Optimal Allocation of Photovoltaic Systems and Energy Storage Systems Considering Uncertainties in the Outputs of Photovoltaics and the Capabilities of Demand Response

Ryusuke Konishi and Masaki Takahashi
Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan
JST, CREST, 4-1-8 Honcho, Kawaguchi, Saitama, 332-0012, Japan

E-mail: ryusuke.konishi@keio.jp, and takahashi@sd.keio.ac.jp

Abstract

It has been required to install a large number of renewable energy (RE), such as photovoltaic systems (PVs) to solve the environmental problem. However, an electricity grid with RE occurs problems of power shortages and surpluses because of uncertainties in generating RE outputs. In addition, demand response (DR) can restrain both shortages and surpluses caused by PV outputs, but, on the other hand, DR may increase the impact of power shortages and surpluses due to the uncertainties of DR capabilities. Energy storage systems (ESSs) can be used to solve supply reliability problems, but the number of installed ESSs should be minimized considering their high cost. This study demonstrates that the optimal allocation of PVs and ESSs can be achieved that satisfies minimization of installed ESSs, the PVs installation target, and the constraints of the probabilistic indices for shortage and surplus to evaluate uncertainties of PV outputs and DR capabilities. The results show that PVs and ESSs are allocated in each bus, satisfying the PV installation target and the constraints of the supply reliability.

Nomenclatures

Indices:

\( i \) Index of buses
\( t \) Index of time

Decision variables:

| \( P_{Gi}(t) \) | Output of generator in bus \( i \) at \( t \) |
| \( C_{PVi} \) | Capacity of PVs |
| \( C_{ESSi} \) | Capacity of mid-level ESSs |
| \( C_{peak}^{ESS} \) | Capacity of peak ESSs |
| \( P_{Ni}(t) \) | Mismatch of power demand and supply in bus \( i \) at \( t \) |
| \( P_{Ei}(t) \) | Scheduling of mid-level ESSs |
| \( P_{peak}^{ESS} \) | Scheduling of peak ESSs |
| \( P_{Bi}(t) \) | Inflow/Outflow power in bus \( i \) at \( t \) |
| \( \theta_{i}(t) \) | Phase angle in bus \( i \) at \( t \) |
| \( S_{i}(t) \) | State of charge of mid-level ESSs in bus \( i \) at \( t \) |
| \( S_{peak}^{i}(t) \) | State of charge of peak ESSs in bus \( i \) at \( t \) |
| \( \alpha_{i}^{l}(t) \) | Relaxation of power shortages for 1 hour in bus \( i \) at \( t \) |
| \( \beta_{i}^{s}(t) \) | Relaxation of power surpluses for 1 hour in bus \( i \) at \( t \) |
| \( EENS_{i,t} \) | Value of Expected Energy Not Supplied for 1 hour in bus \( i \) at \( t \) |
| \( EENU_{i,t} \) | Value of Expected Energy Not Used for 1 hour in bus \( i \) at \( t \) |
| \( C \) | Vector of decision variables related to grid operation |
| \( P \) | Vector of decision variables related to allocation of PV and ESS |

Parameters:

| \( f_{G} \) | Function of fuel cost for each generator |
| \( p_{PV} \) | Price of PVs |
| \( p_{ESS} \) | Price of ESSs |
| \( P_{V_i}^{unit}(t) \) | Output of PV per unit in bus \( i \) at \( t \) |
| \( P_{L_i}(t) \) | Electricity demand in bus \( i \) at \( t \) |
| \( r_{DRi} \) | Rate of participants in DR in bus \( i \) |
| \( P_{DRi}^{unit}(t) \) | Capability of DR per unit in bus \( i \) at \( t \) |
| \( B_{ij} \) | Susceptance between buses \( i \) and \( j \) |
| \( P_{max}^{max} \) | Capacity of transmission line between buses \( i \) and \( j \) |
| \( \alpha_{i,t}, \beta_{i,t} \) | Upper limits of power shortages and surpluses for 1 hour in bus \( i \) at \( t \) |
| \( P_{G_i}^{min} \) | Minimum generator output in bus \( i \) at \( t \) |
| \( P_{G_i}^{max} \) | Maximum generator output in bus \( i \) at \( t \) |
| \( \Delta P_{Gi} \) | Maximum difference of a generator in bus \( i \) at \( t \) for 1 hour |
| \( \sigma_{PV_i}(t) \) | Standard deviation of PV output in bus \( i \) at \( t \) for 1 hour |
| \( \sigma_{DRi}(t) \) | Standard deviation of DR capability in bus \( i \) at \( t \) for 1 hour |
| \( C_{PV}^{target} \) | Installation target of PVs |
\[ \bar{x} \quad \text{Estimated value of stochastic variable } x \]
\[ \bar{x} \quad \text{Mean value of stochastic variable } x \]
\[ g_P(\cdot) \quad \text{Probability density function of Gaussian distribution} \]
\[ \text{erf(\cdot)} \quad \text{Error function} \]
\[ \rho \quad \text{Penalty term for the violation of constraints} \]

1 Introduction

It is essential to install a large number of Photovoltaic systems (PVs) into electricity grids to limit the extent of global warming and to ensure energy security. However, the uncertainty of PV outputs often causes both power shortages and surpluses. There are several methods to solve problems of power shortage and surplus: for example, the installation of energy storage systems (ESSs) and demand response (DR) systems. ESSs, such as batteries and pumped-storage power plants, can reduce power shortage and surplus by discharging and charging with high reliability. However, the number of ESSs installed should be minimized because of their high cost. As well as ESSs, DR systems can be one of the solutions for both power shortage and surplus by producing not only energy saving but also energy spending which implies increasing demand to reduce power surpluses. An example of promoting energy spending in DR is to provide coupons for food delivery services, which encourages people to remain at home. Although the cost of DR can be cheaper than the installation of ESSs, the disadvantage of DR is that its capabilities have uncertainties because of the variability in participants’ behaviors. Consequently we need to focus on both the uncertainties using PV outputs and the capabilities of DR.

Previous researches[1]-[4] have discussed the optimal allocation of PVs and the operation of grids with renewable energy resources, and the optimal strategy of DR considering the uncertainty in their capabilities. Our review of the literature revealed no studies have focused on both the uncertainties using PV outputs and the capabilities of DR to determine the optimal allocation of PVs and ESSs, especially in spending energy. This study determines the optimal allocation of PVs and minimized ESSs while limiting power shortages and surpluses caused by uncertainties in PV outputs and the capabilities of DR, and meeting a target level for the installation of PVs. To introduce limits to power shortages and surpluses under constraints, it is necessary to evaluate the power shortages and surpluses given the uncertainties in operating an electricity grid. Therefore, this research proposes the improvement of two probabilistic indices to take into account the uncertainties of using PV outputs and the capabilities of DR: Expected Energy Not Supplied (EENS)\(^5\) and Expected Energy Not Used (EENU)\(^6\). This study demonstrates that the optimal allocation of PVs and minimized ESSs can be achieved while satisfying the constraints of the supply reliability indices (EENS and EENU) and the PV installation targets.

2 Optimal Allocation Problem of PVs and ESSs

This section first describes a formulation for optimizing the allocation of PVs and ESSs without uncertainties. This formulation is expanded from a multi-period optimal power flow, and can determine not only the grid operation, but also facility allocation. To simplify the problem and reduce the calculation time, the power system is approximated as a DC circuit in which each bus has the same voltage. The time step for grid operation is assumed to be 1 hour. (QP)

\[
\begin{align*}
\min_{C_P, C_{PV}, C_{ESS}} & \sum_{t=1}^{24} \sum_{i=1}^{N} P_{G_i}(t) + c_{PV} \sum_{i=1}^{N} C_{PV_i} + p_{ESS} \sum_{i=1}^{N} C_{ESS_i}, \\
\text{s.t.} & \sum_{i=1}^{N} C_{PV_i} \geq C_{PV_{\text{target}}}, \\
& P_{Ni}(t) = P_{G_i}(t) + C_{PV_i} \cdot P_{\text{unit}}(t) + P_{Ki}(t) - P_{Bi}(t) - (1 + r_{\text{DRi}}) \cdot P_{\text{unit}}(t) - P_{Li}(t), \\
& -\alpha_i \leq P_{Ni}(t) \leq \beta_i, \\
& P_{\text{min}} \leq P_{G_i}(t) \leq P_{\text{max}}^G, \\
& |P_{G_i}(t) - P_{G_i}(t)| \leq \Delta P_{G_i}, \\
& |B_{ij}(\theta_i(t) - \theta_j(t))| \leq |P_{\text{Bij}}^P|, \\
& P_{Bi}(t) = \sum_{j \neq i} B_{ij}(\theta_i(t) - \theta_j(t)), \\
& S_i(t + 1) - S_i(t) = -P_{Ei}(t), \\
& 0 \leq S_i(t) \leq C_{ESS_i}
\end{align*}
\]

Equation (1) is the objective function for optimizing the allocation of PVs and ESSs and minimizing the costs of grid operation and facility allocation of the PVs and ESSs. The cost of grid operation indicates a fuel cost which is modeled by a quadratic function. Equation (2) is a constraint for the installation target of PVs in a grid. Here, equations (3)-(10) are constraints related to grid operation, and apply for all \(i, t\). Equation (3) is a power flow constraint that considers the PV outputs and scheduling of ESSs, where negative and positive values of \(P_{Ni}\) denote power shortage and surplus, respectively. \(P_{Ni}\) is limited by equation (4), considering both shortages and surpluses. Moreover, this research employs two assumptions when giving \(\alpha_i(t), \beta_i(t)\): no
power shortages and surpluses in buses without demands, and the same upper limits of power shortages and surpluses for all buses and times. Equations (5) and (6) are constraints for conventional plants, and represent the upper and lower limits of generator output and the upper and lower limits of generation rate of change, respectively. Equation (7) gives a constraint on the transmission capacity limits, and equation (8) is the relationship between the phase angle in a bus and the transmission power using the DC approximation. Equations (9)–(10) are constraints related to ESSs: relational expressions for charging/discharging and state of charge, relational expressions for the state of charge and ESS capacity, and limits for charging/discharging over one hour, respectively.

The optimization problem (QP) is formulated as a quadratic programming problem, because the objective functions are quadratic and the constraints are linear based on the assumption of a DC circuit. Moreover, the PV outputs are considered to be deterministic in this step.

### 3 Probabilistic Indices

This research employs probabilistic indices to evaluate power shortages and surpluses caused by uncertainties in PV outputs and DR capabilities. In this section, we define a probabilistic index and describe its application.

First, to simplify the calculation of indices, \( \tilde{P} \) is set as shown in equation (11), which is derived from equation (3) with uncertainties.

\[
\tilde{P}_t(t) = \bar{P}_{\text{NL}}(t) + P_{\text{L}}(t) \\
= P_{\text{G}}(t) + C_{\text{PV1}} \cdot \bar{P}_{\text{PV}}^{\text{unit}}(t) + P_{\text{E}}(t) \\
- P_{\text{B}}(t) - r_{\text{DR}} \cdot \bar{R}_{\text{DR}}^{\text{unit}}(t) \cdot P_{\text{L}}(t) \\
(11)
\]

This stochastic variable includes both uncertainties in PV outputs and DR capabilities. We denote the mean and standard deviation of \( \tilde{P} \) as \( \bar{P} \) and \( \sigma_P \), respectively. Moreover, this assumption models \( \bar{P} \) and \( \sigma_P \) as shown in equations (12) and (13).

\[
\bar{P}_t(t) = \bar{P}_{\text{NL}}(t) + P_{\text{L}}(t) \\
\bar{P}_{\text{NL}}(t) = P_{\text{G}}(t) + C_{\text{PV1}} \cdot \bar{P}_{\text{PV}}^{\text{unit}}(t) + P_{\text{E}}(t) \\
- P_{\text{B}}(t) - r_{\text{DR}} \cdot \bar{R}_{\text{DR}}^{\text{unit}}(t) \cdot P_{\text{L}}(t) \\
(12)
\]

\[
\sigma_P(t) = \sqrt{\sigma_{\text{PV1}}(t)^2 + \sigma_{\text{DR}}(t)^2} \\
(13)
\]

Second, to simplify the formulation, we assume that the PV outputs and DR capabilities for 1 hour follow a Gaussian distribution. Therefore, the probability density function can be expressed as

\[
g_P(x, \bar{P}, \sigma_P) = \frac{1}{\sqrt{2\pi\sigma_P^2}} \exp\left(-\frac{(x - \bar{P})^2}{2\sigma_P^2}\right) \\
(14)
\]

Using this assumption for the probability density function, two probabilistic indices are formulated: Expected Energy Not Supplied (EENS) [5] and Expected Energy Not Used (EENU) [6]. The former measures the incidence of power shortages, and the latter is a metric for power surpluses. Based on the probability density function of \( \tilde{P} \), Fig. 1 shows the amount of shortages and surpluses, where the probability of shortages and surpluses are denoted in blue and red, respectively.

Equations (15) and (16) describe the formulation of EENS and EENU, which are defined as the expectations of power shortages or surpluses given the inherent uncertainties. These indices are introduced into the constraints of our optimization problem with uncertainties.

\[
\text{EENS}_t(\bar{P}, \sigma_P, P_{\text{L}}) = \int_{-\infty}^{\bar{P}_L} (P_{\text{L}} - x) \cdot g_P(x, \bar{P}, \sigma_P) \, dx \\
= \frac{\bar{P} - P_{\text{L}}}{2} \left[ \text{erf}\left(\frac{\bar{P} - P_{\text{L}}}{\sqrt{2}\sigma_P}\right) - 1\right] \\
+ \frac{1}{\sqrt{\pi}} \cdot \frac{2\sigma_P^2}{2\sigma_P^2} \cdot \exp\left(-\frac{(\bar{P} - P_{\text{L}})^2}{2\sigma_P^2}\right) \\
(15)
\]

\[
\text{EENU}_t(\bar{P}, \sigma_P, P_{\text{L}}) = \int_{P_{\text{L}}}^{\infty} (x - P_{\text{L}}) \cdot g_P(x, \bar{P}, \sigma_P) \, dx \\
= \frac{P_{\text{L}} - \bar{P}}{2} \left[ \text{erf}\left(\frac{P_{\text{L}} - \bar{P}}{\sqrt{2}\sigma_P}\right) - 1\right] \\
+ \frac{1}{\sqrt{\pi}} \cdot \frac{2\sigma_P^2}{2\sigma_P^2} \cdot \exp\left(-\frac{(P_{\text{L}} - \bar{P})^2}{2\sigma_P^2}\right) \\
(16)
\]

Here, \( \text{erf}(\cdot) \) denotes the error function

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt \\
(17)
\]
4 Optimization Problem Considering Uncertainties

This section describes an optimization problem including uncertainties in PV outputs and DR capabilities. The formulation can be modeled as a two-stage stochastic program with recourse (SP).

\[
\begin{align*}
\min_{\mathbf{C}} & \quad F(\mathbf{P}) + p_{\text{PV}} \sum_{i=1}^{N} C_{\text{PV}i} + p_{\text{ESS}} \sum_{i=1}^{N} C_{\text{ESS}i} \\
\text{s.t.} & \quad (2) \quad (19) \\
F(\mathbf{P}) = & \min_{P_i, \alpha_i', \beta_i'} \left[ \sum_{t=1}^{24} \sum_{i=1}^{N} (f_G + \rho(\alpha_i', t + \beta_i', t)) \right] \\
\text{s.t.} & \quad \tilde{P}_{Ni}(t) = P_{Gi}(t) + C_{\text{PV}i} \cdot P_{\text{unit}}(t) + P_{El}(t) \\
& \quad - P_{Bi}(t) - (1 + r_{\text{DRi}} \cdot \tilde{R}_{\text{unit}}(t)) P_{Li}(t) (21) \\
& \quad EENS_i(P, \sigma_P, P_L) \leq \alpha_i - \alpha_i' (22) \\
& \quad EENU_i(P, \sigma_P, P_L) \leq \beta_i - \beta_i' (23) \\
& \quad (5) - (10)
\end{align*}
\]

Equation (18) minimizes the total cost of grid operation and facility allocation considering the given uncertainties. As we are accounting for the uncertainties, the power flow constraint changes from equation (3) to equation (20), which includes probabilistic variables. Moreover, EENS and EENU are employed to formulate constraints on the power shortages and surpluses, as shown in equations (22) and (23). \(\alpha_i'(t)\) and \(\beta_i'(t)\) are introduced to ensure that the shortage and surplus constraints have feasible solutions in all cases. These variables play the role of relaxing EENS and EENU, and are minimized in equation (18) as a penalty function.

Although we have formulated this optimization problem with stochastic variables, it can be reformulated without stochastic variables because the EENS and EENU constraints are deterministic in terms of \(\tilde{P}\) and \(\sigma_{PV}\) based on equations (15) and (16).

Therefore, the optimization problem can be written as follows:

\[
\begin{align*}
\min_{\mathbf{C}, \tilde{P}} & \quad \sum_{t=1}^{24} \sum_{i=1}^{N} (f_G(P_{Gi}(t)) + \rho(\alpha_i'(t) + \beta_i'(t))) \\
\text{s.t.} & \quad \tilde{P}_{Ni}(t) = P_{Gi}(t) + C_{\text{PV}i} \cdot P_{\text{unit}}(t) + P_{El}(t) \\
& \quad - P_{Bi}(t) - (1 + r_{\text{DRi}} \cdot \tilde{R}_{\text{unit}}(t)) P_{Li}(t) (25) \\
& \quad (2), (5) - (10), (22), (23)
\end{align*}
\]

We achieved to rewrite one-stage problem, but it is difficult to solve problem (CSP) securing the global convergence because this has nonlinear constraints including an exponential function. To solve the problem (CSP), we use a “Block Coordinate Descent Method”. We derive a two-stage optimization problem in (CSP-1) and (CSP-2), which are solved alternately. The optimized variables derived from the first stage are used as parameters in the second stage, and vice versa.

\[
\begin{align*}
\text{(CSP-1)} & \quad \min_{\mathbf{C}} \sum_{t=1}^{24} \sum_{i=1}^{N} (f_G(P_{Gi}(t))) \quad (26) \\
\text{s.t.} & \quad (5) - (10), (22), (23), (25).
\end{align*}
\]

\[
\begin{align*}
\text{(CSP-2)} & \quad \min_{\mathbf{C}, \tilde{P}} \sum_{t=1}^{24} \sum_{i=1}^{N} (f_G(P_{Gi}(t)) + \rho(\alpha_i'(t) + \beta_i'(t))) \\
& \quad + \rho \sum_{t=1}^{24} \sum_{i=1}^{N} (\alpha_i'(t) + \beta_i'(t)) \quad (27) \\
\text{s.t.} & \quad (10), (22), (23), (25),
\end{align*}
\]

In these problems(CSP-1) and (CSP-2), the objective functions and decision variables are separately from (CSP). Moreover, decision variables in (CSP-1) are operational factors, such as \(P_G\), and on the other hand, decision variables in (CSP-2) are ones related to facility allocation, such as \(C_{PV}\) and \(C_{ESS}\).

To accelerate the process of solving (CSP-1), some constraints should be reformulated. In particular, equations (22) and (23) should be converted from nonlinear constraints including exponential functions to linear constraints using the inverse functions of EENS and EENU, as shown in equation (28). This is because \(EENS_i^{-1}(\sigma_P, \alpha)\) and \(EENU_i^{-1}(\sigma_P, \beta)\) are linearized when \(C_{PV}\) and \(\sigma_{PV}\) are fixed.

\[
\begin{align*}
- EENS_i^{-1}(\sigma_P, \alpha) - \frac{\rho_{\text{peak}}(t)}{2} & \leq \tilde{P}_{Ni,d}(t) \\
\leq EENU_i^{-1}(\sigma_P, \beta) + \frac{\rho_{\text{El}}(t)}{2} \quad (28)
\end{align*}
\]

Here, the new variable \(\rho_{\text{El}}(t)\) controls the scheduling of ESSs following sudden changes due to the uncertainties in PV outputs and DR capabilities. These ESSs not only play the role of peak power plants, but are also slack variables to ensure (CSP-1) has feasible solutions, similar to \(\alpha'\) and \(\beta'\). In addition to equation (28), the scheduling and capacities of ESSs for peak demand must satisfy the constraints given by equations (29) and (30).

\[
\begin{align*}
S_{\text{peak}}(t + 1) - S_{\text{peak}}(t) & = - P_{\text{peak}}(t) \quad (29) \\
0 & \leq S_{\text{peak}}(t) \leq C_{\text{peak}} \quad (30)
\end{align*}
\]

Therefore, (CSP-1) can be converted to (CSP-1’), a quadratic programming problem, as follows:

\[
\begin{align*}
\text{(CSP-1')} & \quad \min_{\mathbf{C}, \tilde{P}} \sum_{t=1}^{24} \sum_{i=1}^{N} (f_G(P_{Gi}(t)) + \rho(\alpha_i'(t) + \beta_i'(t))) \\
& \quad \text{s.t.} \quad (5) - (10), (22), (23), (25),
\end{align*}
\]
To distinguish the two types of ESSs ($P_{\text{peak}}(t), C_{\text{ESS}i}$ and $P_{E_{i}}(t), C_{\text{ESS}i}$), we call those that play the role of peak power plants “peak ESSs” ($P_{\text{peak}}(t), C_{\text{ESS}i}$), and refer to those that are included in the case that does not consider uncertainties as “mid-level ESSs” ($P_{E_{i}}(t), C_{\text{ESS}i}$), because the latter can be regarded as mid-level power plants compared with the peak ESSs.

As the objective function is convex, the convergence of (CSP) is guaranteed by the results of [7].

5 Simulation

This section presents simulation results for the optimization problems developed above. We conducted simulations for an OPF using Matpower [8].

5.1 Modeling and Assumptions

5.1.1 Power System Model Assumptions

This research considers the IEEE 14 Bus System [9] shown in Fig. 2 as a power system to operate and allocate facilities. In addition, we use the IEEE Reliability Test System (RTS) [10] because this contains data for the annual electricity demand load, which are required when considering changes across seasons and days of the week. Thus, we can obtain the allocation of PVs and ESSs considering time-series changes in electricity demand.

5.1.2 PV Model Assumptions

This research models PV outputs using actual data from 1 March, 2013, to 28 February, 2014, published by California ISO [11]. However, these data include increases in the number of PVs installed in California, so it was necessary to modify the data using the installation capacities of PVs in order to determine the actual PV outputs per unit capacity at a certain time. This research compensates for the PV outputs using data from California Solar Statics [12].

5.1.3 DR Model Assumptions

This research considers both capabilities of saving and spending energy because both capabilities are important when a large number of PVs is installed into a power grid. The former is for the case of power shortages by raising an electricity price, and the latter is for the case of power surpluses by reducing an electricity price.

As described in Section 3, it is assumed that the uncertainty of DR capabilities per one unit, $R_{\text{init}}(t)$, is given for each 1 hour. To give the average and standard deviation of DR capabilities $R_{\text{DR}}(t)$, this research assumes that the value $\mu + 2\sigma$ is regarded as the maximum DR capability and that the minimum capability is zero. Fig. 3 shows the maximum DR capabilities for each hour, using the relative values against the maximum demand in each bus, and the average and standard deviation of DR capabilities are calculated by these values. Here, these DR capabilities are common to both saving and spending energy effects.

5.1.4 Date for Considering Grid Operation

This research determines the installed allocation of PVs and ESSs through 1 year, so it is necessary to consider long-term changes in electricity demand and PVs outputs. Moreover, it is important to consider the most severe cases of shortages and surpluses. To determine the allocation of facilities, this paper focuses on the grid
operation for the day when the most surplus occurred and the demand was low and PV output high because ESSs are necessary mainly to restrain power surpluses. We used data from the demand on the 266th day in IEEE RTS as the minimum demand, and the PV output on 27 September 2013 as the maximum PV output in September, by analyzing both annual data. Figs. 4 and 5 show the mean value and standard deviation of PV output per unit in this date.

5.2 Simulation Results
This section explains the simulation results for the optimal allocation of PVs and ESSs. This study considered two parameter sets for the DR capabilities: the first set $\sum_{i=1}^{N} r_{DR_i}$ to 50%, whereas the second set this quantity to 10%. The upper limits of both EENS and EENU were set to 1.0 MWh/year, and the installation target for PVs was set to 100 MW. Both simulations ended within 20 iterations in a block coordinate descent.

Fig. 6 shows the allocation of PVs. The total amount of PVs was 100MW, satisfying the target. The allocation results in both of the DR cases were the same. Because of the same constraints on all buses with electricity demands, power shortages and surpluses were allocated to each bus evenly. Hence, the PVs were approximately allocated to each bus, and no consideration was given to losses over transmission lines with adequate capacities.

Fig. 7 shows the allocation of mid-level ESSs and peak ESSs in the case of $\sum_{i=1}^{N} r_{DR_i} = 50\%$. Fewer mid-level ESSs were allocated to buses 1, 7, and 8, because no PVs were allocated in these buses. The total size of the installed mid-level ESSs was 429 MWh. In contrast, peak ESSs were allocated to buses with electricity demands, and those with higher demands had more peak ESSs. This is because the DR capabilities expand the standard deviation of power shortages and surpluses in a power grid, which could increase the amount of peak ESSs, and because DR capabilities depend on the electricity demand. The total size of the installed peak ESSs was 141 MWh.

Fig. 8 shows the allocation of the two types of ESSs in the case of $\sum_{i=1}^{N} r_{DR_i} = 10\%$. The pattern of allocation was similar to the previous case, but the total size of the mid-level ESSs was 771 MWh and the size of the peak ESSs was 21 MWh. The reason why the former increased and the latter decreased is that the lower DR capabilities had both a positive impact on the deviation of power shortages/surpluses and a negative impact on the mean power demand. In other words, the amount of installed mid-level ESSs and peak ESSs are depending on the standard deviation and mean value of power mismatch of demand and supply respectively, which includes not only DR capabilities but also PV outputs. Moreover, the two types of ESSs have a trade-off relationship, and therefore it is possible to optimize the rate of DR participation. We intend to determine this optimal rate in future work.

6 Conclusions
This research derived the optimal allocation of PVs and ESSs in a power grid with uncertain PV outputs and variable DR capabilities to reduce the requirement for installed ESSs. To formulate the optimization problem, we employed two probabilistic indices, EENS and EENU, to deal with both power shortages and surpluses. As a result, the problem was formulated as
a two-stage stochastic program with recourse including EENS and EENU. A block coordinate descent method was used to solve this optimization problem. The simulation results not only showed the optimal allocation of PVs and ESSs, but also suggested the optimal rate of DR participation. Our future work will seek the optimal rate of DR participation, and formulate the problem with an AC circuit model that can consider losses over transmission lines.

Acknowledgment

This work was supported in part by the Program for Leading Graduate Schools, “Science for Development of Super Mature Society” and Japan Science and Technology Agency (JST), CREST.

References


