\quad + G_0 dw_0(t), \end{aligned} \quad (7)$$

where $w_0(t) = [w^T(t), w_\theta^T(t)]^T$, and

$$\begin{aligned} A_0(t) &= \begin{bmatrix} A & B\Phi(t) \\ 0 & 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} C \\ 0 \end{bmatrix}, \\ G_0 &= \begin{bmatrix} G & 0 \\ 0 & G_\theta \end{bmatrix}. \end{aligned}$$

The observation process is rewritten in terms of $z(t)$ as

$$dy(t) = H_\theta z(t) dt + R dv(t) \quad (8)$$

with $H_\theta = [H \ 0]$.

The identification of θ may be performed by the Kalman filter constructed for (7) and (8). Unfortunately, this approach dooms to yield poor and unsatisfactory identification results in θ . Unsuccessful results in the conventional approach are caused entirely by the lack of information about the unknown exogenous input $\xi(t)$ or the parameter vectors $\{b_i\}$.

3. Introduction of Pseudomeasurement

How can we obtain any information about the unknown parameter vector θ ? Up to here we have an only information that it is constant. In order to get more information about θ we make the most of system dynamics.

To do this, consider the solution of (1),

$$\begin{aligned} x(t) &= e^{At} x(0) + \left[\int_0^t e^{A(t-\tau)} B \Phi(\tau) d\tau \right] \theta \\ &\quad + \int_0^t e^{A(t-\tau)} C u(\tau) d\tau + \{\text{noise}\} \end{aligned} \quad (9)$$

in which the term concerning to the system noise is simply denoted by $\{\text{noise}\}$ because this is not essential to get direct relation between θ and $x(t)$. In principle, we could determine θ from this if the state $\{x(\cdot)\}$ is completely known (except $\{\text{noise}\}$). However, the true state is not available. Keep in mind that only the couple of control input and observation data $\{u(t), y(t)\}$ are available. So, in order to get the relation between θ and $\{y(t)\}$, upon-multiplying both sides of (9) by the matrix $H dt$ to obtain

$$\begin{aligned} Hx(t) dt &= \left[\int_0^t H e^{A(t-\tau)} B \Phi(\tau) d\tau \right] \theta dt \\ &\quad + H e^{At} x(0) dt \\ &\quad + \left[\int_0^t H e^{A(t-\tau)} C u(\tau) d\tau \right] dt + \{\text{noise}\} \end{aligned} \quad (10)$$

in which the L.H.S. should be equal to $dy(t) - R dv(t)$. Therefore, we have the relation between θ and the observation data,

$$\begin{aligned} dy(t) &= \left[\int_0^t H e^{A(t-\tau)} B \Phi(\tau) d\tau \right] \theta dt \\ &\quad + \left[\int_0^t H e^{A(t-\tau)} C u(\tau) d\tau \right] dt + \{\text{noise}\} \end{aligned} \quad (11)$$

in which the $\{\text{noise}\}$ term reflects now the collection of system noise, observation noise and the random initial state $x(0)$. The term $H e^{At} x(0)$ becomes disappearing as time passes as far as the system matrix A is stable. So, this term is regarded and included in the $\{\text{noise}\}$ term.

Subtracting the second term in the R.H.S. from

both sides of (11), we have

$$\begin{aligned} dy(t) - \left[\int_0^t H e^{A(t-\tau)} C u(\tau) d\tau \right] dt \\ = \left[\int_0^t H e^{A(t-\tau)} B \Phi(\tau) d\tau \right] \theta dt \\ + \{\text{noise}\}. \end{aligned} \quad (12)$$

Note that the L.H.S. is a known quantity, and hence let it $dy_p(t)$, i.e.,

$$\begin{aligned} dy_p(t) = dy(t) \\ - \left[\int_0^t H e^{A(t-\tau)} C u(\tau) d\tau \right] dt. \end{aligned} \quad (13)$$

Then, (12) can be rewritten, denoting $\theta = \theta(t)$ as

$$dy_p(t) = H_p(t) \theta(t) dt + R_p dv_p(t), \quad (14)$$

where

$$H_p(t) = \int_0^t H e^{A(t-\tau)} B \Phi(\tau) d\tau, \quad (15)$$

and the term $R_p dv_p(t)$ is introduced with a newly introduced m -vector standard Wiener process $v_p(t)$ to describe the $\{\text{noise}\}$ term.

At this stage, it should be emphasized that (14) can be regarded just as an observation model for the state $\theta(t)$. In this sense, the newly defined $dy_p(t)$ is called the pseudomeasurement for unknown parameter vector θ . Clearly, $\{dy_p(t)\}$ is $\mathcal{Y}_t$ -measurable ($\mathcal{Y}_t = \sigma\{y(\tau), 0 \leq \tau \leq t\}$).

Here, it is noted that we may have another version of (14). From (11) we have formally

$$\begin{aligned} \theta(t) dt = H_p^\dagger(t) \left\{ dy(t) \right. \\ \left. - \left[\int_0^t H e^{A(t-\tau)} B \Phi(\tau) d\tau \right] dt \right\} + \{\text{noise}\}, \end{aligned} \quad (16)$$

where $H_p^\dagger(t)$ is the pseudoinverse of $H_p(t)$ and is given by a modified form from the Moor-Penrose generalized inverse as

$$H_p^\dagger(t) = H_p^T(t) [H_p(t) H_p^T(t) + \varepsilon I_m]^{-1} \quad (17)$$

in which εI_m (ε : small positive constant) is introduced to prevent the divergence of the inverse matrix near $t = 0$. If we define the first term in the R.H.S. of (16) as the pseudomeasurement $dy_p(t)$, i.e.,

$$\begin{aligned} dy_p(t) = H_p^\dagger(t) \left\{ dy(t) \right. \\ \left. - \left[\int_0^t H e^{A(t-\tau)} B \Phi(\tau) d\tau \right] dt \right\}, \end{aligned} \quad (18)$$

Then we have, instead of (14), the following version,

$$dy_p(t) = \theta(t) dt + R_p dv_p(t). \quad (19)$$

This form of pseudomeasurement was extensively used in [2]-[5]. However, in this case the generation of pseudomeasurement needs the computation of inverse matrix besides the pseudoinverse. Which form of (14) or (19) is useful depends on the relevant case and it will be the user's choice.

4. Identification of Unknown Input

Augmenting the pseudomeasurement (14) with the original observation process to form the vector $y_0(t) = [y^T(t), y_p^T(t)]^T \in R^{2m}$, we obtain the augmented observation process:

$$dy_0(t) = H_0(t) z(t) dt + R_0 dv_0(t) \quad (20)$$

where

$$H_0(t) = \begin{bmatrix} H & 0 \\ 0 & H_p(t) \end{bmatrix}$$

$v_0(t) = [v^T(t), v_p^T(t)]^T$, and $R_0 = \text{block diag } \{R, R_p\}$.

Then, we have the following new time-varying system formulation:

$$\left. \begin{aligned} dz(t) &= A_0(t) z(t) dt + C_0 u(t) dt \\ &\quad + G_0 dw_0(t) \\ dy_0(t) &= H_0(t) z(t) dt + R_0 dv_0(t). \end{aligned} \right\} \quad (21)$$

Based on (21), the Kalman filter can be constructed to obtain the estimate of b_i ($= b_i(t)$):

$$\begin{aligned} d\hat{z}(t|t) &= A_0(t) \hat{z}(t|t) dt + C_0 u(t) dt \\ &\quad + P(t|t) H_0^T(t) \{R_0 R_0^T\}^{-1} \{dy_0(t) \\ &\quad - H_0(t) \hat{z}(t|t) dt\} \end{aligned} \quad (22)$$

$$\begin{aligned} \dot{P}(t|t) &= A_0(t) P(t|t) + P(t|t) A_0^T(t) + G_0 Q_0 G_0^T \\ &\quad - P(t|t) H_0^T(t) \{R_0 R_0^T\}^{-1} H_0(t) P(t|t), \end{aligned} \quad (23)$$

where $Q_0 = \text{block diag } \{Q, Q_b\}$.

Fig. 1. Behaviour of identified components $\{b_{ik}\}$ ($i = 1, 2; k = 0, 1, 2, 3$).

5. Simulation Studies

The following third-order system is considered in which the input $u(t)$ is assumed to be zero, i.e., $u(t) = 0$ for all t :

$$dx(t) = Ax(t)dt + B\xi(t)dt + Gdw(t), \quad (24)$$

where $w(t)$ is a noise process with variance $Q = \text{diag}\{90, 120, 105\}$, and

Fig. 2. Estimations of $x_1(t)$ (top), $x_2(t)$ (middle) and $x_3(t)$ (bottom).

$$A = \begin{bmatrix} -5 & 5 & 10 \\ 0 & -10 & 5 \\ 5 & 0 & -15 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4.5 \end{bmatrix}.$$

The true exogenous input $\xi(t) \in R^2$ was assumed as follows:

$$\xi_1(t) = 3 + \frac{3}{2}t - \frac{1}{2}t^2 - \frac{1}{4}t^3 + \frac{1}{24}t^4 + \frac{1}{80}t^5 - \frac{1}{720}t^6$$

$$\xi_2(t) = 3 + t - \frac{3}{8}t^2 - \frac{1}{9}t^3 + \frac{1}{32}t^4 + \frac{1}{120}t^5 - \frac{1}{960}t^6.$$

Fig. 3. Identification features of $\xi_1(t)$ (top) and $\xi_2(t)$ (bottom).

Fig. 4. Unsuccessful results of identification $\xi_1(t)$ (top) and $\xi_2(t)$ (bottom) by the conventional method based on (7) and (8).

For the observation process, $y(t) \in R^2$, the ma-

trices was set as

$$H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

For identifying the unknown input $\xi(t)$, it is assumed that

$$\xi_i(t) = \sum_{k=0}^3 b_{ik} \phi_k(t) \quad (i = 1, 2).$$

As for the basic functions $\phi_k(t)$, although there are many possibilities in selecting them, we select the following simple polynomial of order $p = 3$:

$$\phi_k(t) = (t - t_i)^k \quad \text{for } t_i \leq t < t_{i+1}, \quad (25)$$

where $\{t_i\}$ are equi-partitioned times such that $t_0 = 0$ and $t_{i+1} = t_i + 1$ [sec] ($i = 0, 1, 2, \dots$).

Then, these are produced as the outputs of

$$\begin{bmatrix} \dot{\phi}_0(t) \\ \dot{\phi}_1(t) \\ \dot{\phi}_2(t) \\ \dot{\phi}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} \phi_0(t) \\ \phi_1(t) \\ \phi_2(t) \\ \phi_3(t) \end{bmatrix}$$

or

$$\dot{\phi}(t) = E\phi(t),$$

$$\phi_0(0) = 1, \quad \phi_k(0) = 0 \quad (k = 1, 2, 3),$$

where $\phi(t) = [\phi_0(t), \phi_1(t), \phi_2(t), \phi_3(t)]^T$ and $E \in R^{4 \times 4}$ is clear.

Such a polynomial as (25) is employed for an unknown input by Meditch and Hostetter [8], Mita and Sakami [9], and O'Reilly [10] to derive observers for linear systems with unknown input. Unfortunately, in these works the input model (3) with (25) is not necessarily treated directly to derive the observer, but it is used only to guarantee the smoothness of the input $\xi(t)$ up to the p th order. Here, we employ (25) and identify its unknown coefficient vectors $\{b_k\}$. In this case, the unknown vectors b_k and θ are given by

$$b_k = \begin{bmatrix} b_{1k} \\ b_{2k} \end{bmatrix} \quad (k = 0, 1, 2, 3)$$

$$\theta = [b_{10}, b_{20}, b_{11}, b_{21}, b_{12}, b_{22}, b_{13}, b_{23}]^T.$$

For the model of $\theta(t)$, the matrices concerned with the $w_\theta(t)$ -process were assumed as

$$G_\theta = \text{diag} \{0.4, 0.3, 0.1, 0.2, 0.05, 0.06, 0.003, 0.004\},$$

$$Q_\theta = \text{diag} \{8, 6, 2, 4, 1, 1.2, 0.06, 0.08\}.$$

Furthermore, as for the pseudomeasurement process (14), the matrix R_p was set as $R_p = \text{diag} \{3, 2\}$.

The initial value for the Kalman filter (22) and (23) were set as $\hat{z}(0|0) = 0$ and $P(0|0) = \text{diag} \{3, 4, 5, 1, 2, 1, 2, 1, 2, 1, 2\}$.

Figures 1, 2 and 3 are simulation results. Figure 1 depicts the behaviors of identified components $\{b_{ik}\}$, and the estimation of the states $x_i(t)$ ($i = 1, 2, 3$) and identification processes for the unknown inputs $\xi_i(t)$ ($i = 1, 2$) are illustrated in Figs. 2 and 3 in which solid curves and dotted ones depict the identification or estimation and their true values, respectively. From Figs. 2 and 3 we see that both identification and state estimation may be performed fairly well.

Figure 4 shows the result of identification of the unknown inputs $\xi_1(t)$ and $\xi_2(t)$ using the Kalman filter constructed based on the models (7) and (8) in which no idea of pseudomeasurement is used. As suspected, in this case the identification of unknown input is quite unsuccessful.

6. Conclusion

The identification of exogenous input whose form is quite unknown has been investigated based on the polynomial model. The pseudomeasurement defined in this paper is slightly different from that defined in [2]-[5]. It is defined in such a way to avoid the computation of inverse or pseudoinverse of a matrix as (14).

REFERENCES

- [1] A. Ohsumi: An Outlook on the Use of Pseudomeasurement in System Identification, *Proc. 37th SICE Symp. on Control Theory*, Kirishima, Sep. 2008, pp. 91-96.
- [2] A. Ohsumi, T. Kimura and M. Kono: Identification of Unknown Exogenous Input of Stochastic Linear Systems via Pseudomeasurement Approach, *Proc. of 39th ISCIE Int. Symp. on Stochastic Systems Theory and Its Applic.*, Saga, Nov. 2007, pp. 1-7.
- [3] T. Kimura, A. Ohsumi and M. Kono: Observer-based Identification of Unknown Exogenous Input via Pseudomeasurement Approach, *Proc. SICE Annual Conf. 2008*, Aug. 2008, pp. 2660-2665.
- [4] T. Kimura, A. Ohsumi and M. Kono: Identification of Unknown Exogenous Input to Stochastic Linear Systems Using Pseudomeasurements, *Trans. ISCIE*, vol. 21, no. 12 (2008), pp. 390-399 (in Japanese).
- [5] A. Ohsumi, T. Kimura and M. Kono: Kalman Filter-based Identification of Unknown Exogenous Input of Stochastic Linear Systems via Pseudomeasurement Approach, *Int. J. of Innovative Computing, Information and Control*, vol. 5, no. 1 (2009) (in press).
- [6] P. L. Bogler: Tracking a Maneuvering Target Using Input Estimation, *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-23 (1987), pp. 298-310.
- [7] A. Ohsumi, S. Komiyama, K. Kashiwagi, M. Watanabe, and T. Takatsu: Detection of Polluted Load and Identification of its Discharged Location and Magnitude for Polluted River, *Int. J. of Innovative Computing, Information and Control*, vol. 4, no. 1 (2008), pp. 63-77.
- [8] J. S. Meditch and G. H. Hostetter: Observers for Systems with Unknown and Inaccessible Inputs, *Int. J. Control*, vol. 19, no. 3 (1974), pp. 473-480.
- [9] T. Mita and Y. Sakami: Design of an Input ESTimator by Quasi-Observer, *Trans. SICE*, vol. 13, no. 1 (1977), pp. 1-6 (in Japanese).
- [10] J. O'Reilly: Minimal-order Observers for Linear Multivariable Systems with Unmeasurable Disturbances, *Int. J. Control*, vol. 28, no. 5, (1978), pp. 743-751.