Estimation of Precautionary Demand in USA

Yoji Morita and Shigeyoshi Miyagawa
Department of Economics, Kyoto Gakuen University
Kameoka, Kyoto 621-8555, Japan
E-mail: morita-y@kyotogakuen.ac.jp

Abstract

In 2001, the Bank of Japan (BOJ) adopted “quantitative monetary easing”. Since short term interest rates became almost zero, the operating target of monetary policy was changed from interest rate to the monetary base, where monetary base is defined as the sum of “Cash” and “Reserve at the BOJ”. Honda et al [1] showed that “Reserve at the BOJ” in (2001,2006) is effective to the economy through a transmission path in a stock market, where impulse responses in VAR model are used in monthly data of Japan. Decomposing money into transaction demand and precautionary one, and estimating precautionary one, Morita and Miyagawa [2] tried to show that increasing “Reserve at the BOJ” makes GDP increased through the stock market in quarterly data.

In this paper, the method of estimating precautionary demand in Japan is extensively improved and applied to the case of USA. Using precautionary demand estimated in the whole interval (1980m01, 2012m02), the quantitative easing at Federal Reserve Board (FRB) is shown to be effective during the period (2006m06, 2010m02).

1 Introduction

1.1 Crisis in USA and nontraditional easing policy during (2007,2010)

The subprime problem in 2007 and Lehman crisis in 2008 caused serious depressions in the world economy. Many central banks set interest rates around zero, and carried out “nontraditional monetary policies” in large scales. Generally speaking, operation of short term interest rates based on for example Taylor rule is called “traditional monetary policy”, while in financial crisis of these days a traditional monetary policy has no room to operate around zero interest rates, and hence, many central banks were forced to adopt nontraditional monetary policies.

FRB decreased Federal Funds rate (FF rate) from 2% at Lehman crisis (2008m09) to 0-0.25% (2008m12). Furthermore, additional easing policies were done by operations of buying long term government bond, Residential Mortgage-Backed Securities (RMBS) and agency debt. FRB called these policies as “Credit Easing”.

In this paper, we estimate precautionary demand in (1980,2012). Defining “adjusted money” by “money” minus “precautionary demand”, we show a cointegrating relation between adjusted money and industrial production. Analyzing relationship between reserves and adjusted money in the period (2006,2010), we show that there is a transmission path from reserves to economic activity through a stock market, where we adopt reserves as St.Louis adjusted reserves stated later.

2 Data Properties

Variables and their symbolic notations are given in Table 1, where a business cycle \( u(t) > 0 \) means the magnitude of booming economy, while \( u(t) < 0 \) means that of depressions. See Fig.1.

<table>
<thead>
<tr>
<th>Table 1: List of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( adjressl(t) = \text{St.Louis adjusted reserves} )</td>
</tr>
<tr>
<td>( m2sl(t) = M2 \text{ money stock} )</td>
</tr>
<tr>
<td>( indpro(t) = \text{industrial production index} )</td>
</tr>
<tr>
<td>( cpilfesl(t) = \text{consumer price index, less food &amp; energy} )</td>
</tr>
<tr>
<td>( lnSP500(t) = \ln(\text{S&amp;P 500 index}) )</td>
</tr>
<tr>
<td>( lnhouse(t) = \ln(\text{CPI:housing index}) )</td>
</tr>
<tr>
<td>( napm(t) = \text{ISM manufacturing, PMI composite index} )</td>
</tr>
</tbody>
</table>

\( u(t) = -napm(t) + 50 \)

Fig. 1: Business cycle \( u(t) \), St.Louis adjusted reserves \( adjressl(t) \), industrial production index \( indpro(t) \) and \( m2sl(t) \).
2.1 Unit root test in (1980m01, 2012m02)

Two kinds of unit root tests are carried out. One is DF-GLS (ERS) test with unit root as the null hypothesis and the other is KPSS test with stationarity as the null hypothesis. The results are shown in Table 2. Every variable except $u(t)$ is shown to be nonstationary.

![Table 2: Unit root test in (1980m01, 2012m02)](image)

* * *, ** and * denote significance levels of 1%, 5% and 10% respectively.

3 Estimation of Precautionary Demand

3.1 Formulation of precautionary demand

Precautionary demand is defined as money which is held without usage. Although Keynes appointed that precautionary demand increases when GDP increases, anyone did not analyze the behavior of precautionary demand increases when GDP increases, rather than by $u(t)$. Notice that a magnitude of anxiety $u(t)$ varies together with a business cycle and is affected by money. Hence, we regard money as a function of GDP.

(p-4) $\text{money}_{\text{prec}}$ depends on reserves at FRB when quantitative easing policy is adopted.

Using these properties, we shall attack to formulate $\text{money}_{\text{prec}}$ in USA:

![Equation](image)

where the 2nd term on the RHS represents the effect of the reserves, the 3rd term means that $\text{money}_{\text{prec}}$ is a function of the nominal industrial production ($\text{indpro}(t) \times \text{cpilfesl}(t)$), the 4th term means that the business cycle $u(t)$ multiplied by nominal industrial production effects to $\text{money}_{\text{prec}}$.

**Remark-1** In the analysis of Japan’s economy [2], a business cycle (corresponding to $u(t)$ in USA) was quarterly given by BOJ’s TANKAN, and so we had to use quarterly GDP instead of monthly $\text{indpro}$. We assumed in [2] that $\text{money}_{\text{prec}}$ is a linear function of GDP and business cycle by setting $c_1 > 0$ and $c_2 \equiv 1$ and does not contain reserves in it ($c_4 \equiv 0$). Reserves are set in a system as an exogenous variable.

**Remark-2** In this paper, using USA monthly data, we extend parameters from a linear case $c_1 > 0$ and $c_2 \equiv 1$ in Japan to a nonlinear case $c_1 < 0$ and $c_2 < 0$ in USA which can assure property (p-1) and exhibits a larger value of log-likelihood function for estimation. Furthermore, reserves are directly contained in $\text{money}_{\text{prec}}$ by setting $c_4 \neq 0$. The improved method in this paper was applied to Japan’s economy, but a sample size due to quarterly data showed bad results. Even in USA case, quarterly data may cause a trouble.

**Remark-3** In (p-4) and Assumption, we use a total amount of reserves. For further study, it may be necessary to discriminate contents of reserves such as short term government bonds, long-term ones and RMBS.

3.2 Maximum likelihood estimation in (1980m01,2012m02)

Letting $y(t)$ and $\text{rm}_{2,\text{adj}}(t)$ be defined by

\[
y(t) = \ln(\text{indpro}(t)), \quad (2)
\]

\[
\text{rm}_{2,\text{adj}}(t) = \ln(m2s(t) - \text{money}_{\text{prec}}(t)), \quad (3)
\]

we regress $\Delta y(t) = y(t) - y(t - 1)$ by the regressors in Table 3. Since a nonlinearity of $c_2$ makes optimization procedures difficult, we fix, at first, the value of $c_2$ during (-10,2) and carry out maximum likelihood estimation procedures numerically. Secondly, changing the value of $c_2$ with 0.01 step, we continue MLE procedures, and finally, the optimal value of $c_2$ was determined as $c_2 = -7.24$ which maximizes log-likelihood functions among various values of $c_2$. The objective function can be given by log-likelihood function of $\Delta y(t)$-process. For economy of space, we shall omit to describe it.

![Table 3: lag length](image)

Using these properties, we shall attack to formulate $\text{money}_{\text{prec}}$ in USA:

![Equation](image)
Table 3: Regressors of $\Delta y(t)$

<table>
<thead>
<tr>
<th>coefficient</th>
<th>std. error</th>
<th>z-statistic</th>
<th>prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>1312.005</td>
<td>129.800</td>
<td>10.107</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-0.485</td>
<td>0.1547</td>
<td>-3.134</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.4146</td>
<td>0.1502</td>
<td>2.760</td>
</tr>
<tr>
<td>$c_4$</td>
<td>1.804</td>
<td>1.142</td>
<td>1.578</td>
</tr>
</tbody>
</table>

Exchange rate and rate of government bond (10 years) are tried as regressors, but they are not significant.

Table 4: Estimation results ($c_2 = -7.24$)


where $\Pi = \sum_{i=1}^{k} \Pi_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$. When the rank of $\Pi$ is $r_\pi < p$, then $\Pi$ has a representation of

$$\Pi = \alpha \beta' \quad \text{with} \quad \alpha_i (p \times r_\pi).$$

The above relation is called “cointegration property” and implies that $\beta' x(t)$ becomes stationary, while $x(t)$ itself is nonstationary, that is, there are $r_\pi$ kinds of linear combinations each of which is stationary although every element of $x$ is nonstationary. $\beta' x(t)$ is called “error correction term” and is denoted by $ect(t)$.

Although $\Delta y(t)$ is regressed by $\Delta rm2_{adj}(t - i)$, $\Delta lnSP500(t - i)$, $\Delta lnhouse(t - i)$ and $\Delta y(t - i)$, a cointegration relationship can be shown later between $rm2_{adj}$ and $y$. In this case, a stationary $\beta' x(t) \equiv ect(t)$ can be written in the form:

$$ect(t) \equiv rm2_{adj}(t) + \beta_0 + \beta_1 y(t) \quad (7)$$

$$= n(t), \quad (8)$$

where $n(t)$ is a stationary noise process. Letting $n(t) \equiv 0$, the following relation is called a money demand function in long-run equilibrium:

$$rm2_{adj}(t) = -\beta_0 - \beta_1 y(t). \quad (9)$$

Calculating $rm2_{adj}$ from the estimated $m_{prec}$ in the preceding section, we have

$$ect(t) = rm2_{adj}(t) - 1.433 - 1.373 y(t). \quad (10)$$

When we write Eq.(5) in componentwise, then $\Delta y(t)$ should be regressed by regressors $\Delta x(t - i)$ and $ect(t - 1)$. This fact imposes us to correct regressors of maximum likelihood estimation of $m_{prec}$ in 3.2.

4  System Model with Cointegration

4.1 Cointegration formulation

Consider the $p$-dimensional autoregressive process $x(t)$ defined by the equations(Johansen [3]).

$$x(t) = \sum_{i=1}^{k} \Pi_i x(t - i) + \Phi D(t) + \varepsilon(t), \quad (4)$$

where the deterministic terms $D(t)$ can contain constant, a linear term, seasonal dummies and so on. Assume that $x(t)$ belongs to $I(1)$ class, that is, $x(t)$ is nonstationary and $\Delta x(t)$ is stationary. Equation (4) can be written in VEC (Vector Error Correction) equation:

$$\Delta x(t) = \Pi x(t - 1) + \sum_{i=1}^{k-1} \Gamma_i \Delta x(t - i) + \Phi D(t) + \varepsilon(t), \quad (5)$$

4.2 Maximum likelihood estimation of precautionary demand in the presence of $ect(t)$

$\Delta y(t)$ should be regressed by $ect(t - 1)$, in addition to regressors given in Table 3. Maximum likelihood estimation is carried out by the following procedures:

**Estimation Procedures in [1980m01,2012m02]**

(i) An initial estimate of $rm2_{adj}^{(0)}$ is given in Eq.(3), using $c_i$ ($i = 1, \cdots, 4$) in Table 4.

(ii) An initial estimate of $ect(t - 1)^{(0)}$ is given in Eq.(10), with $rm2_{adj}$ replaced by $rm2_{adj}^{(0)}$.

(iii) Maximum likelihood estimation procedures are carried out with $ect(t - 1)^{(0)}$ as an additional regressor of $\Delta y(t)$, where $c_2^{(0)} = -7.24$. Parameters inside $ect(t - 1)^{(0)}$ are not renewed in procedures (iii) and (iv), but fixed.

(iv) Changing $c_2$ with step size 0.01, the optimal $c_2^{(1)}$ is determined such that log-likelihood function of $\Delta y(t)$ takes the maximum value, where optimal $c_i$’s are denoted by $c_i^{(1)}$ ($i = 1, \cdots, 4$).
(v) Using \( c_i^{(1)} \) obtained in procedure (iv), precautionary demand and \( rm2_{adj}^{(1)} \) in Eqs.(1) and (3) are calculated. Letting \( x = (rm2_{adj}^{(1)})' \), VEC equation (5) is estimated and an error correction term \( ect(t-1)^{(1)} \) in Eq.(7) is determined.

(vi) Go to (iii) with \( c_2^{(0)} \) and \( ect(t-1)^{(0)} \) replaced by \( c_2^{(1)} \) and \( ect(t-1)^{(1)} \) respectively and iterate calculations (iv) and (v) till \( c_2^{(j)} \) converges to \( c_2^{(j+1)} \).

Estimation Results ( moneyprec and ect(t) )

\[
\text{moneyprec}(t) = 1263.529 \\
-0.0355 \ast (indpro(t) \ast cpi_fes(t)/10000)^{-9.54} \\
+0.2938 \ast u(t) \ast indpro(t) \ast cpi_fes(t)/100 \\
+1.576 \ast adjressl(t) \\
\text{ect}(t) = rm2_{adj}(t) - 1.179 \ast y(t) - 2.282
\]

![Fig. 3: m2sl, m2sl - moneyprec(with ect) and m2sl - moneyprec(without ect)](image)

Table 5: Cointegration test of \((rm2_{adj}, y)\) in (1980m1,2012m2)

<table>
<thead>
<tr>
<th>Test for the number ( r_c ) of cointegrating vectors</th>
<th>( E ) values</th>
<th>( 0.0365 )</th>
<th>( 0.00525 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypo.</td>
<td>( r_c = 0 )</td>
<td>( r_c &lt; 1 )</td>
<td></td>
</tr>
<tr>
<td>( \lambda_{max} )</td>
<td>14.198</td>
<td>2.007</td>
<td></td>
</tr>
<tr>
<td>( \lambda_{trace} )</td>
<td>16.206*</td>
<td>2.007</td>
<td></td>
</tr>
<tr>
<td>( p(\lambda_{max}) )</td>
<td>0.0512</td>
<td>0.1566</td>
<td></td>
</tr>
<tr>
<td>( p(\lambda_{trace}) )</td>
<td>0.0390</td>
<td>0.1566</td>
<td></td>
</tr>
</tbody>
</table>

Adjustment Coefficients \( \alpha \)

| \( \Delta rm2_{adj} \) | -0.0268 |
| \( \Delta y \) | 0.00109 |

Cointegrating coefficients \( \beta^* \)

<table>
<thead>
<tr>
<th>( rm2_{adj} )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>-1.1792</td>
</tr>
</tbody>
</table>

* denotes rejection of hypothesis at 5% significance level. \( p(\lambda_{max}) \) and \( p(\lambda_{trace}) \) are \( p \)-values of \( \lambda_{max} \) and \( \lambda_{trace} \) respectively. A lagged difference is decided as \( p = 4 \).

Figure 3 depicts nominal money \( m2sl \) and \( m2sl - moneyprec \) in Eq.(11), and for comparison, \( moneyprec \) in Table 4 without error correction term is also shown together. We can see that there is a little difference between \( moneyprec \) with and without \( ect(t-1) \). From Table 5 we can regard that there is one cointegrating relation between \( x(t) = (rm2_{adj}(t), y(t))' \), though \( p(\lambda_{max}) = 0.0512 \) is greater than the value of 0.05. It should be noted that \( rm2_{adj} \) obtained from Eq.(11) becomes nonstationary by unit root test.

5 Effect of Increased Reserves in (2006m6, 2010m2)

The increased reserves at FRB raise M2 money which is decomposed into transaction demand money and precautionary demand money, where the former money directly connects to economic activity and the latter does not because the latter money is only kept without consumption. So, in order to investigate the effect of non-traditional monetary policy, we have to know whether or not the increased reserves raise transaction demand (i.e., \( rm2_{adj} \) in Eq.(3) with \( moneyprec \) given by Eq.(11).

5.1 Relation between \((u(t), rm2_{adj}(t), adjressl(t))\) in (2006m6, 2010m2)

Letting \( x(t) = (u(t), rm2_{adj}(t), adjressl(t))' \), the effect of reserves \( (adjressl(t)) \) to \( rm2_{adj}(t) \) is investigated in (2006m6, 2010m2). Since in this period \( rm2_{adj}(t) \) and \( u(t) \) are shown to be stationary, the model should be VAR without cointegration:

\[
\Delta x(t) = a_0 + \sum_{i=1}^{3} A_i \Delta x(t - i) + \varepsilon(t).\tag{13}
\]

Structural VAR Description of Eq.(13) makes no use of prior theoretical ideas about how variables \( x(t) \) are expected to be related. In order to overcome this difficulty, structural VAR model is usually introduced in the following form:

\[
B_0 \Delta x(t) = b_0 + \sum_{i=1}^{3} B_i \Delta x(t - i) + \nu(t),\tag{14}
\]

where \( \nu(t) \) is vector white noise. Multiplying \( B_0^{-1} \) on both sides of Eq.(14), we obtain Eq.(13) as a reduced form, where

\[
a_0 = B_0^{-1} b_0 \tag{15}
\]

\[
A_i = B_0^{-1} \tag{16}
\]

\[
\varepsilon(t) = B_0^{-1} \nu(t) \tag{17}
\]

Letting \( \Omega = E(\varepsilon(t)\varepsilon(t)') \) and \( D = E(\nu(t)\nu(t)') \), Eq.(17) implies

\[
\Omega = B_0^{-1} E(\nu(t)\nu(t)')(B_0^{-1})' = B_0^{-1} D(B_0^{-1})'.\tag{18}
\]
Equation (18) is estimated by assuming that $B_0$ and $D$ are of the form:

$$B_0 = \begin{pmatrix} 1 & 0 & 0 \\ b_{21} & 1 & 0 \\ b_{31} & b_{32} & 1 \end{pmatrix}, \quad D = \begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$$

**Impulse response** Figure 4 depicts impulse responses with a shock at each element of $v(t)$ added to (14).

Letting $v(t) \equiv (v_u(t), v_{rm2adj}(t), v_{adjressl}(t))^\top$, shock1 in Fig.4 means an impulse shock of $v_u(t)$ at $t = 1$, shock2 is that of $v_{rm2adj}(1)$ and shock3 is that of $v_{adjressl}(1)$. The 1st column shows responses of $x(t)$ to shock1, the 2nd column shows those to shock2, and the 3rd column shows those to shock3:

- The 1st column:
  1. (upper figure) $v_u(t) \rightarrow u(\downarrow)$,
  2. (middle figure) $v_u(t) \rightarrow rm_{2adj}(\downarrow)$ (increased anxiety implies decreased money-flow to industrial production),
  3. (lower figure) $v_u(t) \rightarrow adjressl(\downarrow)$ (increased anxiety of business cycle makes reserves increased at FRB).

- The 2nd column:
  1. $v_{rm2adj}(\uparrow) \rightarrow u(\uparrow)$ (not significant),
  2. $v_{rm2adj}(\uparrow) \rightarrow rm_{2adj}(\uparrow)$,
  3. $v_{rm2adj}(\uparrow) \rightarrow adjressl(\downarrow)$ (increased money-flow to industrial production makes reserves decreased).

- The 3rd column:
  1. $v_{adjressl}(\uparrow) \rightarrow u(\downarrow)$ (increased reserves decrease anxiety of business cycle),
  2. $v_{adjressl}(\uparrow) \rightarrow rm_{2adj}(\uparrow)$ (increased reserves raise money-flow to industrial production after 4 months),
  3. $v_{adjressl}(\uparrow) \rightarrow adjressl(\uparrow)$.

5.2 Relation between $(rm_{2adj}(t), lnSP500(t), lnhouse(t), y(t))$ in (2006m6,2010m2)

Since $rm_{2adj}(t)$ is stationary and $y(t)$ is nonstationary in this period, VAR model in growth rate system should be considered without cointegration. Furthermore, we use structural VAR model as similarly as in 5.1. Equation (14) is considered with $x(t)$ replaced...
by \( x(t) = (rm2_{adj}(t), \ln SP500(t), \ln house(t), y(t))' \). Hence, \( v(t) \) in Eq.(14) is given by \( v(t) = (\nu_{rm2_{adj}}(t), v_{lnSP500}(t), v_{lnhouse}(t), \nu_y(t))' \). Impulse responses of \( x(t) \) to shocks \( v(t) \) are depicted in Fig.5, where each of shock1, \ldots, shock4 implies element of \( v(1) \) respectively.

- The 1st column: \( rm2_{adj}(\uparrow) \rightarrow \ln SP500(\uparrow) \).
- The 2nd column: \( \ln SP500(\uparrow) \rightarrow y(\uparrow) \) after 3 months. It should be noted that on the 1st and 2nd columns, \( \ln house \) seems to be insignificant, because in this period \( \ln house \) was not recovered.
- The 3rd column: subprime problem and Lehman crisis decreased rapidly \( \ln house \) which caused further fall of \( \ln SP500 \) and \( y \).

Consequently, Figs.4 and 5 show us a transmission path: \( \text{adjressl}(\uparrow) \rightarrow rm2_{adj}(\uparrow) \rightarrow \ln SP500(\uparrow) \rightarrow y(\uparrow) \).

6 Conclusions

Precautionary demand is estimated in USA during (1980,2012). Assuming that precautionary demand depends on (i) economic activity, (ii) business condition and (iii) reserves at FRB, we obtain a good estimation of that demand so that \( \text{money}_{adj} = \text{money} - \text{money}_{prec} \) has a cointegrating property with industrial production. Then, efficiency of easing policies is shown such that in (2006,2010) increased reserves made a larger \( \text{money}_{adj} \) which gave a plus shock to the stock market, and that increased S&P500 raised up the industrial production.

References

