Estimation of Parameters in the Linear Stochastic Dynamical System Driven by Colored Noise Sequence

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Abstract

In this paper, an estimation procedure of parameters such as a damping coefficient and a natural frequency in the linear stochastic dynamical system (LSDS) driven by colored noise sequence is proposed based on a methodology applied bi-variate autoregressive (AR) modeling procedure. Ship’s roll motion, in which external disturbances such as waves and winds have strong frequency dependent, is treated as the subject. The verification of the proposed procedure is implemented by using results of onboard experiments that a ship ran in an octagon. Obtained findings are reported.

1 Introduction

A ship’s roll motion in waves and winds is treated as the typical linear stochastic dynamical system (LSDS) driven by colored noise sequence, since external disturbances such as waves and winds have strong frequency dependent and, they do not satisfy a characteristic of a white noise sequence. In this study, parameters are a damping coefficient and a natural frequency included in LSDS. Moreover, in the case of ship motions there is the feature in which the external disturbances can not measure generally. We would like to emphasize that our problem is special and is different from the problem of the general parameter estimation. Here, it should be noted that, of course, when the external disturbances satisfy the characteristics of the white noise sequence, as well known the procedure of Bartlett[1] can be used.

By the way in the field of naval architecture, first of all, the issue in which the stochastic characteristics of the external disturbances are not the white noise sequence was treated by Yamanouchi[2]. Both of the ship’s roll motion and the external disturbances such as waves and winds were modeled as the linear discrete autoregressive (DAR) process in his research work. That is, the modeling was done by substituting the DAR model concerning the ship’s roll motion in the DAR model concerning the external disturbances, and the LSDS was approximated as the time series model. However, in this research he treated the LSDS as the 2nd order DAR process, and it can be considered that this assumption is not enough theoretically. After that, in Ohtsu and Kitagawa[3], it was modeled as the linear continuous autoregressive process (CAR), and the parameter estimation can be done by using Kalman filter. They treated the external disturbances as the continuous time process, although the basic idea is same with the research of Yamanouchi[2]. However, it should be noted that the validity of this assumption is unknown, because there is not the guarantee that the physical process of the external disturbances is driven by the white noise sequence. On the other hand, Iseki and Ohtsu[4] focused on the procedure of parameter estimation in the linear CAR model, and proposed the method to use the IIR digital filter. As well as Iseki and Ohtsu[4], Terada and Kitagawa[5] focused on the procedure of parameter estimation in the linear CAR model, and proposed the method to use the Monte Carlo filter which is a kind of the particle filter. As well as Ohtsu and Kitagawa[3], these methods has the problem concerning the modeling of the external disturbances.

In order to solve above-mentioned problems, we propose the novel modeling procedure. That is, the LSDS is treated as the continuous time process and is discretized by using the analytic solution, and also the external disturbances is treated as the DAR process. After that, as well as Yamanouchi[2], the modeling can be done by substituting the analytic solution concerning the ship’s roll motion in the DAR model concerning the external disturbances. In order to verify the
proposed procedure, onboard experiments that a ship ran in an octagon were carried out. The results were compared with the procedure of Yamanouchi[2], and it can be confirmed that the proposed procedure is more effective than the results by one of Yamanouchi[2]. We explain the detail below.

2 Modeling for parameter estimation

In general, when we assume that a ship is the rigid body and we consider the coordinate system shown in Fig. 1, then we can deal the ship motion in waves and winds as the motion of 6-DOF (Degrees Of Freedom). Moreover, by assuming that the motion is enough small, the motion can be divided between the longitudinal motion and the transverse motion, and they can be dealt as the independent mode each other. That is, the roll motion as the single DOF motion. There is basic knowledge in the field of the naval architecture[6].

Moreover, by assuming that the motion is enough small, winds as the motion of 6-DOF (Degrees Of Freedom). Fig. 1, then we can deal the ship motion in waves and winds as the motion of 6-DOF (Degrees Of Freedom). From this theoretical background, it can be allowed that we treat the roll motion as the single DOF motion. There is basic knowledge in the field of the naval architecture[6].

Therefore, consider that the following ship’s roll motion equation:

\[
\ddot{x}(t) + 2\alpha \dot{x}(t) + \omega^2 x(t) = u(t),
\]

where, \(x(t)\) is a roll angle of the ship, \(\alpha\) is a damping coefficient, \(\omega(=2\pi f)\) is a natural angular frequency, \(f\) is a natural frequency and \(u(t)\) is an external disturbances, respectively. Here, as mentioned before, \(u(t)\) is treated as the stochastic process and does not satisfy the assumption of the white noise sequence, since the characteristics of the roll motion change with the frequency characteristic of the external disturbances such as waves and winds. For simply, we express Equation 1 in the following vector form:

\[
x_t = K x_t + Bu_t
\]

where,

\[
x_t = [\dot{x}(t), x(t)]^T, \quad u_t = [u(t), 0]^T,
\]

\[
K = \begin{pmatrix}
-2\alpha & -\omega^2 \\
1 & 0
\end{pmatrix}, \quad B = \begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}.
\]

As well known, a discrete model on an analytic solution of the linear dynamical stochastic model shown in the Equation 2 can be written by as follows:

\[
x_n = \text{EXP}[K\Delta t] \cdot x_{n-1} + Bu_n
\]

where,

\[
A \equiv \text{EXP}[K\Delta t] = \frac{e^{a\Delta t}}{b} \times \begin{pmatrix}
b \cos b\Delta t + a \sin b\Delta t & -(a^2 + b^2) \sin b\Delta t \\
\sin b\Delta t & b \cos b\Delta t - a \sin b\Delta t
\end{pmatrix},
\]

and, \(x_n = [\dot{x}_n, x_n]^T\), respectively. Here \(a\) and \(b\) are the real part and the imaginary part of the eigenvalue of \(K\), respectively. And also, \(Bu_n\) is a two–dimensional colored noise sequence, which is obtained by the stochastic integral. Since the term of the noise is not the white noise sequence, it is necessary to transform the colored noise sequence into a white noise sequence in order to deal with the problem stochastically. To do the whitening, Yamanouchi (1956) showed how to use the DAR process: in Equation 3, let

\[
\epsilon_n \equiv Bu_n.
\]

Then, suppose that \(\epsilon_n\) can be approximated by the following \(m\)-th order DAR process.

\[
\epsilon_n = \sum_{i=1}^{m} D_i \epsilon_{n-i} + w_n, \quad (\epsilon_n = w_n \text{ for } i = 0),
\]

where \(w_n\) is a 2\times2 Gaussian white noise sequence with \(N(0, \text{diag}(\sigma_1^2, \sigma_2^2))\) and \(D_n\) indicates a 2\times2 autoregressive coefficient matrix. On the other hand, the following relation is evident.

\[
\begin{cases}
\epsilon_n = x_n - Ax_{n-1}, \\
\epsilon_{n-1} = x_{n-1} - Ax_{n-2}, \\
\vdots \\
\epsilon_{n-m} = x_{n-m} - Ax_{n-m-1}.
\end{cases}
\]

Therefore, by substituting Equations 6 into Equation 5, we can obtain the following two dimensional \((m+1)\)-th order bi–variate autoregressive (AR) model.

\[
x_n = \sum_{i=1}^{m+1} C_i x_{n-i} + w_n.
\]
Here \( C_i \) \( (i = 1, \ldots, m + 1) \) is the bi-variate AR coefficient matrix, which is expressed as follows:

\[
\begin{align*}
C_1 &= D_1 + A, \\
C_2 &= D_2 - D_1 A, \\
& \vdots \\
C_m &= D_m - D_{m-1} A, \\
C_{m+1} &= -D_m A.
\end{align*}
\] (8)

Therefore, the parameter estimation can be realized by solving Equation 8 after a fitting of the bi-variate AR model. In this case, as to the relationship between the elements of \( A \) and the real part and the imaginary part \( \{a, b\} \) of the eigenvalue of \( K \), the following relations are satisfied.

\[
\begin{align*}
A_{1,1} + A_{2,2} &= 2e^{a\Delta t} \cos(b\Delta t) \\
A_{1,2} &= -\left( a^2 + b^2 \right) A_{2,1},
\end{align*}
\] (9)

where, \( A_{i,j} \) is the element \((i, j)\) of \( A \). And, as to the relationship between the real part and the imaginary part \( \{a, b\} \) of the eigenvalue of \( K \) and the unknown parameters \( \{\alpha, f\} \), the following relations are satisfied.

\[
\begin{align*}
\alpha &= a \\
f &= \sqrt{a^2 + b^2}/2\pi
\end{align*}
\] (10)

3 Method of the parameter estimation

In this section, the method of the parameter estimation is described. First of all, in order to estimate parameters, we implement the fitting of the bi-variate AR model according to Kitagawa[7]. Here, we used the AR modeling procedure based on the least-squares method and the Akaike Information criterion(AIC)[8]. After that, from the calculated AR coefficients, the solution of Equation 8 is obtained based on a downhill simplex method[9]. Then, eigenvalues \( a \) and \( b \) of matrix \( K \) in Equation 9 is calculated by using a genetic algorithm[10]. Finally, the unknown parameters \( \alpha \) and \( f \) are obtained from Equation 10.

In this paper, we omit the explanation of the AR modeling procedure, the downhill simplex method and the genetic algorithm, since these methods are recognized widely. See the references for their details.

4 Onboard experiments

In order to examine the effectiveness of the proposed procedure, onboard experiments were carried out. The sample ship is a fisheries research vessel "Taka–maru" belonging to our institute. Table 1 shows the principal dimensions of Taka–maru and Fig. 2 shows the general arrangement of Taka–maru. The time series for the parameter estimation, which was measured by a micro electro mechanical systems(MEMS) type gyro sensor at a sampling interval 1.0[sec], is the data in which the ship ran in an octagon. The experimental sea area is the Tateyama bay off. The measurement time of one experiment is 240[sec]. Experimental conditions in this case are summarized in Table 2. In this table, the definition of the direction accords to a geographic coordinate system and characteristics of waves were observed by ship officers. From this table, it can be seen that the ship encountered waves of various direction. In the field of the naval architecture, it is well known that the unknown parameters which are the damping coefficient and the natural frequency are almost constant without depending on the wave direction, although it should be noted that to be exact the magnitude of the damping force depends on the amplitude of the roll angle[6]. That is, if the amplitude of the roll angle is less than the 10[deg.], then this is satisfied. Therefore, it means that the results of parameter estimation are reliable if their values are almost constant without depending on the wave direction. This information is very important for the verification of the proposed procedure. From Fig. 3 to Fig. 10 shows the measured time series. In these figures, a black line and a red line indicate the roll angle and the roll rate, respectively. From these figures, it can be seen that the roll angle and the roll rate are large in the beam sea such as Fig. 6 and Fig. 10, and are small in the head sea and the following sea such as Fig. 4 and Fig. 8. In general, the amount of these variables depends on the wave height and the wave period. As to the results of Fig. 9, the roll angle and the roll rate are also small. It is considered that the reason is influence of the wave height, since the wave height in this data was 1.0[m].

Where, we would like to emphasize that the damping coefficient and the natural frequency are the almost constant value, although the characteristics of the roll motion are changed by the encounter wave direction. And, we show that the proposed method can estimate the damping coefficient and the natural frequency as the constant values, in spite of the characteristics of roll motion obtained as the output of the system are different against the wave direction.

<table>
<thead>
<tr>
<th>Table 1: Principal dimensions of Taka-maru</th>
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<tbody>
<tr>
<td>Length between perpendiculars: ( L )</td>
</tr>
<tr>
<td>Breadth: ( B )</td>
</tr>
<tr>
<td>Mean draft: ( d )</td>
</tr>
<tr>
<td>Block coefficient: ( C_b )</td>
</tr>
<tr>
<td>Metacentric height: GM</td>
</tr>
</tbody>
</table>

5 Results and discussions

First of all, we examined characteristics in a frequency domain based on the spectral analysis to use the AR modeling procedure. In the AR modeling procedure, the AR model from the 1st order to the 20th order was used in order to fit to the time series. The best model
order determined by the AIC is summarized with the value of AIC in Table 3. It can be seen that the stable estimate can be performed, since the best model in each data exists in the setted range. Fig. 11 shows the spectral density of the roll angle. In this figure, the horizontal axis indicates the frequency, and the vertical axis indicates the spectral density. And also, each colored line indicates the results corresponding to the experimental data number. From this figure, it can be seen that the results in the beam sea such as Fig. 6 indicated by the blue line and Fig. 10 indicated by the gray line are large and are the unimodal spectral density. In general, the roll motion of the ship does not depend on the frequency characteristics of the external disturbances such as waves and winds, although the pitch and
the heave motion of the ship depend on one because the proportion of the length and the breadth is large. In other words, as to the pitch and the heave motion, there is the peak frequency of the roll angle near the natural frequency. It can be confirmed that the peak frequency in results of each data exists around the 0.17[Hz] which is the natural frequency of the sample ship obtained from the condition of GM shown in Table 1. However, the results of “data 1”, “data 2” and “data 3” have the bimodal spectral density, because the bottom of the ship was exposed to the air under the navigation in the following seas. It should be noted that these case is the singular situation.

In this study, in order to verify the proposed procedure, the results were compared with the results of the procedure of Yamanouchi[2](Yamanouchi method). The estimated damping coefficient and natural frequency are shown in Fig. 12 and Fig. 13, respectively. In these figures, the horizontal axis indicates the experimental data number, and the vertical axis indicates the estimated result. And also, the symbol ”o” indicates the result based on Yamanouchi method, and the symbol ”△” indicates the result based on the proposed procedure. As shown in Fig. 12, it can be seen that the estimated damping coefficient based on the proposed procedure is stable compared with the Yamanouchi method. As mentioned before, the damping coefficient is almost constant without depending on the wave direction, although it should be noted that to be exact the magnitude of the damping force depends on the amplitude of the roll angle. Therefore, we can judge that the results of the proposed procedure are accurate compared with the Yamanouchi method. As shown in Fig. 13, it can be seen that the results of the estimated natural frequency based on the proposed procedure are also stable compared with the Yamanouchi method. As mentioned before, the natural frequency is almost constant without depending on the wave direction as well as the damping coefficient. Therefore, we can also judge that the results of the proposed procedure are accurate compared with the Yamanouchi method. From these results, we can consider that the proposed procedure is effective in order to estimate the parameters in the LSDS driven by colored noise sequence. It means that the bi–variate AR model is one of the good approximation on the LSDS driven by colored noise sequence.

<table>
<thead>
<tr>
<th>Data num.</th>
<th>Model order</th>
<th>AIC</th>
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<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>-3609.3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>-3775.9</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>-3834.3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>-3677.5</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
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</tr>
<tr>
<td>6</td>
<td>9</td>
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</tr>
<tr>
<td>7</td>
<td>13</td>
<td>-3703.3</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
<td>-3586.0</td>
</tr>
</tbody>
</table>
Fig. 11: Results of spectral analysis based on AR modeling procedure with respect to the roll angle

Fig. 12: Estimates of the damping coefficient

Fig. 13: Estimates of the natural frequency

6 Conclusions

We proposed the novel estimation procedure of parameters in the linear stochastic dynamical system (LSDS) driven by colored noise sequence based on a methodology applied bi–variate AR modeling procedure. As the result of the verification based on the analysis of the data obtained from onboard experiments, we were able to confirm the effectiveness of proposed procedure as compared with the procedure of Yamanouchi\cite{Yamanouchi} (Yamanouchi method). Main conclusions are summarized as follows:

1. The LSDS driven by colored noise sequence can be approximated by the bi–variate autoregressive model.

2. The parameters included in the dynamical system can be related with the autoregressive coefficient, and can be calculated from the autoregressive coefficient.

3. In the field of the naval architecture, it is well known that the parameters such as the damping coefficient and the natural frequency are almost constant without depending on the wave direction. Since the results of the estimated parameters have satisfied this belief, we can judge that the results of the proposed procedure are accurate compared with the Yamanouchi method.

4. The proposed procedure is the powerful tool to estimate the parameters on the LSDS driven by colored noise sequence.

As the future task, we need to perform more verification based on model ship experiments in the marine dynamics basin belonging our institute.

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References


