Statistical Models and ML Positioning Using Received Signal Powers in Sensor Networks

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Abstract

In this paper, we present statistical models and ML (maximum likelihood) positioning algorithms using received signal powers in sensor networks. The purpose of this study is to develop the indoor positioning system with utilizing the IEEE Std 802.15.4 [1] based wireless sensor network.

The distance between nodes can be presumed by using the RSSI (received signal strength indicator) of the wireless data communication in the sensor network. Therefore, it becomes possible to presume the position of the sensor node by using the RSSI from a certain sending source, namely from the base station with already measured position.

The variation of the RSSI is large because of the influence by the measurement environment. Therefore, it is necessary to acquire a lot of the RSSI data to improve the position estimation. In this paper, we present the models of RSSI of radio signal propagation applying the Rayleigh distribution and gamma distribution. We propose a positioning algorithm based on the ML method from the probability density functions of the signal powers.

1 Introduction

Wireless sensor networks can supply sensing data to applications that adapt to the user’s circumstances in a ubiquitous computing environment. Their systems are applied in a variety of fields, such as commodities management, energy monitoring, and a wireless attraction system. Thus, position information is very important in sensor networks.

Therefore, sensor nodes send sensing data to a base station for data collection. If they are appropriately designed, sensor nodes can work autonomously to measure temperature, humidity, acceleration, and so on. In addition, sensor locations are important too, because sensing data are meaningless if the sensor location is unknown in various applications.

The most popular method to acquire position information is GPS (Global Positioning System). GPS provides highly accurate position and velocity. The main factor limiting the use of GPS is the requirement for line-of-sight between the receiver antenna and the satellites. Therefore, a method of the indoor measurement applying GPS, pseudolite method and IMES (Indoor Messaging System) are proposed [2, 3]. However, positioning of pseudolite method is difficult due to multipath signals and cycle slips in indoor environment. Furthermore, the cost of the infrastructure maintenance rises because a tightly-synchronized signal is necessary. On the other hand, IMES method is constructed by using the same frequency of the GPS signal and the modulation method. It is expected as a technique of a seamless measurement using GPS though there are problems of the measurement time and the operation method, etc.

As alternative of GPS, some methods to acquire position information without GPS have been studied [4, 5, 6]. Consequently, in indoor positioning an Inertial Navigation System (INS) provides position, velocity and attitude autonomously at a rate of several tens of Hz. However, its errors are accumulated owing to drift of IMU (Inertial Measurement Unit).

The positioning method using wireless communication, especially, a methods using radio property such as received signal power, time of arrival (TOA), directional antenna (Cell-ID) and angle of arrival (AOA).

In this paper, location estimation method which utilizes the RSSI obtained as a by-product of the data communication between nodes for wireless sensor networks is focused on. But, the RSSI has a larger variation because it is subject to the effects of fading or shadowing. And so, we propose the ML based distance estimation method that can take into account the radio environment. First, we introduce the specification and function of a ubiquitous device which is applied in this paper. Next, we present the models of RSSI of radio propagation and the ML method of estimating the parameters in the models. Furthermore, based on
statistical models of RSSI, ML algorithms are derived for estimating locations. Also we show the experimental results based on our derived method. Finally we mention a brief summary and future work.

2 System equipments

In this paper, we construct a system model to use the following equipment.

IEEE Std 802.15.4 has three frequency bands; 868 MHz (868-868.6 MHz), 915 MHz (902-928 MHz) and 2.4 GHz (2.4-2.4835 GHz) bands, however, only the 2.4 GHz band is allowed in Japan. The bit rate, symbol rate and modulation are 250 kbits/sec, 62.5 ksym-bols/sec and offset-quadrature phase shift keying (O-QPSK), respectively. The 2.4 GHz band is included in industrial, scientific and medical (ISM) band, so it is rich in interference. To mitigate the interference, therefore, direct sequence spread spectrum (DS-SS) with processing gain of 8 is adopted in the standard. The chip rate is 2 Mchips/sec.

We implemented our technique in Ubiquitous Device the “smartMODULE”, which is a sensor network developed by Hitachi Industrial Equipment System Co. Ltd., Japan. The “smartMODULE” is compliant with IEEE 802.15.4 (2.4GHz band). The products include active tags, the base stations, and the relay stations. The sensor network is simply constructed with these products. Fig. 1 shows an example of the system configurations. Active tags with omnidirectional pattern antennas and the base stations with the whip antennas are used in our experiments.

3 Relation between distance and the RSSI

In wireless communications, the RSSI attenuates by multiplication of the distance [7]. Therefore, in indoor environments, relations of the RSSI and the distance are unformulated by “power decay factor” due to near/far effect and “fading distribution”.

3.1 Friis transmission formula

The formula originally proposed by Friis in 1946 [8], gives the relationship between the transmitted signal power $P_t$ and receiver signal power $P_r$ in a one-way, free-space radio link:

$$P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi)^2 d^n}$$

where $G_t$ and $G_r$ are the transmitting and receiving antenna gains, $\lambda$ is the wavelength, $d$ is the distance between antennas and $n$ is the path loss exponent. This model is only assumed as the attenuation model that does not consider the reflection of radio waves. The correct values of antenna gains $G_t$ and $G_r$ are difficult to be obtained due to their large variations depending on circumstances of the nodes.

Fig. 2 shows the Friis equation curves ($G_t = 1, G_r = 1, \lambda = 0.1243$) with different values of $n$, that show a large change. Here, let us show the experimental results between the RSSIs and distances, where the measurement environment is shown in Fig. 3. In experiments, two base stations are located at 1 meter high, and 1 Hz rate measurement data were collected for one minute at several points. Fig. 4 shows the 60 RSSI measurement data at several points that are separately-placed (from 1 to 18 meters) from the base stations.

From Fig. 4, we can observe that a large difference of values of the RSSI measured between each active tag and each base station exist. The distribution of
3.2 Model of radio propagation

The variation of the RSSI is large because of the influence of the environment. In wireless telecommunications, the mean value of RSSI \( \langle R \rangle \) attenuates in proportion to the power \( \langle d \rangle \) of the distance \( \langle d \rangle \) \[7\].

\[
E[R] = \alpha d^\beta, \quad (1)
\]

where \( \alpha \) and \( \beta \) are constants. Here, \( \alpha \) and \( \beta \) in the Eq.(1) are derived from the observation \( R \). However, in indoor environments, the relation of the RSSI and the distance is unformulated by the power decay factor due to near/far effect and the fading distribution. Therefore, we assume RSSI measurement data for each base station as a statistical model. Namely we assume the PDF (probability density function) of the amplitude \( \langle A \rangle \) of the radio wave is assumed as the Rayleigh distribution:

\[
p_A(a) = \frac{a}{\theta} \exp \left\{ -\frac{a^2}{2\theta^2} \right\}, \quad (2)
\]

where \( \theta \) is the mean values of the signal power \( R \equiv A^2 \). Namely

\[
E[R] = E[A^2/2] = \theta. \quad (3)
\]

Then we can easily show by applying the relation of two random variables; \( R = \frac{A^2}{2} \), that the PDF of \( R \) is the exponential distribution as follows:

\[
p_R(r) = \frac{1}{\theta} \exp \left\{ -\frac{r}{\theta} \right\}. \quad (4)
\]

The measurement data of RSSI are usually provided by the average values \( \bar{R} \) of \( \{R_k, k = 1, \cdots, m\} \):

\[
\bar{R} = \frac{1}{m} \sum_{k=1}^{m} R_k,
\]

where we assume each measurement is statistically independent each other. Then the PDF of \( \bar{R} \) can be obtained as the following gamma distribution

\[
p_R(\bar{r}) = \frac{1}{(\bar{r}/\theta)^m \Gamma(m)} \bar{r}^{m-1} e^{-\bar{r}/\theta}. \quad (5)
\]

3.3 ML estimation of parameters of \( \alpha \) and \( \beta \)

Combining (1),(3) and (5), the likelihood function of the \( \alpha \) and \( \beta \) is provided by

\[
L(\alpha, \beta) = \prod_{i=1}^{n} p_R(\bar{r}_i).
\]

Then, the log likelihood function \( l(\alpha, \beta) \) is also given by

\[
l(\alpha, \beta) = \ln L(\alpha, \beta)
\]

\[
= \ln \left[ \prod_{i=1}^{n} \frac{1}{(\bar{r}_i/\theta)^m \Gamma(m)} \bar{r}_i^{m-1} e^{-\bar{r}_i/\theta} \right]. \quad (7)
\]

where, \( \theta_i = \alpha d_i^\beta \). Therefore, the log likelihood function given by the observation \( \{\bar{r}_1, \bar{r}_2, \cdots, \bar{r}_n\} \) in each distance \( d_i \) from the above equation as follows:

\[
l(\alpha, \beta) = \sum_{i=1}^{n} \left[ -m \ln \left( \frac{\theta_i}{\bar{r}_i} \right) - \ln \Gamma(m) + (m - 1) \ln \bar{r}_i - \frac{m \bar{r}_i}{\theta_i} \right]. \quad (8)
\]

Then we have the following relations:

\[
\frac{\partial l}{\partial \alpha} = \sum_{i=1}^{n} \left[ -\frac{m}{\alpha} + \frac{m \bar{r}_i}{(d_i)^2 \alpha^2} \right] = 0 \quad (9)
\]

\[
\frac{\partial l}{\partial \beta} = \sum_{i=1}^{n} -m \ln d_i - \frac{m \bar{r}_i}{\alpha} \cdot \frac{1}{d_i^2} \left( -\ln d_i \right) = 0. \quad (10)
\]


The solutions of the above nonlinear simultaneous equations are derived. From Eq. (9),

\[
\frac{1}{\alpha} \left( \sum_{i=1}^{n} (-m) + \frac{1}{\alpha} \sum_{i=1}^{n} \frac{m \bar{r}_i}{(d_i)^3} \right) = 0.
\]

If \( \alpha \neq 0 \),

\[
\sum_{i=1}^{n} (-m) + \frac{1}{\alpha} \sum_{i=1}^{n} \frac{m \bar{r}_i}{(d_i)^3} = 0.
\]

Also,

\[
\alpha = \frac{1}{n} \sum_{i=1}^{n} \frac{\bar{r}_i}{(d_i)^3}.
\]

(11)

Where, from Eq. (10)

\[
\sum_{i=1}^{n} -m \ln d_i + \frac{m \bar{r}_i}{\alpha} \cdot \frac{1}{d_i} \ln d_i = 0.
\]

(12)

Namely, we define \( \eta \equiv e^{-\beta} \), then we have the following relation as:

\[
\frac{1}{d_i^\beta} = e^{\ln \left( \frac{\bar{r}_i}{\alpha} \right)} = e^{-\beta \ln d_i} = \eta \ln d_i.
\]

(13)

Here, from Eq. (12), (13) we have

\[
n \sum_{i=1}^{n} \bar{r}_i \eta^{ln d_i} \ln d_i - (\sum_{i=1}^{n} \ln d_i) \sum_{k=1}^{n} \bar{r}_k \eta^{ln d_k} = 0.
\]

(14)

Therefore, \( \alpha \) and \( \beta \) can be estimated by solving the nonlinear equation of \( \eta \) in Eq. (14). Then we can obtain

\[
\hat{\beta} = -\ln \hat{\eta},
\]

(15)

\[
\hat{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \frac{\bar{r}_i}{(d_i)^3}.
\]

4 Location estimation

The node position \([x_u \, y_u \, z_u]^T\) is estimated by the distance measured from the multiple base stations \([x_i \, y_i \, z_i]^T; \ (i = 1, 2, \ldots, n)\). The distance between the node and the base station \(i\) is expressed by

\[
d_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2}
\]

(16)

From (5), the likelihood function of the unknown position \([x_u \, y_u \, z_u]^T\) is provided by

\[
L(x_u; y_u; z_u) = \prod_{i=1}^{n} P_{R_i}(\hat{r}_i) |_{\theta=\alpha((x_i-x_u)^2+(y_i-y_u)^2+(z_i-z_u)^2)^{\frac{1}{2}}} ^{2}
\]

(17)

The node position \([x_u \, y_u \, z_u]^T\) is estimated by the ML method. Then, the log likelihood function \(l(x_u, y_u, z_u)\) is also given by

\[
l(x_u, y_u, z_u) = \ln L(x_u, y_u, z_u)
\]

(18)

\[
\left. \frac{\partial l}{\partial x_u} = \sum_{i=1}^{n} \left\{ m \beta [x_i - x_u] + \frac{2m \bar{r}_i (x_i - x_u)}{\alpha [(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2]^{\frac{3}{2}}} \right\} = 0
\]

\[
\left. \frac{\partial l}{\partial y_u} = \sum_{i=1}^{n} \left\{ m \beta [y_i - y_u] + \frac{2m \bar{r}_i (y_i - y_u)}{\alpha [(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2]^{\frac{3}{2}}} \right\} = 0
\]

\[
\left. \frac{\partial l}{\partial z_u} = \sum_{i=1}^{n} \left\{ m \beta [z_i - z_u] + \frac{2m \bar{r}_i (z_i - z_u)}{\alpha [(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2]^{\frac{3}{2}}} \right\} = 0
\]

(20)

5 Experiments

The experiments of estimating the location were carried out in the same environment shown in Fig. 3. The parameters of the active tag are shown in Table 1. We show the estimation results of \( \alpha \) and \( \beta \) applying the RSSI data shown in Fig. 4. Table 2 shows the results of estimated \( \alpha \) and \( \beta \) applying the proposed ML method, and the model derived by Rayleigh distribution [11] and Friis equation Eq. (1) with assuming \( Gt = 1, Gr = 1 \) respectively.
5.1 Location results

We experimented on the location estimation by using the proposed method Eq. (16) and the model of derived by Rayleigh distribution. As shown in Fig. 5, four base stations were located in a 10 [m] square shape. Fig. 6 shows the coordinates of the base stations and tags.

The heights of all the objects, namely the base stations and tags were fixed at 1.0 [m] from the floor. Therefore, in this experiment, the location estimation was implemented in the horizontal (2D) plane. In the experiments, the location of tags plotted by green triangles in Fig. 6 were estimated twice by using two independent datasets. Each dataset was RSSI data collected at 1 [Hz] rate for 60 seconds by the base stations. We compare the difference between the position calculated and the true position.

Table 3: Experimental condition of equipments

<table>
<thead>
<tr>
<th>Device</th>
<th>number</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active tag</td>
<td>1</td>
<td>High:1.0[m]</td>
</tr>
<tr>
<td>Base station</td>
<td>4</td>
<td>High:1.0[m]</td>
</tr>
</tbody>
</table>

Table 4: Experimental results

<table>
<thead>
<tr>
<th>active tag coordinate</th>
<th>first result [m]</th>
<th>first error [m]</th>
<th>second result [m]</th>
<th>second error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ (5.0,5.0)</td>
<td>(6.03,3.03)</td>
<td>2.21</td>
<td>(5.87,3.66)</td>
<td>1.10</td>
</tr>
<tr>
<td>Φ (5.0,0.0)</td>
<td>(1.27,2.78)</td>
<td>4.81</td>
<td>(4.89,1.57)</td>
<td>1.08</td>
</tr>
<tr>
<td>Φ (2.5,2.5)</td>
<td>(2.86,3.52)</td>
<td>1.09</td>
<td>(1.89,3.66)</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Table 5: Experimental results (proposed method)

<table>
<thead>
<tr>
<th>active tag coordinate</th>
<th>first result [m]</th>
<th>first error [m]</th>
<th>second result [m]</th>
<th>second error [m]</th>
</tr>
</thead>
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<td>1.35</td>
</tr>
</tbody>
</table>

Table 6: Location estimation results applying the model of derived by Rayleigh distribution[11]. We can see that location results contain large errors at (5.0, 0.0).

Table 5 shows the location estimation results applying the proposed method (Eq. (20)). In second measurement, We can see that the errors have decreased slightly.

6 Conclusions

In this paper, we considered positioning method by utilizing the model of radio propagation in sensor network. The model was identified from the Rayleigh distribution using the maximum-likelihood method.

We carried out experiments of the estimation of the model parameter and the location. In installation environment, the model of radio propagation was identified accurately, and the location estimation results were the errors of around 1 meter.

However, for the applications of RSSI positioning, more accurate positioning is desired. Therefore, in the
future study, we plan to investigate the relationship between RSSIs and the circumstances around the base stations and tags.

References

[1] Institute of Electrical and Electronics Engineers. IEEE Std 802.15.4-2006, Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (WPANs), 8 September 2006.


