On Noncausal $H_{\infty}$ Tracking Control for Linear-Time Continuous-Time Markovian Jump Systems

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Abstract

In this paper we study the $H_{\infty}$ tracking problems with preview for a class of linear continuous-time Markovian jump systems. The systems are described by the switching systems with Markovian mode transition. The necessary and sufficient conditions for the solvability of the $H_{\infty}$ tracking problem are given by coupled Riccati differential equations with terminal conditions. Correspondingly feedforward compensators introducing future information are given by coupled differential equations with terminal conditions. In this paper we focus on the derivation method of preview compensator dynamics from the point of view of dynamics constraint. We derive the pair of coupled preview compensator dynamics and coupled Riccati differential equations by calculating the first variation of the performance index under the dynamics constraint.

Key Words: Markovian jump systems; Noncausal tracking; Preview compensators; Coupled differential equations; Variational calculus

1 Introduction

It is well known that, for the design of tracking control systems, the preview information of reference signals is very useful for improving the performance of the closed-loop systems, and recently much work has been done for preview control systems.

Particularly, Shaked et al. have presented $H_{\infty}$ tracking control theory with preview for continuous- and discrete-time linear time-varying systems by a game theoretic approach ([2, 12]). It is also very important to consider the effects of stochastic noise or uncertainties for tracking control systems, and so, by Gershon et al., the theory of stochastic $H_{\infty}$ tracking with preview has been presented for linear continuous- and discrete-time systems respectively ([5, 6]). Recently the author has extended their theory to linear jump systems ([10, 11]). All these research results are concerned with the no mode transition cases. Tracking problems with preview for systems with mode transition are also very important issues to research.

Markovian jump systems ([1, 3, 4, 7, 13]) have abrupt random mode changes in their dynamics. The mode changes follow Markov processes. Such systems may be found in the area of power systems, manufacturing systems, communications, aerospace systems, financial engineering and so on. Such systems are classified into continuous-time ([1, 7, 13]) and discrete-time ([3, 4]) systems. The optimal and $H_{\infty}$ control theory has presented for each of these systems respectively ([3, 4, 7, 13]).

The author has presented the noncausal tracking theory for linear switched systems ([9]), which are a type of hybrid systems. However the noncausal tracking theory for the hybrid systems has not been fully investigated. Recently the author has presented the $H_{\infty}$ tracking theory with preview for linear continuous-time Markovian jump systems ([8]).

In this paper we study the $H_{\infty}$ tracking problems with preview by state feedback for a class of linear continuous-time Markovian jump systems on the finite time interval. The author has already presented the necessary and sufficient conditions for the solvability of these $H_{\infty}$ tracking problems and given the control strategies for them ([8]). The necessary and sufficient conditions for the solvability of the $H_{\infty}$ tracking problem with preview are given by coupled Riccati differential equations with terminal conditions. Correspondingly compensators introducing future information are coupled with each other. It is a very important point in the $H_{\infty}$ preview tracking theory for the Markovian jump systems. However it has not been yet given how the coupled preview feedforward compensators are derived in detail. How do we derive the form of the coupled preview compensator dynamics more directly? Why does the preview compensator dynamics have such a form?

In this paper we focus on the design method of preview compensator dynamics from the point of view of dynamics constraint. Even in the case with no mode transitions the derivation of preview compensator dynamics by variational calculus has not been fully investigated. Hence we first describe the derivation of preview compensator dynamics in the case with no mode transitions. Then we derive the pair of coupled preview compensator dynamics and coupled Riccati differential equations in the case with Markovian mode transitions.

Notations: Throughout this paper the superscript ’′” stands for the matrix transposition, $\| \cdot \|$ denotes the Euclidian vector norm and $\| u \|_R^2$ also denotes the weighted norm $\nu^TRv$. $O$ denotes the matrix with all zero components.
2 Problem formulation

Let \((\Omega, \mathcal{F}, \mathcal{P})\) be a probability space and, on this space, consider the following linear continuous-time system with reference signal and Markovian mode transitions:

\[
\dot{x}(t) = A(mt)x(t) + B_1(mt)w(t) + B_2(mt)u(t) + B_3(mt)r_c(t),
\]

\[
x(0) = x_0, \quad m_0 = i_0
\]

where \(x \in \mathbb{R}^n\) is the state, \(w \in \mathbb{R}^p\) is the exogenous disturbance, \(u \in \mathbb{R}^m\) is the control input, \(z_c \in \mathbb{R}^{k_c}\) is the controlled output, \(r_c(\cdot) \in \mathbb{R}^{k_r}\) is a known or measurable reference signal, \(x_0\) is an unknown initial state and \(i_0\) is a given initial mode.

\{m_i\} is a homogeneous Markov process with right transition probabilities:

\[
P(m_{t+\delta} = j|m_i = i) = \{ \pi_{ij} \Delta + o(\Delta) \quad i \neq j \quad 1 + \pi_{ii} \Delta + o(\Delta) \quad i = j \}
\]

where \(\pi_{ij} \geq 0\) is the transition rate from the state \(i\) to \(j\), \(i \neq j\), and \(\pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij} < 0\). Corresponding to each mode \(i\), we define \(A_i := A(mt = i)\), \(B_{2,i} := B_2(mt = i)\), \(B_{3,i} := B_3(mt = i)\), \(C_{1,i} := C(mt = i)\), \(D_{12,i} := D_{12}(mt = i)\) and \(D_{13,i} := D_{13}(mt = i)\), respectively. We assume that these matrices are constant for each \(i\). We also assume that they are of compatible dimensions.

For this system (1), we assume the following condition:

A1:

\[
D_{12}(m_i)D_{13}(m_i) > O
\]

The \(H_\infty\) tracking problems we address in this section for the system (1) are to design control laws \(u(\cdot) \in L_2[0,T]\) over the finite horizon \([0,T]\), using the information available on the known part of the reference signal \(r_c(\cdot)\) and minimizing the sum of the energy of \(z_c(\cdot)\), for the worst case of the initial condition \(x_0\) and the disturbances \(w(\cdot) \in L_2[0,T]\). Considering the average of the performance indices over the statistics of the unknown part of \(r_c\), we define the following performance index:

\[
J_T(x_0,u,w) := -\gamma^2 \alpha_0 R^{-1} x_0 - \gamma^2 \|w\|^2 + \mathbb{E}\left\{\int_0^T E_{R_{\Delta}}(\|z_c(s)\|^2) ds\right\}
\]

where \(\gamma^2 > 0\) is the attenuation factor, \(R_{\Delta}\) means expectation over \(R_{\Delta + h}, h\) is the preview length of \(r_c(\cdot)\), and \(R_{\Delta} \) denotes the future information on \(r_c\) at time \(s\), i.e., \(R_{\Delta} := \{r_c(t); s < t \leq T\}\).

We consider two types of tracking problems according to the information structures (preview lengths) of \(r_c\) as follows.

i) \(H_\infty\) Fixed-Preview Tracking:

In this case, it is assumed that, at the current time \(t\), \(r_c(s)\) is known for \(s \leq \min(T,t + h)\), where \(h\) is the preview length.

ii) \(H_\infty\) Tracking of Noncausal \((r_c(\cdot))\):

In this case, the signal \(\{r_c(\cdot)\}\) is assumed to be known \textit{a priori} for the whole time interval \(t \in [0,T]\). Notice that the type ii) is an extreme case of the type i).

In order to solve these two problems, we formulate the following differential game problem for the system (1) and the performance index (2).

The \(H_\infty\) (Game Theoretic) Tracking Problem by State Feedback:

Find \(\{u^*,\}\) and \(x_0^*\) satisfying the following (saddle point) condition:

\[
J_T(x_0,u^*,w) \leq J_T(x_0^*,u^*,w^*) \leq J_T(x_0^*,u,w^*)
\]

where the control strategy \(u^*(\cdot), 0 \leq t \leq T\), is based on the information \(R_{t+h} := \{r_c(t); 0 \leq t \leq t + h\}\) with \(0 < h < T\).

3 Design of Tracking Controllers

Now we consider the coupled Riccati equations

\[
\dot{X}_i + A_i'X_i + X_iA_i + C_{1,i}'C_{1,i} + \frac{1}{T^2}X_iB_{1,i}B_{1,i}'X_i - S_i'R_i^{-1}S_i + \sum_{j=1}^{N} \pi_{ij}X_j = 0, \quad i = 1, \cdots, N
\]

where \(R_i = D_{12,i}D_{13,i}\), \(S_i(t) = B_{2,i}'X_i(t) + D_{12,i}C_{1,i}\).

Remark 3.1. Note that these coupled Riccati equations are the same as those for the standard \(H_\infty\) disturbance attenuation problem of linear continuous-time Markovian jump systems neither considering any exogeneous reference signals nor any preview information ([19]).

The infinitesimal operator \(\mathcal{L}_u\) denotes

\[
\mathcal{L}_u \mathcal{E}_{R_{\Delta}}(V(x,i,t)) = \lim_{\Delta t \to 0^+} \frac{1}{\Delta t} \left[ \mathbb{E}[\mathcal{E}_{R_{\Delta+t}}(V(x(t + \Delta t),m_{t+\Delta t},t + \Delta t))] - \mathbb{E}[\mathcal{E}_{R_{\Delta}}(V(x(t),i,t))] \right]
\]

where

\[
\mathbb{E}[\mathcal{E}_R(\frac{\partial V}{\partial x}) + (A_{ix} + B_{1,i}w + B_{2,i}u + B_{3,i}r_c(t))\frac{\partial V}{\partial x} + \sum_{j=1}^{N} \pi_{ij}V(x,j,t)] \}
\]

for a scalar function \(V(x,i,t)\). This operator \(\mathcal{L}_u\) is also called the averaged derivative at point \((x(t),x,m_t = i, t)\) ([19]).

We obtain the following necessary and sufficient condition for the solvability of the \(H_\infty\) tracking problem by state feedback for (1) and (2) and a saddle point strategy for it.
Theorem 3.1 Consider the system (1) and the performance index (2). Suppose A1. Then the $H_\infty$ Tracking Problem by State Feedback for (1) and (2) is solvable if and only if there exist matrices $X_i(t)$, $i = 1, \ldots, N$, satisfying the conditions $X_i(0) < \gamma^2 R^{-1}$ and $X_i(T) = O$ such that the coupled Riccati equations (3) hold over $[0, T]$. Moreover a saddle point strategy for the tracking problem (1) and (2) is given by

$$x_0^* = [\gamma^2 R^{-1} - X_i(0)]^{-1} \theta_i(0)$$

$$w^* = \gamma^2 B_{1,i} X_i x + C_{\theta,i} \theta_i$$

$$u^* = -\tilde{R}^{-1}_i \tilde{S}_i x - C_{u,i} r_c - C_{\theta,i} \theta, \quad \text{for } m_t = i, \ i = 1, \ldots, N$$

where

$$C_{\theta,i} = -\gamma^2 B_{1,i}^T, \quad C_{u,i} = \tilde{R}^{-1}_i B_{2,i}$$

$$C_{\theta,i} = \tilde{R}^{-1}_i D_{12,i} D_{13,i}$$

$$\theta_i(t), \ i = 1, \ldots, N, \ t \in [0, T]$$

$$\theta_i(t) = -A_i(t) \theta_i(t) + B_i(t) r_c(t) + \sum_{j=1}^{N} \pi_{ij} \theta_j(t), \ \text{for } m_t = i, \ i = 1, \ldots, N$$

where

$$A_i = A + \frac{1}{\gamma^2} B_{1,i} B_{1,i}^T - B_{2,i} \tilde{R}^{-1}_i \tilde{S}_i$$

$$B_i = -(X_i B_{3,i} + C_{1,i} D_{13,i}) + \tilde{S}_i C_{u,i}$$

and $\theta_{ci}(t)$ is the 'causal' part of $\theta_i(t)$ at time $t$. This $\theta_{ci}(t)$ is the expected value of $\theta_i(t)$ given by

$$\hat{\theta}_{ci}(s) = -\tilde{A}_i(s) \theta_{ci}(s) + \tilde{B}_i(s) r_c(s) - \sum_{j=1}^{N} \pi_{ij} \theta_{cj}(s), \quad t \leq s \leq t_f$$

$$\theta_{ci}(t_f) = 0$$

Moreover, the value of the game is

$$J_T(x_0^*, u^*, w^*) = E \left\{ \int_0^T E_{R_c} \{ \| \tilde{R}^{1/2}_i(m_t) C_{\theta,i}(s) \theta_i(s, m_t) \|^2 \} ds \right\} + J_c(r_c)$$

where $\theta_{ti}(t) = \theta_i(t) - \theta_{ci}(t), \ \theta_{ti}(t, m_t) = i = 1, \ldots, N, \ t \in [0, T]$.

$$J_c(r_c) = \gamma^2 E \{ R_{R_c} \| \theta_i(0) \|^2 \}$$

$$+ E \left\{ \int_0^T E_{R_c} \{ \delta J_T(x, r_c, m_t) + 2 \theta_i(s, m_t) B_3(m_t) r_c(s) \theta_i(s, m_t) \} ds \right\}$$

$$+ E \left\{ \delta J_T(x, r_c, m_t) + 2 \theta_i(s, m_t) B_3(m_t) C_{\theta,i}(m_t) r_c(s) \theta_i(s, m_t) \} ds \right\}$$

$$+ \gamma^2 \| \theta_i(s, m_t) \|^2 + 2 \theta_i(s, m_t) B_3(m_t) C_{\theta,i}(m_t) r_c(s) \theta_i(s, m_t) \} ds \right\}$$

$\delta J_T(x, r_c, m_t) = \| D_{13}(m_t) r_c(t) \|^2$

$$- \| \tilde{R}^{1/2}_i(m_t) C_{\theta,i}(m_t) r_c(t) \|^2,$$

$C_{\theta,i}(m_t) = \gamma^2 A_i(m_t)$

$C_{\theta,i}(m_t) = \tilde{R}^{-1}_i(m_t) B_{2,i}(m_t)$

$C_{\theta,i}(m_t) = \tilde{R}^{-1}_i(m_t) D_{12,i} D_{13,i}(m_t)$

Moreover, the following results hold using

$$K_{x,i} = -\tilde{R}^{-1}_i \tilde{S}_i, \ K_{r,e,i} = -\tilde{R}^{-1}_i D_{12,i} D_{13,i}$$

and $P_0 = [R - \gamma^2 X_i(0)]^{-1}$.

Moreover, the following results hold using

$$K_{x,i} = -\tilde{R}^{-1}_i \tilde{S}_i, \ K_{r,e,i} = -\tilde{R}^{-1}_i D_{12,i} D_{13,i}$$

$$K_{\theta,i} = -\tilde{R}^{-1}_i B_{2,i}.$$

i) The control law for the $H_\infty$ fixed-preview tracking is

$$u_{s1} = K_{x,i} x + K_{r,e,i} r_c + K_{\theta,i} \theta_i$$

with $\theta_{ci}(t)$ given by (5). Moreover, $J_T(x_0^*, u_{s1}, w^*)$ coincides with (6).

ii) The control law for the $H_\infty$ tracking of noncausal $r_c(t)$ is

$$u_{s2} = K_{x,i} x + K_{r,e,i} r_c + K_{\theta,i} \theta_i$$

with $\theta_i(t)$ given by (4) and

$$J_T(x_0^*, u_{s2}, w^*) = J_c(r_c).$$

(Proof)

Sufficiency: We have already presented the proof of the sufficiency for the solvability of this $H_\infty$ noncausal tracking problem. Refer to [8].

Necessity: It will explained in detail in the next two sections. As explained in [8], because of arbitrariness of the reference signal $r_c(t)$, by considering the case of $r_c(t) \equiv 0$, one can also easily deduce the necessity for the solvability of the $H_\infty$ tracking problem. (Q.E.D.)

Remark 3.2 The compensator dynamics (5) in the fixed preview case has the same form as the compensator dynamics (4), while the terminal conditions in these two cases are different.

4 -No Mode Transition Case-

We first consider the case with no mode transitions, i.e., the single mode case.

We assume that the signal $\{r_c(t)\}$ is known a priori for the whole time interval $t \in [0, T]$. Notice that this situation is the same as that in the noncausal $H_\infty$ tracking problem considered by Shaked et. al.([12]).

Also notice that in this case the expectation $E_{R_c}$ is not necessary.

In this case we can regard the dynamics

$$\dot{x}(t) = A x(t) + B_1 u(t) + B_2 u(t) + B_3 r_c(t)$$
as the constraint over the time interval \([0, T]\). Then we can define the following Lagrangian
\[
J_T(x_0, u, \lambda) := -\gamma^2 x_0^2 R^{-1} x_0 - \gamma^2 \|w\|^2
\]
\[
+ \int_0^T \|z(s)\|^2 ds + \|C_1 x(s) + D_{12} r_c(s)\|^2_{Q_T}
\]
\[
+ \int_0^T \lambda'(s) \{Ax(s) + B_1 w(s) + B_2 u(s) + B_3 r_c(s) - \dot{x}(s)\} ds
\]
where \(Q_T \geq 0\) and \(\lambda(\cdot)\) is a Lagrange multiplier. We calculate the first variation of \(J_T\) with regard to \(x, w\) and \(\lambda\) and obtain
\[
\delta J_T = -2\gamma^2 \delta x_0^2 R^{-1} x_0 - \int_0^T 2\gamma^2 \delta w' w ds
\]
\[
+ \int_0^T 2\delta x'(s) C_1 \{C_1 x(s) + D_{12} u(s) + D_{13} r_c(s)\}
\]
\[
+ 2\delta u'(s) D_{12} \{C_1 x(s) + D_{12} u(s) + D_{13} r_c(s)\} ds
\]
\[
+ \delta x'(T) \{C_1 Q_T C_1 x(T) + 2C_1 Q_T D_{13} r_c(T)\}
\]
\[
+ \int_0^T 2\delta \lambda'(s) \{Ax(s) + B_1 w(s) + B_2 u(s) + B_3 r_c(s) - \dot{x}(s)\} ds
\]
\[
+ \int_0^T 2\delta \lambda' x(s) ds.
\]
Let \(\delta J_T = 0\) and then we obtain the following conditions:
(i) the conditions of optimality
\[
w^*(s) = \gamma^{-2} B_1' \lambda^*(s)
\]
\[
u^*(s) = -R^{-1} (D_{12} C_1 x^*(s) + D_{12} D_{13} r_c(s) + B_2 \lambda^*(s))
\]
(ii) the canonical equations
\[
\dot{x}^*(s) = Ax^*(s) + B_1 w^*(s) + B_2 u^*(s) + B_3 r_c(s),
\]
\[
\dot{\lambda}^*(s) = -A' \lambda^*(s) - C_1' \{C_1 x(s) + D_{12} u^*(s) + D_{13} r_c(s)\}
\]
(iii) the boundary conditions
\[
x_0^* = \gamma^{-2} R \lambda^*(0)
\]
\[
\lambda(T) = C_1 Q_T C_1 x(T) + 2C_1 Q_T D_{13} r_c(T)
\]
From the boundary conditions, we set
\[
\lambda(t) = \theta(t) + X(t) x(t).
\]
Then we have the control strategy
\[
u^*(t) = -R^{-1} \{D_{12} C_1 + B_2 X(t)\} x^*(t)
\]
\[
+ D_{12} D_{13} r_c(t) + B_2 \theta(t).
\]
Notice that
\[
\lambda(t) = \dot{\theta}(t) + \dot{X}(t) x(t) + X(t) \dot{x}(t)
\]
\[
= \theta(t) + \dot{X}(t) x(t) + X(t) \{A^* x(t) + B_1 w^*(t) + B_2 u^*(t) + B_3 r_c(t)\}.
\]
Substituting this and \(\lambda(t) = \theta(t) + X(t) x(t)\) into the canonical equation (9), we obtain the following equality.
\[
\dot{\theta}(s) + \dot{X}(s) x(s) + X(s) \{A x(s) + B_1 w^*(s) + B_2 u^*(s) + B_3 r_c(s)\}
\]
\[
= -A' \{\theta(s) + X(s) x(s)\}
\]
\[
- C_1 \{C_1 x(s) + D_{12} u^*(s) + D_{13} r_c(s)\}
\]
By the arbitrariness of \(x\), with regard to the first order terms of \(x\), we obtain the following Riccati equation which give the necessary conditions for the solvability of the \(H\_\infty\) noncausal tracking problem.
\[
\dot{X} + A' X + X A + C_1 C_1 + \frac{1}{\gamma^2} X B_1' X
\]
\[
- S^T \tilde{R}^{-1} S = 0, \ X(T) = C_1 Q_T C_1 (10)
\]
where
\[
\tilde{R} = D_{12} D_{12}, \ \tilde{S}(t) = B_2 X(t) + D_{12} C_1.
\]
With regard to the terms without including \(x\) we obtain
\[
\begin{align*}
\dot{\theta}(t) &= -A' \theta(t) + \tilde{B}(t) r_c(t) \\
\theta(T) &= 2C_1 Q_T D_{13} r_c(T)
\end{align*}
\]
where
\[
\tilde{A} = A + \frac{1}{\gamma^2} B_1' X - \tilde{B} \tilde{R}^{-1} \tilde{S}
\]
\[
\tilde{B} = -(X B_3 + C_1' D_{13}) + \tilde{S}' C_u, \ C_u = \tilde{R}^{-1} D_{12} D_{13}
\]
which gives the preview compensator dynamics with the terminal condition.

**Remark 4.1** Notice that, in the case of \(Q_T = 0\), the terminal conditions reduce to \(X(T) = 0\) and \(\theta(T) = 0\).

Also notice that in this case the above Riccati differential equation (10) and the preview compensator dynamics (11) with terminal conditions coincide with the ones for the noncausal \(H\_\infty\) tracking problem by U Shaked et al. ([12]).

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**5 - Mode Transition Case**

In this section we consider the multi modes case.

We assume that the signal \(\{r_c(t)\}\) is known for the whole time interval \(t \in [0, T]\). Notice that in this case the expectation \(E_{\theta_c}\) is not necessary as the case with no mode transitions. In this case we can regard the dynamics
\[
\dot{x}(t) = A(m_i) x(t) + B_1 (m_i) w(t)
\]
\[
+ B_2 (m_i) u(t) + B_3 (m_i) r_c(t)
\]
as the constraint over the time interval \([0, T]\). Then we can define the following Lagrangian

\[
J_T(x_0, u, w) := -\gamma^2 x_0^R - R^1 x_0 - \gamma^2 \int_0^T \|w(s)\|^2 ds
\]

\[
+ \mathbb{E}\left\{ \int_0^T \left\{ \|z_c(s)\|^2 \right\} ds
+ \|C_1(m_s)x(s) + D_{12}(m_s)r_c(s)\|^2
+ \int_0^T 2\lambda'(m_s)(s)\{A(m_s)x(s) + B_1(m_s)w(s)
+ B_2(m_s)u(s) + B_3(m_s)r_c(s) - \dot{x}(s)\} ds \right\}
\]

where \(Q_T \geq 0\) and \(\lambda_{m(t)}(t)\) is a Lagrange multiplier. Using the relationship

\[
\lambda'(t)x(t) = \dot{\lambda}'(t)x(t) + \sum_{j=1}^N \pi_{ij} \lambda'_j(t)x(t)
\]

and Dynkin's formula, we obtain the following partial integration formula.

\[
\mathbb{E}\left\{ \int_0^T \lambda'_m(s)x(s)ds \right\} = \left[ \lambda'_m(s)x(s) \right]_0^T
\]

\[
- \mathbb{E}\left\{ \int_0^T \lambda'_m(s)x(s) + \sum_{j=1}^N \pi_{mj}\lambda'_j(s)x(s)ds \right\}
\]

We calculate the first variation of \(J_T\) with regard to \(x\), \(w\) and \(\lambda_{m(t)}\) and obtain

\[
\delta J_T = -2\gamma^2 \delta x_0^R - R^1 \delta x_0 - \int_0^T 2\gamma^2 \delta w'w'ds
\]

\[
+ \mathbb{E}\left\{ \int_0^T 2\delta x'(s)C'_1(m_s)\{C_1(m_s)x(s)
+ D_{12}(m_s)u(s) + D_{13}(m_s)r_c(s)\}
+ 2\delta w'(s)D_{12}(m_s)\{C_1(m_s)x(s)
+ D_{13}(m_s)r_c(s)\}ds
+ \delta x'(T)\{C'_1(m_T)Q_T C_1(m_T)x(T)
+ 2C'_1(m_T)Q_T D_{13}(m_T)r_c(T)\}
+ \int_0^T 2\lambda'_m(s)\{A(m_s)x(s) + B_1(m_s)w(s)
+ B_2(m_s)u(s) + B_3(m_s)r_c(s) - \dot{x}(s)\} ds
+ \int_0^T 2\delta x'(s)A'(m_s) + \delta w'(s)B'_1(m_s)
+ \delta u'(s)B'_2(m_s)\} \lambda_m(s)ds
- 2\lambda'_m(T)\delta x(T) + 2\lambda'_m(0)\delta x(0)
\]

Let \(\delta J_T = 0\) and then we obtain the following conditions:

(i) the conditions of optimality

\[
w^*(s) = \gamma^{-2}B'_1(m_s)\lambda'_m(s)
\]

\[
u^*(s) = -\tilde{R}_1^{-1}\{D_{12}(m_s)x^*(s)
+ \tilde{D}_{13}(m_s)r_c(s) + B_2(m_s)\}
\]

(ii) the canonical equations

\[
x^*(s) = A_1x^*(s) + B_1w^*(s) + B_2u^*(s) + B_3r_c(s)
\]

\[
\lambda'_m(s) = -A'_1\lambda'_m(s) - C'_1(m_s)x(s) + D_{12}(m_s)x^*(s)
+ D_{13}(m_s)r_c(s)\}
\]

(iii) the boundary conditions

\[
x^*_0 = \gamma^{-2}R\lambda'_m(0)
\]

\[
\lambda_m(T) = C_{14}Q_T C_1(m_T)x(T) + 2C_{14}Q_T D_{13}(m_T)r_c(T)
\]

From the boundary conditions, we set

\[
\lambda_m(t) = \theta(t, m_t) + X(t, m_t)x(t)
\]

and obtain the form of noncausal control input \((8)\) directly. Next we calculate

\[
\mathbb{L}_u[\lambda'_m(s)x(s)]
\]

\[
= \lambda'_m(s)x(s) + \lambda'_m(s)x(s) + \sum_{j=1}^N \pi_{ij}\lambda'_j(s)x(s)
\]

\[
= -\theta'_1(s) + x'(s)X_1(s) + \theta'_2(s) + \theta'_3(s)x(s)
\]

\[
= -\theta'_1(s) + x'(s)X_1(s) + \theta'_2(s) + \theta'_3(s)x(s)
\]

\[
= \theta'_1(s)x(s) + \theta'_2(s)x(s) + \theta'_3(s)x(s) + x'(s)X_1(s)x(s)
\]

\[
= \theta'_1(s)x(s) + \theta'_2(s)x(s) + \theta'_3(s)x(s) + x'(s)X_1(s)x(s)
\]

and

\[
\mathbb{L}_u[\theta'_1(s)x(s) + x'(s)X_1(s)x(s)]
\]

\[
= \theta'_1(s)x(s) + \theta'_2(s)x(s) + \theta'_3(s)x(s) + x'(s)X_1(s)x(s)
\]

\[
= \theta'_1(s)x(s) + \theta'_2(s)x(s) + \theta'_3(s)x(s) + x'(s)X_1(s)x(s)
\]

\[
= \theta'_1(s)x(s) + \theta'_2(s)x(s) + \theta'_3(s)x(s) + x'(s)X_1(s)x(s)
\]

\[
= \theta'_1(s)x(s) + \theta'_2(s)x(s) + \theta'_3(s)x(s) + x'(s)X_1(s)x(s)
\]

(12)
Since the right hand sides of the two equalities (12) and (13) are equal, by the arbitrariness of \( x \), with regard to the second order terms of \( x \) we obtain the set of coupled differential Riccati equations (3), which gives the necessary condition for the solvability of the \( H_\infty \) non-causal tracking problem. With regard to the first order terms of \( x \) we also obtain the compensator dynamics (4) introducing the known information of the reference signal \( r_c(\cdot) \).

\[ \text{6 Concluding Remarks} \]

In this paper we have presented the \( H_\infty \) tracking control theory considering the preview information by state feedback for the linear continuous-time Markovian jump systems, which are a class of stochastic switching systems. The compensators introducing the preview information of the reference signal are coupled with each other.

The author had presented the solution of the \( H_\infty \) preview tracking control theory by state feedback for the linear continuous-time Markovian jump systems. However it has not been fully investigated why the preview compensator dynamics has such form. Therefore in this paper he has presented the direct derivation method of the form of the preview compensator dynamics by using the stochastic variational calculus with dynamics constraints. Notice that, in the case with no mode transitions, the form of the preview compensator dynamics by the derivation method presented in this paper corresponds to the one by another derivation method by U. Shaked et al. ([12]).

It is well known that there does not exist any general partial integral formula for stochastic processes. Therefore we need to introduce each partial integral formula for each stochastic process. In this paper we introduce the partial integral formula for the jump processes.

The derivation method of preview compensator dynamics presented in this paper can be applied to the various types of systems and problem settings. They will be reported elsewhere.

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References


