Dual State-Parameter Estimation of ECG Signals with Recursive Bayesian Filters

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Keywords: Electrocardiogram, parameter estimation, ensemble Kalman filter, dual state-parameter estimation, joint state-parameter estimation.

Abstract

The Electrocardiogram (ECG) is used for diagnosing heart conditions by recording the small electric waves generated during heart activity. Gois et al. have recently proposed a mathematical model to describe heart rhythms considering three-coupled Van der Pol oscillators, and indicated that the heart rhythms of the cardiac diseases can be shown by changing three coupling parameters. In this paper, we propose two methods for simultaneous estimation of the states and the parameters of Gois ECG model. For the joint state-parameter estimation, we constitute the ensemble Kalman filter from this model to detect the cardiac diseases by estimating the coupling parameters as the state variables in Gois’s model. For the dual space-parameter estimation, in addition, we apply the dual ensemble Kalman filter to estimate the coupling parameters. Finally, we compare the estimation results of these filters.

1 Introduction

Gois et al.[1] have recently proposed a mathematical model to describe heart rhythms considering three-coupled Van der Pol oscillators, and indicated that the heart rhythms of cardiac diseases can be shown by changing three coupling parameters. We focused on two coupling parameters from the sinoatrial (SA) node to the atrioventricular (AV) node, and from the AV node to the His-Purkinje (HP) complex in Gois’s model. The normal ECG have both coupling parameters, while the ventricular flutter occurs if the coupling from the SA node to the AV node is cut, and the sinus bradycardia appears if the coupling from the AV node to the HP complex is disconnected. Then, to detect cardiac diseases, we proposed a joint estimation method of the internal states and the coupling parameters of Gois model [2]. The Bayesian filtering have been used to denoise ECG signals [3], but we introduced the recursive Bayesian filtering to ECG diagnosis by estimating the coupling parameters. We designed the ensemble Kalman filter for an augmented model including the coupling parameters as internal states to detect the cardiac diseases by estimating the coupling parameters as the state variables in Gois’s model [2]. Though the filter could estimate the coupling parameters for the estimation using initial-time response including transient state, the filter could not estimate the coupling parameters in the case of the steady state ECG data. An alternative approach for simultaneous estimation is the dual estimation method. The dual estimation was first suggested in [4, 5] and the ensemble Kalman filter is employed for the dual estimation [4, 6]. The dual estimation can be understood as a general EM algorithm: E-step uses Kalman filter for a state estimation; whereas M step performs model parameter estimation [7]. In this paper, we design a dual ensemble Kalman filter to detect the cardiac diseases by estimating the coupling parameters with the steady state ECG signals. The simulations are performed in the case of the normal ECG, the sinus bradycardia, and the ventricular flutter. The estimation performance of the dual estimation is compared with that of the joint estimation. In particular, we used two types of the ECG data calculated by the discrete Van der Pol oscillators, which are the initial-time response including the transient state and the steady state response except for the transient state [2]. In the former case, both filter succeeded to estimate the connection parameters of the normal ECG, the sinus bradycardia, and the ventricular flutter. In the latter case, however, the ensemble Kalman filter using the joint estimation failed to estimate the parameters.

2 VdP model of heart dynamics and cardiac diseases

2.1 Electrocardiogram(ECG)

Cardiac electric signals on an intracellular level may be recorded with a microelectrode, which is inserted inside a cardiac muscle cell. The ECG is a measure of the extra-cellular electric behavior of the cardiac muscle tissue. The propagation wave-front of the cardiac electrical signal through the body presents a very com-
plicated shape. In general, the signal contains P-Wave, PR-Interval, QRS-Complex, ST-Interval, and T-Wave.

2.2 VdP model of heart dynamics

The general heartbeat dynamics can be approximated by the coupling of Van der Pol (VdP) oscillators of a different heart region signal. The normal cardiac rhythm is primarily generated by the SA node, which is considered as the normal pacemaker[1]. Besides, the AV node is another pacemaker. Each one of these presents an actuation potential that is fundamental to the heart dynamics, but not necessarily the most expressive to compose the ECG signal[1]. Moreover, the third oscillator that represents the pulse propagation through the ventricles, which physiologically represents the His-Purkinje complex, composed by the His bundle and the Purkinje fibers[1]. Gois et al.[1] proposed a mathematical model to describe heart rhythms considering three modified Van der Pol oscillators:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -a_{SA}x_2(x_1 - w_{SA1})(x_1 - w_{SA2}) - x_1(x_1 + d_{SA})(x_1 + e_{SA}) + p_{SA}\sin(\omega_{SA}t) + k_{SA-\text{AV}}(x_1 - x_3) + k_{SA-\text{HP}}(x_1 - x_5) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -a_{AV}x_4(x_3 - w_{AV1})(x_3 - w_{AV2}) - x_3(x_3 + d_{AV})(x_3 + e_{AV}) + p_{AV}\sin(\omega_{AV}t) + k_{AV-SA}(x_3 - x_1) + k_{AV-\text{HP}}(x_3 - x_5) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -a_{HP}x_6(x_5 - w_{HP1})(x_5 - w_{HP2}) - x_5(x_5 + d_{HP})(x_5 + e_{HP}) + p_{HP}\sin(\omega_{HP}t) + k_{HP-SA}(x_5 - x_1) + k_{HP-\text{AV}}(x_5 - x_3).
\end{align*}
\]

The state variables and the parameters are defined as follows: \(x_1, x_2\) : SA membrane flow, voltage, \(x_3, x_4\) : AV membrane flow, voltage, \(x_5, x_6\) : HP membrane flow, voltage, \(a_{\bullet\bullet}\) : damping coefficient for each node, \(\rho_{\bullet\bullet}\) : voltage range for each node, \(\omega\) : frequency of external signal, \(\tau\) : vanishing all others.

The ECG signal is built from the composition of these internal states as

\[
\text{ECG} = a_0 + a_1 x_1 + a_2 x_3 + a_5 x_5.
\]

This equation can be rewritten by the vector form:

\[
\dot{z} = f(z, \dot{z}) + \Gamma(t) + Kz
\]

where

\[
\begin{align*}
x &= [x_1 \ x_3 \ x_5]^T, \quad \dot{z} &= [x_2 \ x_4 \ x_6]^T \\
\Gamma(t) &= [p_{SA}\sin(\omega_{SA}t) \ p_{AV}\sin(\omega_{AV}t) \ p_{HP}\sin(\omega_{HP}t)]^T
\end{align*}
\]

\[
K = \begin{bmatrix}
k_{SA-\text{AV}} + K_{SA-\text{HP}} & -k_{SA-\text{AV}} & -k_{SA-\text{AV}} & k_{AV-SA} + k_{AV-\text{HP}} & -k_{AV-\text{SA}} & k_{AV-\text{HP}} & -k_{AV-\text{AV}} & -k_{SA-\text{HP}} & -k_{AV-\text{HP}} \\
-k_{AV-\text{SA}} & k_{AV-SA} + k_{AV-\text{HP}} & -k_{AV-\text{SA}} & k_{AV-\text{HP}} & -k_{AV-\text{AV}} & -k_{SA-\text{HP}} & -k_{AV-\text{HP}} & k_{HP-SA} + k_{HP-\text{AV}}
\end{bmatrix}.
\]

2.3 Relationship between coupling parameters and cardiac diseases

We focus on two coupling parameters from SA node to AV node and from AV node to HP complex, because these parameters affect cardiac diseases. From the results in Gois et al.[1], the coupling parameters are given as \(k_{AV-SA} \neq 0, k_{HP-AV} \neq 0\), and the other parameters are zeros.

The following parameters are select as in [1]: \(a_{SA} = 3, w_{SA1} = 0.2, w_{SA2} = -0.19, d_{SA} = 3, e_{SA} = 4.9, a_{AV} = 3, w_{AV1} = 0.1, w_{AV2} = -0.1, d_{AV} = 3, e_{AV} = 3, a_{HP} = 5, w_{HP1} = 1, w_{HP2} = -1, d_{HP} = 3, e_{HP} = 7, \alpha_0 = 0.1, \alpha_1 = 0.1, \alpha_2 = 0.05, \alpha_3 = 0.4, \tau_{SA-\text{AV}} = 0.8, \tau_{SA-\text{HP}} = 0.1\), vanishing all others.

We perform the simulation in the case of the normal ECG, the sinus bradycardia, and the ventricular flutter. Since we cannot get the cardiac rhythms for the coupling parameters given in Gois et al.[1], we find them via our MATLAB/Simulink model.

- Normal ECG (NM)

The normal ECG is generated when the coupling parameters \(k_{AV-SA} = 5, k_{HP-AV} = 20\). Fig. 1 shows the normal ECG response.

- Ventricular flutter (VF)

The ventricular flutter is generated when the coupling parameters \(k_{AV-SA} = 0, k_{HP-AV} = 20\). Fig. 2 shows the ECG response in the ventricular flutter.

- Sinus bradycardia (SB)

The sinus bradycardia is generated when the coupling parameters \(k_{AV-SA} = 50, k_{HP-AV} = 0\). Fig. 3 shows the ECG response in the sinus bradycardia.
Fig. 2: Simulated abnormal ECG without SA-AV coupling (VF).

Fig. 3: Simulated abnormal ECG without AV-HP coupling (SB).

2.4 Problem statement : detection of cardiac diseases

We can detect the cardiac diseases such as the ventricular flutter and the sinus bradycardia by the values of the coupling parameters \( k_{AV-SA} \) and \( k_{HP-AV} \). In particular, in the case of the ventricular flutter, the coupling from the SA node to the AV node, i.e. \( k_{AV-SA} = 0 \), and in the case of the sinus bradycardia, \( k_{HP-AV} = 0 \). Thus, we estimate the coupling parameters \( k_{AV-SA} \) and \( k_{HP-AV} \) by using the ensemble Kalman filter.

3 Joint estimation method

An easy way to estimate states and parameters simultaneously is to augment the state-space regarding the parameters as the part of the states.

3.1 Augmented state-space model

The discrete-time equation of the VdP model can be expressed as

\[
x_{t+1} = f_t(x_t) + w_t, \quad y_t = h_t(x_t) + v_t
\]

where \( t \) is a sampling time, \( x_t \in \mathbb{R}^n \) is internal states, \( y_t \in \mathbb{R}^p \) is an ECG output, \( w_t \in \mathbb{R}^n \) is a system noise, and \( v_t \in \mathbb{R}^p \) is a measurement noise.

To estimate the internal states and the coupling parameters, the augmented state variables are defined as

\[
x_t = \begin{bmatrix} x_t \\ \theta_t \end{bmatrix}^T
\]

where \( \theta_t \) is given by

\[
\theta_t = \begin{bmatrix} k_{AV-SA} \\ k_{HP-AV} \end{bmatrix}^T.
\]

We employ the particle filter and the ensemble Kalman filter as Bayesian Filter Algorithms to estimate the states and the connection parameters.

3.2 Simulation results

We can detect the cardiac diseases such as the ventricular flutter and the sinus bradycardia by the values of the coupling parameters \( k_{AV-SA} \) and \( k_{HP-AV} \). In particular, in the case of the ventricular flutter, the coupling from the SA node to the AV node, i.e. \( k_{AV-SA} = 0 \), and in the case of the sinus bradycardia, \( k_{HP-AV} = 0 \). Thus, we estimate the coupling parameters \( k_{AV-SA} \) and \( k_{HP-AV} \) by using the ensemble Kalman filter. Fig. 4 and Fig. 5 show the estimate results of the ECG wave by EnKF for the ventricular flutter (VF) and the sinus bradycardia (SB), respectively.

Fig. 4: Estimated VF by EnKF.

Fig. 5: Estimated SB by EnKF.

3.2.1 Estimation using initial-time response including transient state

Fig. 6 and Fig. 7 show the estimate results of the coupling parameter \( k_{AV-SA} \) and \( k_{HP-AV} \) using the initial-time response for the ventricular flutter (VF) and the sinus bradycardia (SB), respectively, where the results of the coupling parameters are plotted by repeating 10 times. The initial values of coupling parameters are selected as zeros.

3.2.2 Estimation using steady state response

In order to apply the ensemble Kalman filter to the real data in the future, we also try to estimate the coupling parameters using the steady state response excepting the transient state of the VdP model. However,
the joint estimation with the EnKF fails to estimate the coupling parameters for any initial values.

Fig. 6: Estimated coupling parameters of VF by EnKF; top : $k_{AV-SA}(=0)$, bottom : $k_{HP-AV}(=20)$.

Fig. 7: Estimated coupling parameters of SB by EnKF; top : $k_{AV-SA}(=50)$, bottom : $k_{HP-AV}(=0)$.

4 Dual estimation method

The unknown parameters of the designed filters are estimated as the state variables. Since we need only the parameter estimates, the estimation mechanism of the parameters should separate with that of the internal states. We design the dual ensemble Kalman filter to estimate the coupling parameters in the manner of the method in [4].

4.1 State model

The discrete-time equation of the VdP model can be expressed as

$$x_{t+1} = f_t(x_t, \theta_t) + w_t, \ y_t = h_t(x_t, \theta_t) + v_t.$$  

where $t$ is a sampling time, $x_t \in \mathbb{R}^n$ is the internal states of the VdP oscillators, $\theta_t$ is the unknown parameters of the VdP oscillators, $y_t \in \mathbb{R}^p$ is an ECG output, $w_t \in \mathbb{R}^n$ is a system noise, and $v_t \in \mathbb{R}^p$ is a measurement noise. To detect the cardiac diseases, the unknown parameters are selected as

$$\theta_t = \begin{bmatrix} k_{AV-SA} & k_{HP-AV} \end{bmatrix}^T.$$  

4.2 Dual state-parameter estimation with EnKF

Parameter samples are made as follows:

$$\theta_{fj}^{t+1} = \theta_{aj}^t + \tau_{j_t}^t, \tau_{j_t}^t \in \mathbb{N}(0, \Sigma_{\theta}^t) (2)$$

where $j_t$ is the $i$-th ensemble member. Using the forecasted parameters, a model state ensemble and predictions are made, respectively:

$$x_{fj}^{t+1} = f(x_{aj}^t, \theta_{fj}^{t+1}), \hat{y}_{j}^{t+1} = h((x_{fj}^{t+1}, \theta_{fj}^{t+1}). (3)$$

Updating the parameter ensemble members is made by the Kalman filter equation:

$$\theta_{aj}^{t+1} = \theta_{fj}^{t+1} + K_{\theta}^{t+1}(y_{jt}^{t+1} - \hat{y}_{j}^{t+1}) (4)$$

where the Kalman gain for the parameter estimation is given by

$$K_{\theta}^{t+1} = \Sigma_{\theta}^{t+1}[\Sigma_{yy}^{t+1} + \Sigma_{y}^{t+1}]^{-1}. (5)$$

Using the updated parameter, the new state forecast and prediction are generated:

$$x_{fj}^{t+1} = f(x_{aj}^t, \theta_{aj}^{t+1}), \hat{y}_{j}^{t+1} = h((x_{fj}^{t+1}, \theta_{aj}^{t+1}) (6)$$

Model state ensemble is similarly updated as follows:

$$x_{aj}^{t+1} = x_{fj}^{t+1} + K_{x}^{t+1}(y_{jt}^{t+1} - \hat{y}_{j}^{t+1}) (7)$$

where the Kalman gain for the state estimation is given by

$$K_{x}^{t+1} = \Sigma_{x}^{t+1}[\Sigma_{yy}^{t+1} + \Sigma_{y}^{t+1}]^{-1}. (8)$$

4.3 Simulation results

We perform the simulations in the case of the normal ECG, the sinus bradycardia, and the ventricular flutter using initial-time response including transient state and using steady state response, respectively.

Since the parameters are constant and positive, in this simulation, the parameter samples are restricted as follows:

$$\theta_{fj}^{t+1} = Pr\left(a\theta_{aj}^t + (1-a)\theta_{aj}^t + \tau_t \right) (9)$$

$$\tau_t^j \in \mathbb{N}(0, \Sigma_{\theta}^t) (10)$$
where $\bar{\theta}_a$ is the mean of the estimated parameter samples, $a$ is a weight, and $Pr$ is the projection operator which restricts to the nonnegative region.

Fig. 8 and Fig. 9 show the estimate results of the ECG waves by the dual EnKF for the ventricular flutter (VF) and the sinus bradycardia (SB), respectively.

Fig. 8: Estimated VF by dual EnKF; top : using initial time response, bottom : using steady state response.

Fig. 9: Estimated SB by dual EnKF; top : using initial time response, bottom : using steady state response.

4.3.1 Estimation using initial-time response including transient state

Fig. 10 and Fig. 11 show the estimate results of the coupling parameter $k_{AV-SA}$ and $k_{HP-AV}$ by the dual EnKF using the initial-time response for the ventricular flutter (VF) and the sinus bradycardia (SB), respectively. The initial values of coupling parameters are selected as zeros.

4.3.2 Estimation using steady state response

Fig. 12 and Fig. 13 show the estimate results of the coupling parameter $k_{AV-SA}$ and $k_{HP-AV}$ by the dual EnKF using the steady state response for the ventricular flutter (VF) and the sinus bradycardia (SB), respectively. Since the transient state has continued until 2 sec, each simulation is started from 2 sec. The initial values of coupling parameters are selected as zeros.

Fig. 10: Estimated coupling parameters of VF by dual EnKF using initial-time response; top : $\hat{k}_{AV-SA}(= 0)$, bottom : $\hat{k}_{HP-AV}(= 20)$.

Fig. 11: Estimated coupling parameters of SB by dual EnKF using initial-time response; top : $\hat{k}_{AV-SA}(= 50)$, bottom : $\hat{k}_{HP-AV}(= 0)$.

5 Discussion

Real ECG data are measured as steady-state time sequences. In order to apply the ECG diagnosis by estimating the coupling parameters, it is desirable to
estimate the parameters using steady-state responses. The joint estimation with the EnKF fails to estimate the coupling parameters for any initial values. From the simulation results, the dual EnKF seems a more promising estimator.

6 Conclusion

In this paper, we designed the ensemble Kalman filter in order to apply the recursive Bayesian filtering to ECG diagnosis. The filter allows us to estimate the coupling parameters for the estimation using initial-time response including transient state. However, the filter could not estimate the coupling parameters in the case of the steady state ECG data. Next, to improve the estimation accuracy of the coupling parameters using steady state response, the dual ensemble Kalman filter was employed to estimate the coupling parameters. The dual state-parameter estimation allows us to estimate the coupling parameters for both the estimation using initial-time response including transient state and that using the steady state. Comparing the results by the above two methods, we consider that the dual estimation method (the dual EnKF) is better because the estimation using the steady response is successful and it suggests a possibility to apply to the real data of ECG in the future.

References


